EXPERIMENTAL STUDY ON ULTIMATE STRENGTH OF NORMAL SECTIONS IN REINFORCED CONCRETE BEAMS

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Abstract: This paper presents an experimental programme conducted on a number of six reinforced concrete (RC) beams in order to investigate the developments of strain and stress, moment-curvature relationship, failure mode and the ultimate strength on normal sections (USoNS) of this type of basic structural element. The beam specimens were 120mm × 200mm in cross section and 2.2m in length. They were divided into three series with the longitudinal reinforcement ratio varied from 0.42, 0.65 and 0.94%. There were two identical beams in each series. The experimental data were incorporated to validate the results calculated based on various design codes for RC structures including ACI 318- 11, EN 1992-1-1:2004, TCVN 5574:2012 and SP 63.13330.2012. The up-to-date Russian code SP 63.13330.2012, which is currently used as a basis for drafting the new Vietnamese code to replace TCVN 5574:2012, also adopts the plane strain assumption and the simplified stress-strain relationships of materials in the calculation based on non-linear deformation model similar to ACI and EC2. It is shown that such assumption and design procedure for USoNS specified in SP 63.13330.2012 can be applied with reliability for specimens made and tested in Vietnam condition.

Keywords: Strength, bending moment, normal section, beam, reinforced concrete.

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1. Introduction

Reinforced concrete (RC) beams are among the flexural elements commonly used in the structural systems of civil and industrial buildings, bridges, ports, etc. In RC beams, longitudinal reinforcement and stirrups are designed based on the ultimate limit states that to avoid failure occurred on normal sections (due to bending moment) and on inclined sections (due to shear force), respectively.

In the determination of ultimate strength on normal sections (which will be hereafter abbreviated as USoNS) in RC beams, simplified assumptions and calculation principles are developed in various national and regional design codes. However, there have been different approaches among the codes. The previous Russian code [1] as well as the current Vietnamese code for design of concrete structures [2] use stress-based principle whereas the codes of Western countries [3,4] adopt the plane strain assumption to determine the bending moment resistance of RC beams. Recently, the new Russian code [5] has also accepted the plane strain assumption for the calculation based on non- linear deformation model like those of the US [3] and the EU [4]. Since SP 63.13330.2012 is currently used as a basis for drafting the new Vietnamese code to replace TCVN 5574:2012, it is important to study on the reliability of applying SP 63.13330.2012 into Vietnam condition.

This fact motivated the authors to conduct an experimental study on a fair number of test specimens in order to investigate the developments of strain and stress, moment-curvature relationship, failure mode and the USoNS in RC beams. The six beam specimens had identical lengths of 2.2m and cross sections of 120mm×200mm. They were divided into three series with the longitudinal reinforcement ratio varied from 0.42, 0.65 and 0.94%. B25-30 concrete and AlI-AlII-type reinforcement were used. The experimental data are compared to the results calculated based on [2-5]. Test results and the discussions on the parameters affecting the flexural behavior and strength, as well as on the reliability of calculating the USoNS of the test specimens based on SP63 will be presented in the latter part of the paper.

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2. Theoretical ultimate strength on normal sections in RC beams

Fig. 1(a) shows a simply-supported RC beam carrying its self-weight and two symmetric point loads P that are gradually increased from zero till the beam fails by means of bending moment on the normal section A-A at its mid-span.



Figure 1. RC beam subjected to bending moment

The maximum bending moment M_{u} that the beam can sustain at section A-A, so-called the USoNS, is basically formed by the internal-force couple of C and T that are respectively contributed by concrete in compression zone and by longitudinal reinforcement located in the opposite side of the cross section with the application of certain failure criteria (Fig. 1(b)). The theoretical developments of strains and stresses in concrete and reinforcement as well as the calculation of M_{u} based on various national design codes for RC structures mentioned in Section 1 will be presented hereafter.

2.1 TCVN 5574:2012

The Vietnamese code for design of RC structures TCVN 5574:2012 was established based on the previous version of Russian code SNIP 2.03.01-84. The flexural behavior of RC beams is specified in the associated materials of the code as shown in Fig. 2 [1,2,6].



Figure 2. Flexural behavior and strength calculation of RC beams to TCVN 5574:2012

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It can be seen in Fig. 2 that in Stage I, when bending moment is small (M_e), concrete can be considered elastic and normal stress distributes linearly along the normal section A-A in Fig. 1. When moment is increased, plastic strain develops in concrete and the stress contribution becomes nonlinear. A_s tensile stress obt in the extreme fiber of concrete reaches tensile strength R_{bt} , crack occurs. In order to avoid cracking in the beam, bending moment shall not be greater than M_c meaning that the normal tensile stress does not reach R_{bt} in this Stage Ia. When moment is further increased, cracks occur in the tensile zone and develop upward, all the tensile forces are sustained by longitudinal reinforcement with cross-sectional area of A_s with tensile strength $\sigma_s < R_s$ whereas the compressive stress ob at the extreme concrete fiber is still lower than the compressive strength R_b . This Stage II is within service stage with bending moment Mser. If there is too much reinforcement ($\mu_s > \mu_{max}$) that the tensile stress is still small whereas σ_b reaches R_b , the beam would have failed in brittle mode, which is not the expected failure criteria (Stage IIIa). Hence, the longitudinal reinforcement shall be limited ($\mu_s \le \mu_{max}$) so that os reaches R_s at Stage IIa (with the associated yielding moment $M_{y'}$) before σ_b reaches R_b at Stage III, at which ductile failure occurs. Hence, the USONS M_u is specified in TCVN 5574:2012 following the path Stages I \rightarrow II \rightarrow III \rightarrow III

It is noteworthy that there is no material stress-strain relationship and plane strain assumption specified for strength calculation in TCVN 5574:2012. Hence, the code can be referred to be using stress-based principle in the determination of USoNS.

2.2 ACI 318-11 and EN 1992-1-1:2004 (EC2)

Different from SNIP 2.03.01-84 and TCVN 5574:2012, the stress-strain relationships of concrete and reinforcing steel are both explicitly provided in ACI 318-11 and EC2. Figs. 3 and 4 respectively depict the EC2 material models for concrete and reinforcing steel as an example.



(a) Real stress-strain diagram (hot rolled steel) (b) Real stress-strain diagram (Cold worked steel) (c) Idealised and design stress-strain diagrams

Figure 4. Stress-strain relationship of reinforcing steel specified in EC2

The codes also adopt the following assumptions in flexural theory [7,8]: (i) Sections perpendicular to the axis of bending that are plane before bending remain plane after bending (plane strain assumption); (ii) The strain in reinforcement is equal to that in concrete at the same distance to the neutral axis; (iii) The stresses in concrete and reinforcement can be computed from the strains using stress-strain relationships for concrete and reinforcing steel (Figs. 3,4); (iv) The tensile strength of concrete is neglected in flexural strength calculation; (v) Concrete is assumed to fail when a maximum compressive strain reaches a limiting value; and (vi) The compressive stress-strain relationship of concrete may be based on stress-strain curves

or may be assumed to be rectangular, trapezoidal, parabolic or any other shape that results in prediction of strength in substantial agreement of the results of compressive tests.

The flexural behavior of RC beams is specified in ACI 318-11 and EC2 in the form of moment- curvature diagram as shown in Fig. 5. Flexural tension cracking occurs in the section when the stress in the extreme tension fiber reaches the modulus of rupture with the associated cracking moment $M_{\rm a}$. Up to this cracking point (C), the moment-curvature relationship is linear and can be referred to as the uncracked-elastic range of behavior. The moment and curvature at cracking can be calculated directly from elasticity. The yielding point (Y) represents the end of the elastic range of behavior. As the moment applied to the section continues to increase after the cracking point, the tension stress in the reinforcement and the compression stress in the concrete compression zone will steadily increase. Eventually, either the steel or the concrete will reach its respective capacity and start to yield (steel) or crush (concrete). Because the section under consideration here is assumed to be under-reinforced, the steel will yield before the concrete reaches its maximum useable strain. To calculate moment M, and curvature values for the yield point, the strain at the level of the tension steel is set equal to the yield strain. Beyond the yield point, additional points on the moment-curvature relationship can be determined by steadily increasing the maximum strain in the extreme compression fiber until the ultimate point (U) is reached corresponding to a maximum value of compression strains. ACI 318-11 and EC2 respectively specify a maximum useable compression strain of 0.0030 and 0.0035 at which the ultimate moment strength of the section is to be calculated [7,8].



Figure 5. Flexural behavior and strength calculation of RC beams to ACI 318-11 and EC2

The ultimate compression strain at extreme concrete fiber ε_{cu} , coefficient of the equivalent height of concrete compressive stress block β_1 (ACI 318-11) and λ (EC2), and the equivalent concrete ultimate stress σ_{cu} (Fig. 5) specified in ACI 318-11 and EC2 are shown in Table 1 [7-9].

Table 1.	Values	for ca	lculating	М,
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ACI 318-11	ε _{cu} =0.0030;	β ₁ =0.85-0.05(<i>f</i> ′ _c -28MPa)/7MPa;	0.65≤β ₁ ≤0.85;	σ_{cu} =0.85 f'_{c}
EC2	ε _{cu} =0.0035;	λ=0.8-(f _{ck} -50)/400≤1.0;	$\sigma_{cu} = \eta 0.85 f_{ck} / \gamma_c;$	η =1.0-(f_{ck} -50)/200≤1.0

2.3 SP 63.13330.2012

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SP 63.13330.2012 [5] is a set of design principle rules for concrete and reinforced concrete structures issued by Ministry of Regional Development of the Russian Federation in 2012. This is the current design code in Russia. The code provides similar calculation procedure for ultimate strength M_u on normal sections like that of TCVN 5574:2012 (Fig. 2). The only difference is that the limiting value ξ_R of the relative height of the compression zone is now purely dependent on strain $\varepsilon_{s,el}$ of tensile reinforcement at stress equal to R_s and strain ε_{b2} of compressive reinforcement at stress equal to R_b .

At the same time, SP63 also specifies that the design of RC members based on non-linear deformation model can be performed based on the stress-strain relationships of concrete and reinforcement using flat cross-section hypothesis as a base. Reaching of ultimate strains in concrete and reinforcement is considered as strength condition of a normal section. Bilinear stress-strain diagrams of concrete and reinforcing steel shown in Fig. 6 can be used for simplification [5].



(a) Bi-linear stress-strain diagram of compressive concrete (b) Bi-linear stress-strain diagram of tensile reinforcement

Figure 6. Simplified stress-strain relationship of material specified in SP 63.13330.2012

In Fig. 6, the strain values are specified as follows: $\varepsilon_{b1} = \sigma_{b1}/E_b$, $\varepsilon_{b2} = 0.0035$ for compressive strength class of concrete B60 and lower, $\varepsilon_{s0} = R_s/E_s$, and $\varepsilon_{s2} = 0.015$.

It can be seen that SP63 approaches the ACI 318-11 and EC2 in some aspects so that it also allows for more-flexible and explicit determination of ultimate strength M_u on normal sections. These theoretical results will be validated by the experimental study presented in the next sections.

3. Experimental programme

3.1 Test specimens

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The experimental study presented herein was conducted in the Laboratory of Construction Testing and Inspection (National University of Civil Engineering - NUCE) in July, 2017. A total number of six RC beam specimens were divided into three series and were cast with the details shown in Fig. 7.



Figure 7. Details of beam specimens

All the specimens had identical cross-section of 120×200 mm and the same length of 2.2m. There was 1 Φ 6 (No.2) on the top side for hanging purpose. The triangle stirrups No.3 were ϕ 6@60 along 850mm-segments at two ends and ϕ 6@150 along the mid-span segment. This was to avoid the shear failure of the specimens. Steel plates No.4 were located at supports (R1 and R2) and at point loads (P1 and P2) to avoid local failure. There were three steel strain gauges (S7, S8, S9) fixed on the longitudinal reinforcing bars at mid-span. Another three concrete strain gauges (S10, S11, S12) were attached on the surface of the beam within the compression zone as shown in Fig. 7(b).

The respective amount of 597, 1207, and 430kg of Red river sand, 1-2 size gravel and PC-30 cement as well as 197 liter of clean water were used for the design mixture of 1m³ concrete.

The only difference between the series was the two longitudinal reinforcing bars at bottom (No.1), which were $2\phi 8$, $2\phi 10$ and $2\phi 12$ for Series 1, 2 and 3, respectively. It is noted that both cube and cylinder concrete samples were cast for each beam specimens so that the 28-day concrete strengths specified in various codes could be determined individually. Table 2 shows the categorization and the material properties obtained from material tests for each specimen.

No	Test series	Specimen	Rebar No.1	A _s (mm²)	μ _s (%)	f _{cube} (MPa)	f _{cylinder} (MPa)	f _y (MPa)	
1	Soriaa 1	D1.1-2 þ 8	2 φ 8	100 5	0.42	30.5	23.9	374.0	
2	Selles I	D1.2-2¢8	2 ¢ 8	100.5	0.42	36.9	29.5	409.8	
3	Sorias 2	D2.1-2ф10	2 φ 10	157.1	0.65	37.5	30.0	328.5	
4	Series 2	D2.2-2ф10	2 φ 10		0.05	34.3	26.3	341.2	
5	Soriaa 2	D3.1-2ф12	2φ12	226.2	226.2	0.04	29.1	22.9	406.7
6	6 Series 3	D3.2-2ф12	2φ12	220.2	0.94	19.8	15.6	412.0	

	Table 2.	Categorization	of test s	pecimens
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It should be noted in Table 2 that there was a significant reduction in concrete strength of specimen D3.2-2 ϕ 12. This was due to the poor quality of the concrete casting of the last specimen in an individual batch. Besides, the tensile strength of ϕ 10 re-bars was also lower than those of ϕ 8 and ϕ 12.

3.2 Test set-up and apparatus

The elevation view and an image of the test set-up are shown in Fig. 8.



Figure 8. Test set-up

In the test, the 2.2m-length specimen was singly supported over a span of 2.0m and was subjected to two symmetrical concentrated loads at points P1 and P2 which were both at a distance of 0.75m from the supports R1 and R2. The loads were generated by means of a steel beam SB, which was in turn subjected to a point load P at its mid-span. The load P was from the hydraulic jack HJ and a steel frame installed over the specimen. A load cell LC was fixed between the hydraulic jack and the steel frame to measure the load applied during the test.

A total number of five Linear Variable Deformation Transducers (LVDTs) were installed in the test. LVDTs I1, I3, I2 were to measure the vertical displacements at supports R1, R2, and the beam mid- span, respectively. LVDTs I4 and I5 were for the measurement of concrete strains at extreme tension and compression fibers on the normal section at mid-span of the beam, with the measured lengths of 600 and 150mm, respectively.

All the apparatus were connected to a data logger TDS-530 and a computer to record the test data.

3.3 Test procedure

After the reinforcement tensile strength and the 28-day concrete strength were obtained (Table 2), the theoretical ultimate strengths Mcode on normal section of the test specimens were theoretically predicted based on ACI 318-11, EC2, TCVN 5574:2012 and SP63. It should be noted that all the partial safety factors for material strengths were conventionally set to unity in the calculation of the test specimens. Results are shown in Table 3.

No	Test series	Specimen	ACI 318-11	EC2	TCVN 5574:2012	SP63
1	Sorioo 1	D1.1-2¢8	6.882	6.842	6.934	6.930
2	Selles I	D1.2-2ø8	6.942	6.898	6.995	6.975
3	Soriaa 2	D2.1-2φ10	9.122	9.047	10.885	10.172
4	Series 2	D2.2-2φ10	9.060	8.992	10.067	9.137
5	Corico 2	D3.1-2φ12	15.097	14.790	15.353	15.222
6	Series 3	D3.2-2¢12	14.403	13.964	14.772	14.522

All the tests were conducted at Laboratory of Construction Testing and Inspection (NUCE) after 30 days of the cast of the specimens. For each specimen, the corresponding maximum applied load at failure in the test was predicted as $P_{max}=2(M_{code}-0.125gL^2)/L_1$ where g is the uniformly distributed self- weight of the beam, *L* is the beam span *L*=2.0m, and L1 is the distance from the point loads to the supports $L_1=0.75m$. In the test, a pre-loading of $0.05P_{max}$ was applied and released to eliminate all the gaps existed in the system and to check whether all the apparatus work properly. Then, the load was again increased gradually in an interval of $5\%P_{max}$ and all the data were recorded in every two seconds until the beam specimens failed.

4. Test results and discussions

4.1 Developments of strain and stress on normal section

The development of flexural strains along the mid-span normal section can be observed by apparatus including steel strain gauges S7, S8, concrete strain gauges S9, S10, S11 (Fig. 7), LVDTs I4 at extreme tensile concrete fiber and I5 at extreme compressive concrete fiber (Fig. 8).

Fig. 9 depicts the strain data obtained from specimen D1.1-2 ϕ 8. The following observations can be made: (i) The strains distribute accordingly with the distance from the measured points to the neutral axis; and (ii) Strains at the points having the same distance to neutral axis are similar (S7 vs. S8 and S9 vs. S10).

Similar observations are also obtained from the other specimens. It can be seen that



Figure 9. The development of strain along the normal section of D1.1-2 ϕ 8

strains at both tensile reinforcing bars and extreme compressive concrete fibers develop more rapidly in the specimens having lower ratios of longitudinal reinforcing bars.

The distributions of flexural strains along the mid-span normal section at certain load levels of specimen D1.1-2 ϕ 8 are shown in Fig. 10.

It can be observed from Fig. 10 that the linear strain distribution is more obvious in the initial and the intermediate stages of the test. In the latter stage, although the plane strain assumption is not so accurate due to concrete cracking but it is still within an acceptable tolerance.

The stress development is based on the stress-strain relationships given in the codes.





4.2 Moment-curvature diagrams

The experimental moment-curvature diagrams of the specimens can be constructed based on the following parameters: (i) The applied moment $M=0.5PL_1+0.125gL^2$ where P is the record of the load cell LC; the terms $L_1=0.75m$, g=0.6kN/mand L=2.0m as explained in Section 3.3; and (ii) The curvature is determined from the linear distribution of strains along the normal section.

The theoretical moment-curvature diagrams of the specimens are determined based on ACI 318-11, EC2 and SP63 as introduced in Section 2. It is noted that TCVN 5574:2012 does not produce specific moment-curvature diagram due to the stress-based nature of the code in the USoNS determination.

The experimental and theoretical moment-curvature diagrams of D1.1-2¢8 are shown in Fig. 11, from which relative good agreement between the codes and the test result can be observed.

4.3 Failure mode

Fig. 12 shows the images of all the tested specimens, which all failed with excessive midspan deflection.

Fig. 13 shows the zoom-in images of the front and the rear sides at mid-span area of a typical failed specimen. The ductile failure mode can be clearly observed with excessive normal cracks in tension zone occurred before the crushing at extreme concrete fiber in compression zone.

4.4 Ultimate strength on normal sections

The ACI 318-11, EC2 and SP63 experimental ultimate strengths M_{test} of the test specimens can be determined based on the strain at extreme compressive concrete fiber measured by LVDT I5 (Figs. 8 and 11) when it reached the respective limiting values of 0.0030, 0.0035 and 0.0035 by definition [3-5]. It is difficult to determine the M_{test} of TCVN 5574:2012 since there is no such criteria of limiting concrete compressive strain specified in the code. Hence, the maximum bending moments obtained in the tests can be referred to as the TCVN 5574:2012 experimental ultimate strength. The experimental relationships between the applied bending moment and extreme concrete compressive strain of all the specimens are shown in Fig. 14.



Figure 11. Experimental and theoretical momentcurvature diagrams of D1.1-2\u00f68



Figure 12. Images of failed specimens





Figure 14. Experimental relationship between moment and extreme concrete compressive strain

All the test results of M_{test} are shown in Table 4. The agreement ratios M_{code}/M_{test} between the theoretical value shown in Table 3 and the corresponding experimental value are shown in the brackets.

It can be seen from Table 4 that the mean values of the agreement ratios between the theoretical and the test results of ACI 318-11, EC2, TCVN 5574:2012 and SP63 are 0.980, 0.937, 0.848, and 0.956 with the coefficients of variable (COV) of 0.009, 0.011, 0.043, and 0.014, respectively. Hence, relatively good and conservative agreement was obtained from the experimental programme.

No	Test series	Specimen	ACI 318-11 M _{test} (M _{code} /M _{test})	EC2 M _{test} (M _{code} /M _{test})	TCVN 5574:2012 M _{test} (M _{code} /M _{test})	SP63 M _{test} (M _{code} /M _{test})
1	Sorioo 1	D1.1-2ø8	7.084 (0.971)	7.459 (0.917)	9.484 (0.731)	7.459 (0.929)
2	Selles I	D1.2-2ø8	7.384 (0.940)	7.796 (0.885)	9.671 (0.723)	7.796 (0.895)
3	Sorias 2	D2.1-2ф10	9.146 (0.997)	9.484 (0.954)	13.795 (0.789)	9.484 (0.967)
4	Series 2	D2.2-2¢10	9.221 (0.983)	9.334 (0.963)	10.346 (0.973)	9.334 (0.979)
5	Soriaa 2	D3.1-2ф12	15.409 (0.980)	15.371 (0.962)	16.984 (0.904)	15.371 (0.990)
6	Series 3	D3.2-2ф12	14.246 (1.011)	14.846 (0.921)	15.296 (0.966)	14.846 (0.978)
	Mean		(0.980)	(0.937)	(0.848)	(0.956)
	COV		(0.009)	(0.011)	(0.043)	(0.014)

Table 4. Experimenta	l validation	of ultimate	strength	M _{test}	(kNm)	and M	M_{test}
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The following conclusions are available within the scope of the experimental study presented in this paper:

- It is sufficient to use the limiting value of compressive strain at extreme concrete fiber as a basis to determine of the ultimate strength of normal sections (USoNS) in RC beams;

- The USoNS in RC beams determined by ACI 318-11, EN 1992-1-1:2004, TCVN 5574:2012 and SP 63.13330.2012 are validated in this experimental study with good and conservative agreement; and

- It is safe and reliable to incorporate the plane strain assumption and the calculation based on bilinear deformation model specified in SP 63.13330.2012 into the determination of the USoNS in RC beams cast and tested in Vietnam condition.

This experimental study can also be used to validate the specifications of cracking and deformation of RC beams provided by SP 63.13330.2012. Since the code is currently use as a basis for drafting the new Vietnamese code for design of RC structures to replace TCVN 5574:2012, the studies mentioned will be of importance in design practice.

Acknowledgement

5. Conclusions

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