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FUZZY POSSIBILITY-BASED SAFETY ASSESSMENT OF PORTAL STEEL FRAME

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Abstract

This paper investigates the application of fuzzy possibility theory for assessing the safety of portal steel frames, addressing the limitations of traditional methods such as Load and Resistance Factor Design (LRFD) and Allowable Stress Design (ASD) in handling uncertainties and subjective judgments in structural systems. Portal steel frames, widely used in industrial buildings, are susceptible to various uncertainties in loads, material properties, and geometric dimensions. Unlike probability theory, fuzzy possibility theory offers a robust framework for quantifying the possibility of safety or failure under imprecise or incomplete information, making it ideal for capturing real-world variability. The study establishes a practical procedure for structural fuzzy possibility analysis. It further introduces a new fuzzy possibility degree model that accounts for the importance of information at different membership levels, enhancing the assessment of structural safety compared to existing models. Numerical results demonstrate that the proposed model, operating within an extended possibility measure interval of $(-1, 2)$, provides more refined and reasonable outcomes than traditional models confined to $(0, 1)$, effectively distinguishing between absolute safety, absolute failure, and intermediate cases. Through a case study of a portal steel frame subjected to dead loads, live loads, wind loads, and foundation settlement, the paper evaluates safety and failure possibilities using deterministic and different fuzzy methods. Findings highlight the superiority of the proposed fuzzy possibilistic model in capturing complex uncertainties, though its non-traditional results require careful interpretation. Validating the model against empirical data, exploring sensitivity analyses, and developing normalization methods to bridge traditional and extended possibility frameworks, offer valuable insights for enhancing structural safety assessments in civil engineering.

Keywords: fuzzy possibility theory; structural safety assessment; portal steel frames.

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1. Introduction

Ensuring the safety and reliability of structures is paramount in civil engineering. Traditional safety assessment methods, such as Load and Resistance Factor Design (LRFD) and Allowable Stress Design (ASD), primarily rely on probabilistic approaches, often assuming precise values for input parameters like loads, material properties, and geometric dimensions. While these methods have proven useful, they may not fully capture the inherent uncertainties (e.g., variable wind loads) and subjective judgments that are often present in real-world structural systems. These uncertainties can arise from various sources, including manufacturing variations, environmental conditions, and a lack of precise information. Furthermore, expert knowledge and experience, which play a crucial role in safety assessment, are often difficult to incorporate directly into traditional probabilistic frameworks. Portal steel frames are widely used in industrial and commercial buildings due to their efficiency and ease of construction. Especially, the prefabricated steel frame structure of a single-story is a widely used structure for industrial buildings. This is a type of light steel frame with columns and girders

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made of welded I-shaped steel with outstanding advantages of reducing material costs, automating the manufacturing process, and easy construction and installation. The design specifications of this structure in Vietnam are currently defined in the design standard TCVN 5575:2012 [1] which is based on ASD. The structure is required to be designed to withstand various types of loads including static loads, live loads for roof repairs, crane loads, horizontal and vertical winds, and even earthquakes [2]. The design parameters are assumed to be precise [2–4]. However, like any structural system, these structures are susceptible to various uncertainties that can affect their safety. Accurate safety assessment is crucial to prevent failures and ensure the structural integrity of these frames [5].

To address the limitations of traditional methods in handling imprecise and uncertain information, fuzzy set theory and, more specifically, fuzzy possibility theory offer a powerful alternative. Fuzzy set theory provides a mathematical framework for representing and manipulating uncertain or vague information [6]. Unlike probability theory, which quantifies the likelihood of an event occurring, fuzzy possibility theory quantifies the possibility of an event occurring [7]. This makes it particularly well-suited for dealing with situations where the available information is imprecise or incomplete, and where subjective judgments play a significant role. Existing studies on structural safety assessment using fuzzy possibility have explored various aspects of uncertainty modeling and analysis. Researchers have employed fuzzy sets to represent uncertain parameters like material properties (yield strength, modulus of elasticity), geometric imperfections (out-of-plumbness, residual stresses), and loading conditions (dead loads, live loads, wind loads) [8–10]. Fuzzy structural analysis techniques, including fuzzy finite element methods, have been applied to determine fuzzy structural responses (displacements, stresses, internal forces) [11–13]. These fuzzy responses are then used in conjunction with fuzzy failure criteria to evaluate the possibility of failure [14, 15]. Studies have demonstrated the ability of fuzzy possibility-based methods to capture the combined effect of multiple uncertainties and provide a more comprehensive picture of structural safety compared to traditional probabilistic approaches. Findings often highlight the importance of carefully selecting appropriate membership functions and the need for efficient computational techniques to handle the complexity of fuzzy structural analysis. Some research has also focused on comparing fuzzy possibility results with those obtained from probabilistic methods, illustrating the differences in representing and interpreting uncertainty [16]. Nevertheless, the studies on portal steel frames are rare. Applying fuzzy possibility theory to the safety assessment of portal steel frames allows for the incorporation of uncertainties in a more comprehensive and nuanced manner.

This paper explores the application of fuzzy possibility theory in assessing the safety of portal steel frames. It examines the fundamental concepts of fuzzy possibility, proposes a methodology for fuzzy possibility-based structural safety analysis. Significantly, a new fuzzy possibility degree model is proposed, which reflects the importance of the fuzzy information that has not been considered in the past works. The paper also presents a case study of portal steel frame safety assessment using this approach, comparing and contrasting it with traditional methods. In the numerical investigations, the conditions of safety under dead load, live load, wind load, and foundation settlement are assessed according to TCVN 5575:2012 [1]. Finally, it identifies limitations associated with the current state of research and proposes potential directions for future investigations in this important area of structural safety.

2. Fuzzy possibility

2.1. Fuzzy possibility concept

Consider the limit state function of the structure $G_F = g_F(\mathbf{X})$ with $\mathbf{X} = [X_1, X_2, \ldots, X_n]^T$ being n-dimensional fuzzy input vector, and the membership function $\mu_{X_i}(x_i)$ $(i = 1, 2, \ldots, n)$ corresponds to the i-th fuzzy input X_i. The membership level is denoted by $\alpha \in [0,1]$. The membership function $\mu_{X_i}(x_i)$ essentially defines how each value in the input space is mapped to a degree of membership between 0 and 1. An α-cut of the fuzzy input X_i is an interval variable at the membership level α, i.e., $X_{i\alpha} = \left[\underline{X}_{i\alpha}, \overline{X}_{i\alpha}\right]$, where $\underline{X}_{i\alpha}$ and $\overline{X}_{i\alpha}$ are the lower bound and upper bound of the interval variable $X_{i\alpha}$, respectively. The limit state function G_F corresponding to $X_{i\alpha}$ is also an interval quantity at the membership level α, i.e., $G_{F\alpha} = \left[\underline{G}_{F\alpha}, \overline{G}_{F\alpha}\right]$, where $\underline{G}_{F\alpha}$ and $\overline{G}_{F\alpha}$ are the lower bound and upper bound of the interval variable $G_{F\alpha}$, respectively.

Traditionally, in the fuzzy safety-failure possibilistic model, the structure is considered safe if $G_F > 0$ or failed if $G_F \leq 0$. Cremona and Gao [17] defined the fuzzy failure possibility index π_f as

$$\pi_f = \sup\{\alpha \mid G_F = g_F(\mathbf{X}) \leq 0\} \tag{1}$$

where sup{·} is the supremum. At the α-level, the interval safety and interval failure possibilities are defined, respectively, as

$$h_s(\alpha) = \text{Poss}\{G_{F\alpha} = g_F(\mathbf{X}_\alpha) > 0\} = \begin{cases} 1 & \underline{G}_{F\alpha} > 0 \\ 0 & \underline{G}_{F\alpha} \leq 0 \end{cases} \tag{2}$$

$$h_f(\alpha) = \text{Poss}\{G_{F\alpha} = g_F(\mathbf{X}_\alpha) \leq 0\} = \begin{cases} 1 & \underline{G}_{F\alpha} \leq 0 \\ 0 & \underline{G}_{F\alpha} > 0 \end{cases} \tag{3}$$

where Poss{·} is the possibility of the event; $\mathbf{X}_\alpha \in \left[\underline{\mathbf{X}}_\alpha, \overline{\mathbf{X}}_\alpha\right]$ is the input interval vector at level α. The fuzzy safety possibility π_s and fuzzy failure possibility π_f can be obtained by aggregating the safety possibility $h_s(\alpha)$ and failure possibility $h_f(\alpha)$ at all membership levels α:

$$\pi_s = \text{Poss}\{G_F = g_F(\mathbf{X}) > 0\} = \int_0^1 h_s(\alpha)\,\mathrm{d}\alpha \tag{4}$$

$$\pi_f = \text{Poss}\{G_F = g_F(\mathbf{X}) \leq 0\} = \int_0^1 h_f(\alpha)\,\mathrm{d}\alpha \tag{5}$$

The fuzzy possibility model defined above by Cremona and Gao has certain limitations in the assessment of structural safety. The transition domain, i.e., $\underline{G}_{F\alpha} < 0$ and $\overline{G}_{F\alpha} > 0$, is not considered in this model. Wang et al. [18] introduced the rank-fuzzy possibility model, in which the interval safety and interval failure possibilities are given by

$$h_s(\alpha) = \text{Poss}\{G_{F\alpha} = g_F(\mathbf{X}_\alpha) > 0\} = \frac{\max\left\{0, \text{len}(G_{F\alpha}) - \max\left\{0, -\underline{G}_{F\alpha}\right\}\right\}}{\text{len}(G_{F\alpha})} \tag{6}$$

$$h_f(\alpha) = \text{Poss}\{G_{F\alpha} = g_F(\mathbf{X}_\alpha) \leq 0\} = \frac{\max\left\{0, \text{len}(G_{F\alpha}) - \max\left\{0, \overline{G}_{F\alpha}\right\}\right\}}{\text{len}(G_{F\alpha})} \tag{7}$$

where $\text{len}(G_{F\alpha}) = \overline{G}_{F\alpha} - \underline{G}_{F\alpha}$. The fuzzy safety (failure) possibility can then be computed by Eq. (4) (Eq. (5)). Note that this model gives $\pi_s \in [0,1]$, $\pi_f \in [0,1]$, and $\pi_s + \pi_f = 1$. This rank-fuzzy possibility model is superior to the original model of Cremona and Gao because it does consider the transition domain. However, as shown in the study by Shi and Lu [19], both models cannot distinguish the difference among different absolute safety (failure) cases.

In 2019, Shi and Lu [19] proposed a new model for fuzzy safety possibility and fuzzy failure possibility which overcomes the shortcomings of the traditional models. The α-cut safety possibility and α-cut failure possibility are defined by Shi and Lu, respectively, as

$$h_s(\alpha) = \text{Poss}\left\{G_{F\alpha} = g_F(\mathbf{X}_\alpha) > 0\right\} = \begin{cases} 1 + \dfrac{\underline{G}_{F\alpha}}{\overline{G}_{F\alpha}} & 0 < \underline{G}_{F\alpha} \\ \dfrac{\overline{G}_{F\alpha}}{\overline{G}_{F\alpha} - \underline{G}_{F\alpha}} & \underline{G}_{F\alpha} \le 0 \le \overline{G}_{F\alpha} \\ -\dfrac{\overline{G}_{F\alpha}}{\underline{G}_{F\alpha}} & \overline{G}_{F\alpha} < 0 \end{cases} \tag{8}$$

$$h_f(\alpha) = \text{Poss}\left\{G_{F\alpha} = g_F(\mathbf{X}_\alpha) \le 0\right\} = \begin{cases} -\dfrac{\underline{G}_{F\alpha}}{\overline{G}_{F\alpha}} & 0 < \underline{G}_{F\alpha} \\ -\dfrac{\underline{G}_{F\alpha}}{\overline{G}_{F\alpha} - \underline{G}_{F\alpha}} & \underline{G}_{F\alpha} \le 0 \le \overline{G}_{F\alpha} \\ 1 + \dfrac{\overline{G}_{F\alpha}}{\underline{G}_{F\alpha}} & \overline{G}_{F\alpha} < 0 \end{cases} \tag{9}$$

The fuzzy safety possibility π_s and fuzzy failure possibility π_f are then obtained by substituting Eq. (8) to Eq. (4) and Eq. (9) to Eq. (5), respectively. Note that $\pi_s + \pi_f = 1$ and $\pi_s \in (-1, 2)$; $\pi_f \in (-1, 2)$ according to Eqs. (8) and (9). These fuzzy possibilities can measure the safety (π_s) or failure (π_f) degree. The bigger the π_s (π_f) is, the higher the safety degree (failure degree) of the structure is [19]. An absolute safety (failure) structure corresponds to $0 < \underline{G}_{F\alpha}$ ($\overline{G}_{F\alpha} < 0$), for $\alpha \in [0, 1]$. A structure with $\underline{G}_{F\alpha} \le 0 \le \overline{G}_{F\alpha}$ has an uncertain safety degree.

Nevertheless, the fuzzy safety and failure possibilities obtained by Eq. (4) and (5) do not reflect the importance of the information at different membership levels because the interval safety (failure) possibility calculated at different α levels is of equal importance degree. This does not reflect the nature of the information distribution of the fuzzy limit state function G_F. Therefore, in this study, alternative fuzzy safety and failure possibility measures are introduced as follows.

2.2. *Alternative fuzzy safety-failure possibility degree*

The new fuzzy safety possibility π_s^{new} and new fuzzy failure possibility π_f^{new} are estimated by

$$\pi_s^{\text{new}} = \text{Poss}^{\text{new}}\left\{G_F = g_F(\mathbf{X}) > 0\right\} = \frac{\displaystyle\int_0^1 h_s(\alpha)\,\alpha\,\mathrm{d}\alpha}{\displaystyle\int_0^1 \alpha\,\mathrm{d}\alpha} = 2\int_0^1 h_s(\alpha)\,\alpha\,\mathrm{d}\alpha \tag{10}$$

$$\pi_f^{\text{new}} = \text{Poss}^{\text{new}}\left\{G_F = g_F(\mathbf{X}) \le 0\right\} = \frac{\displaystyle\int_0^1 h_f(\alpha)\,\alpha\,\mathrm{d}\alpha}{\displaystyle\int_0^1 \alpha\,\mathrm{d}\alpha} = 2\int_0^1 h_f(\alpha)\,\alpha\,\mathrm{d}\alpha \tag{11}$$

It can be easily proved that $\pi_s^{\text{new}} + \pi_f^{\text{new}} = 1$ by substituting Eq. (6) or (8) into Eq. (10), and Eq. (7) or (9) into Eq. (11). Since the membership degree α is taken into account, the new fuzzy possibility model better reflects the importance of the information, i.e., the uncertainty of the information at different membership levels.

3. Fuzzy possibility-based safety assessment

This section presents a practical methodology for the safety assessment of portal steel frame using the fuzzy possibility measure. The basic components for the fuzzy safety assessment of the structure include uncertainty modeling; fuzzy structural analysis; fuzzy safety-failure possibility analysis.

3.1. Uncertainty modeling of the input parameters

Using fuzzy sets to represent uncertain input parameters such as loads, material properties, and geometric imperfections. This is done by choosing the membership function $\mu_{X_i}(x_i)$ for each uncertain input parameter X_i. Some common types of membership functions are:

- *Triangular membership function*: This is defined by a triangular shape and is characterized by a peak point (where membership is 1) and two end points (where membership is 0). It's simple and widely used in applications with linear relationships.

- *Trapezoidal membership function*: Similar to the triangular membership function but with a plateau where the membership degree is 1. It's defined by four parameters: two for the feet of the trapezoid and two for the shoulders. It's useful when the system has a range of values with full membership.

- *Gaussian membership function*: Defined by a bell-shaped curve and is characterized by two parameters: the mean and the standard deviation. It's smooth and differentiable, making it suitable for applications requiring smooth transitions.

- *Sigmoidal membership function*: Defined by an S-shaped curve. It's useful in applications where the transition between membership degrees is gradual.

- *Generalized bell membership function*: Defined by three parameters: the width, the slope, and the center. It provides a flexible shape that can be adjusted to fit various kinds of data.

Factors to consider when choosing the membership function for the fuzzy input parameters are nature of the data, computational efficiency, and expert knowledge. Choosing the right membership function is critical as it significantly influences the accuracy and effectiveness of the possibility-based safety assessment. In the numerical example in Section 4, for the sake of simplicity, triangular membership function is assumed for the fuzzy inputs.

3.2. Fuzzy structural analysis

Performing structural analysis with fuzzy input parameters to obtain fuzzy responses. For the fuzzy possibility-based safety assessment, we need to determine the interval $G_{F\alpha}$ of the limit state function G_F at the membership level α. The bounds of $G_{F\alpha}$ are estimated as follows:

$$\underline{G}_{F\alpha} = \min g_F(\mathbf{X}_\alpha), \quad \mathbf{X}_\alpha \in \left[\underline{\mathbf{X}}_\alpha, \overline{\mathbf{X}}_\alpha\right] \tag{12}$$

$$\overline{G}_{F\alpha} = \max g_F(\mathbf{X}_\alpha), \quad \mathbf{X}_\alpha \in \left[\underline{\mathbf{X}}_\alpha, \overline{\mathbf{X}}_\alpha\right] \tag{13}$$

where $\underline{\mathbf{X}}_\alpha$ and $\overline{\mathbf{X}}_\alpha$ are the vectors of lower bounds and upper bounds of the fuzzy inputs at level α; $g_F(\mathbf{X}_\alpha)$ is the limit state function.

Several methods can be used to determine $\underline{G}_{F\alpha}$ and $\overline{G}_{F\alpha}$. In this study, the Taylor's Approximation Method (TAM) [20–23] is suggested to efficiently estimate the limit state interval. The finite element method is applied as the core deterministic structural analysis, where the frame members are modelled as 2D beam elements.

3.3. Fuzzy safety-failure possibility analysis

Determining the possibility of safety π_s^{new} and the possibility of failure π_f^{new} based on the fuzzy limit state function. This procedure is performed as follows:

Step 1: Discretize the membership of the inputs into m levels α_j $(j = 1, 2, \ldots, m)$ and determine the input interval vector $\mathbf{X}_{\alpha_j} = \left[\underline{\mathbf{X}}_{\alpha_j}, \overline{\mathbf{X}}_{\alpha_j}\right]$ at each level α_j by using the membership functions $\mu_{X_i}(x_i)$ $(i = 1, 2, \ldots, n)$.

Step 2: Estimate the output interval $G_{F\alpha_j} = \left[\underline{G}_{F\alpha_j}, \overline{G}_{F\alpha_j}\right]$ for each level α_j through fuzzy structural analysis with TAM method.

Step 3: Using Eqs. (6) and (7) or Eqs. (8) and (9) to compute the α_j-cut safety possibility $h_s\left(\alpha_j\right)$ and failure possibility $h_f\left(\alpha_j\right)$.

Step 4: Based on Eq. (10) and (11), the fuzzy safety and failure possibilities are determined as follows:

$$\pi_s^{\text{new}} = \frac{\sum_{j=1}^{m} h_s\left(\alpha_j\right)\alpha_j}{\sum_{j=1}^{m} \alpha_j} \tag{14}$$

$$\pi_f^{\text{new}} = \frac{\sum_{j=1}^{m} h_f\left(\alpha_j\right)\alpha_j}{\sum_{j=1}^{m} \alpha_j} \tag{15}$$

4. Numerical applications and case study

4.1. Comparison of different fuzzy possibility models

In this investigation, the membership level of the fuzzy inputs is discretized into 100 uniform portions α_j $(j = 1, 2, \ldots, 100)$. The fuzzy safety possibility and fuzzy failure possibility are estimated by the use of existing methods (Wang et al. [18] and Shi and Lu [19]) and the proposed model presented in Section 3.3.

a. Absolute safety

A limit state function $G_F = g_F(\mathbf{X}) = X_1 + X_2$ is considered, in which X_1 and X_2 are two triangular fuzzy numbers with parameters $(\underline{X}_1, \hat{X}_1, \overline{X}_1)$ and $(\underline{X}_2, \hat{X}_2, \overline{X}_2)$, respectively. The fuzzy safety possibility and fuzzy failure possibility obtained by different methods for different parameters are given in Table 1. The membership functions of G_F are depicted in Fig. 1. Fig. 1 shows that $G_F > 0$ for all cases, implying absolutely safe condition. Clearly, the safety redundancies for the five cases depicted in Fig. 1 vary. It is straightforward to see that a higher output corresponds to a safer structure.

Table 1. Fuzzy safety and failure possibilities output for absolute safety cases

	Case 1	Case 2	Case 3	Case 4	Case 5
$(\underline{X}_1, \hat{X}_1, \overline{X}_1)$	(0, 1, 2)	(0, 1, 2)	(1, 3, 5)	(3, 4, 5)	(5, 6, 6.5)
$(\underline{X}_2, \hat{X}_2, \overline{X}_2)$	(0, 1, 2)	(1, 2, 3)	(1, 2, 3)	(2, 3, 4)	(4, 5, 6)
Wang et al. [18]	1.00000	1.00000	1.00000	1.00000	1.00000
	0.00000	0.00000	0.00000	0.00000	0.00000
Shi and Lu [19]	1.38743	1.53315	1.56726	1.75939	1.85410
	−0.38743	−0.53315	−0.56726	−0.75939	−0.85410
Proposed	1.54973	1.69206	1.69206	1.83448	1.90162
	−0.54973	−0.69206	−0.69206	−0.83448	−0.90162

The data in Table 1 demonstrate that the proposed safety-failure possibilistic models provide a more effective assessment of structural safety, aligning closely with intuitive understanding. Specifically, the safety rankings for the five cases are as follows: Case 5 > Case 4 > Case 3 > Case 2 > Case 1. In contrast, the model by Wang et al. fails to differentiate between these varying levels of structural safety redundancy, yielding identical results across all five cases. Compared with the results by Shi and Lu, the proposed model gives higher fuzzy safety possibility values.

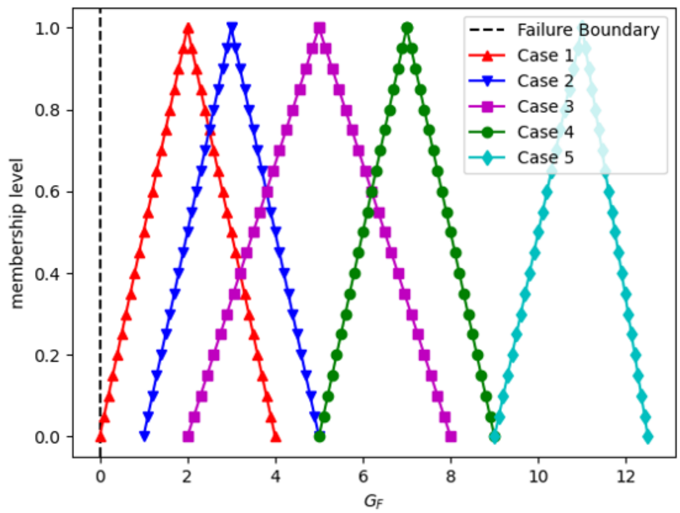


Figure 1. Membership function of G_F for the absolute safety cases

b. Absolute failure

Consider the limit state function in the previous section (absolute safety case) with the input parameters given in Table 2. Fig. 2 indicates that the structure reaches complete failure under the conditions specified in Table 2. Table 2 also presents the fuzzy safety possibilities and fuzzy failure possibilities calculated using the new possibility model and the existing ones. Additionally, Fig. 2 illustrates the membership functions of the limit state function G_F corresponding to five sets of fuzzy input parameters. In this example, the structure exhibits complete failure. As depicted in Fig. 2, all results of G_F across the five cases are negative; however, the extent to which they fall below zero varies. From Fig. 2, it is intuitively apparent that Case 1 is safer than Case 5, as the output for Case 1 exceeds that of Case 5. This observation is supported by the new fuzzy failure possibilities, calculated as $\pi_f^{new} = 1.54973$ for Case 1 and $\pi_f^{new} = 1.95447$ for Case 5, confirming that Case 1 is

Table 2. Fuzzy safety and failure possibilities output for absolute failure cases

	Case 1	Case 2	Case 3	Case 4	Case 5
$(\underline{X}_1, \hat{X}_1, \overline{X}_1)$	(−2, −1, 0)	(−3, −2, −1)	(−5, −3, −2)	(−6, −5, −4)	(−8, −7.5, −7)
$(\underline{X}_2, \hat{X}_2, \overline{X}_2)$	(−2, −1, 0)	−2, −1.5, 0)	(−3, −2, −1)	(−4, −3, −1)	(−7, −6.5, −6)
Wang et al. [18]	0.00000 1.00000	0.00000 1.00000	0.00000 1.00000	0.00000 1.00000	0.00000 1.00000
Shi and Lu [19]	−0.38743 1.38743	−0.55312 1.55312	−0.63939 1.63939	−0.73162 1.73162	−0.93182 1.93182
Proposed	−0.54973 1.54973	−0.68745 1.68745	−0.74339 1.74339	−0.81621 1.81621	−0.95447 1.95447

indeed safer than Case 5, aligning with the intuitive assessment. The proposed new fuzzy safety and failure possibility measures provide more reasonable outcomes compared to Wang et al. model. The safety ranking for the five cases is as follows: Case 1 > Case 2 > Case 3 > Case 4 > Case 5. In contrast, Wang et al. fuzzy possibilistic model fails to distinguish the varying degrees of safety across these cases. Furthermore, the proposed model provides clearer fuzzy safety and failure possibility results compared to the model by Shi and Lu.

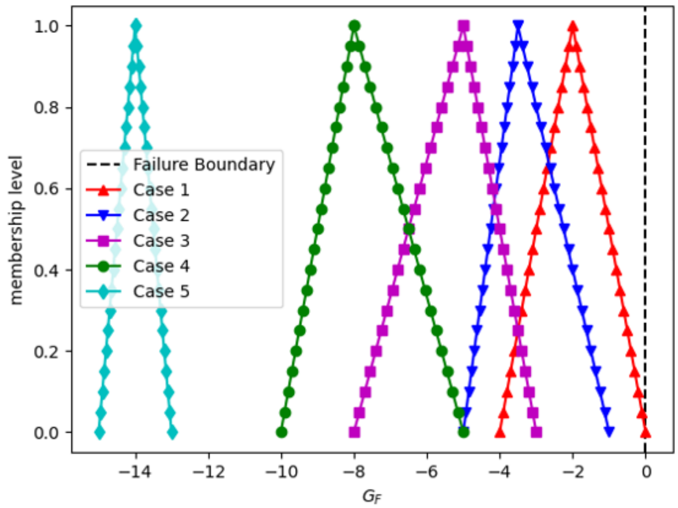


Figure 2. Membership function of G_F for the absolute failure cases

c. Safety-failure cases

The limit state function presented in previous cases remains under consideration. Table 3 lists the fuzzy safety possibilities and fuzzy failure possibilities derived from the different possibility models for various parameter sets. Additionally, Fig. 3 displays the membership functions of G_F across different cases. Table 3 reveals that the safety degree rankings provided by the new fuzzy possibilistic model and the other possibilistic models are identical, following the order: Case 5 > Case 4 > Case 3 > Case 1 > Case 2. This consistency highlights the effectiveness of the proposed fuzzy possibilistic model. A straightforward intuitive observation from Fig. 3 clearly shows distinctions of the membership functions.

Table 3. Fuzzy safety and failure possibilities output for safety-failure cases.

	Case 1	Case 2	Case 3	Case 4	Case 5
$(\underline{X}_1, \hat{X}_1, \overline{X}_1)$	(−1, 0, 1)	(−1, −0.5, 0)	(0,0.5,1)	(−1, 0.5, 3)	(−0.5, 2, 3)
$(\underline{X}_2, \hat{X}_2, \overline{X}_2)$	(−1, 0, 1)	(−1, −0.5, 1)	(−1, 0.5, 1)	(−0.5, 1, 3)	(0, 2, 5)
Wang et al. [18]	0.50495	0.10291	0.89709	0.93825	0.99634
	0.49505	0.89709	0.10291	0.06175	0.00366
Shi and Lu [19]	0.50000	−0.11630	1.11630	1.11721	1.34931
	0.50000	1.11630	−0.11630	−0.11721	−0.34931
Proposed	0.50000	−0.33441	1.33441	1.38641	1.52160
	0.50000	1.33441	−0.33441	−0.38641	−0.52160

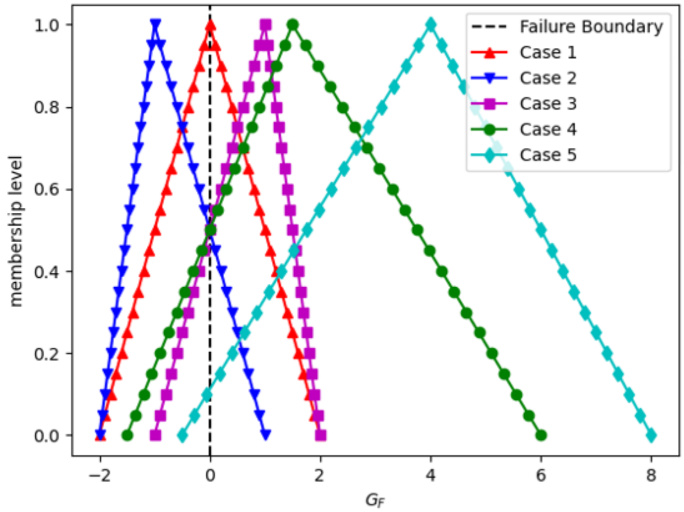
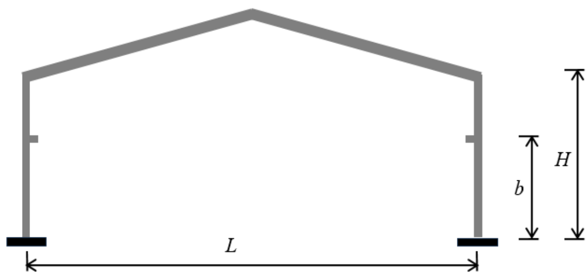


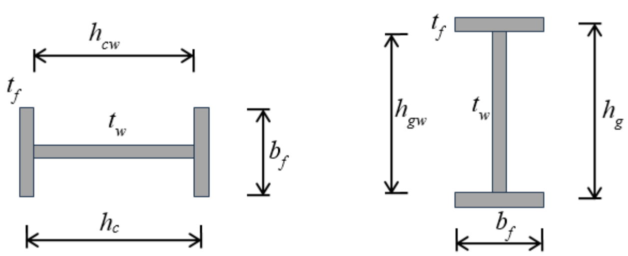
Figure 3. Membership function of G_F for the safety-failure cases

4.2. *Fuzzy portal steel frame*

Consider a portal steel frame with girders and columns made of prefabricated I-shaped steel [4]. The geometric dimensions of the frame include: (i) Frame span L; (ii) Crane rail elevation b; (iii) Column top elevation H; (iv) Roof slope angle α (Fig. 4(a)). The cross-section and dimensions are shown in Fig. 4(b). The frame is subjected to dead load, live load, crane load, wind load, and support settlement. The structure is analyzed according to the planar frame model, with rigid nodal connections. The loads and internal force combinations are in accordance with TCVN 2737:2023 standard [9]. TCVN 5575:2012 is used for the safety assessment of girders and columns.



(a) Layout of the portal steel frame



(b) Cross-section of the columns and girders

Figure 4. Geometry and cross-section details of the portal steel frame [4]

a. Input data

The nominal values of the input parameters are given in Table 4. For numerical investigation in this study, the loads (roof dead load, roof live load and wind pressure), and material properties (modulus of elasticity and strength of steel) are uncertain parameters and assumed as triangular fuzzy numbers. The other input parameters are considered crisp.

Table 4. Nominal values of input parameters for the portal steel frame [4]

Parameter	Notation	Nominal value	Unit
Span	L	24	m
Height	H	10	m
Step	B	6	m
Crane rail elevation	b	6.3	m
Roof slope angle	α	5.71	Degree
Height of column web	h_{cw}	0.4	m
Height of girder web	h_{gw}	0.5	m
Flange width	b_f	0.24	m
Web thickness	t_w	0.008	m
Flange thickness	t_f	0.014	m
Modulus of elasticity	E	2.1×10^8	kN/m^2
Strength of steel	f	2.1×10^5	kN/m^2
Steel density	ρ	7.85	Ton/m^3
Roof dead load	q_0	0.31	kN/m^2
Roof live load	p_0	0.3	kN/m^2
Wind pressure	w_0	0.95	kN/m^2
Crane load	Q	10	Ton
Differential settlement	Z	0.08	m
Support rotation	φ	0.5	Degree

b. Deterministic structural analysis

Structural analysis is conducted to evaluate internal forces—such as bending moment, shear force, and axial force—at the sections of columns and girders. Additionally, it assesses the horizontal displacement at the column top. For this steel frame, various methods like the analytical approach or the finite element method (FEM) can be employed for structural analysis. In this study, the finite element method is utilized.

The internal forces are calculated at specific sections of the structural members, including four column sections (column base, section below the column shoulder, section above the column shoulder, and column top) and two girder end sections, for each load case. Subsequently, the critical internal forces at these sections are determined by applying load combinations as per TCVN 2737:2023:

- Basic combination 1: Includes permanent loads (e.g., dead load and differential settlement) combined with a single live load.

- Basic combination 2: Incorporates permanent loads along with selected temporary adverse loads.

The horizontal displacement at the column top is computed for three nominal load combinations, as outlined in TCVN 5575:2012: dead load combined with wind load, dead load combined with crane load, and dead load combined with 0.5 wind load plus crane load.

c. Limit state function

The safety (failure) condition for the frame is checked according to limit state I (for strength) and limit state II (for serviceability).

Limit state I: Columns and girders are evaluated as bending-compression members in accordance with TCVN 5575:2012. If the relative eccentricity, m_e, exceeds 20, they are assessed solely as bending members.

Limit state II: The frame is analyzed for horizontal displacement, with checks performed at the column top elevation. For structures utilizing a heavy crane, the maximum allowable horizontal displacement is determined based on TCVN 5575:2012. For light and medium cranes, the limit for horizontal displacement is set to not exceed 1/300.

Thus, the limit state function is defined as:

$$g_F(E, f, q_0, p_0, w_0) = \min\left(s_c, s_g, \frac{H}{300} - \delta^{\max}\right) \tag{16}$$

where s_c, s_g are the safety measures of column and of girder, respectively; $\delta^{\max}$ is the maximum horizontal displacement at column top.

The safety measure s_c is determined according to TCVN 5575:2012, as the minimum value of:

- Strength of column: $f - \sigma_c^{\max}$, where $\sigma_c^{\max}$ is the maximum stress at column sections.
- Overall buckling of column: $f - \sigma_c^{\text{buckling}}$, where $\sigma_c^{\text{buckling}}$ is the maximum buckling stress.
- Buckling of column web: $n_{cw} - \dfrac{h_{cw}}{t_w}$, where n_{cw} is the allowable slenderness for column web.
- Buckling of column flange: $n_{cf} - \dfrac{b_0}{t_f}$, where n_{cf} is the allowable slenderness for column flange, $b_0 = 0.5\left(b_f - t_w\right)$.

The safety measure s_g is determined according to TCVN 5575:2012, as the minimum value of:

- Strength of girder: $f - \sigma_g^{\max}$, where $\sigma_g^{\max}$ is the maximum stress at girder sections.
- Buckling of girder web: $n_{gw} - \dfrac{h_{gw}}{t_w}$, where n_{gw} is the allowable slenderness for girder web.
- Buckling of girder flange: $n_{gf} - \dfrac{b_0}{t_f}$, where n_{gf} is the allowable slenderness for girder flange.

d. Fuzzy safety assessment

The fuzzy safety assessment for the portal steel frame is carried out, focusing on the safety and failure possibilities under specific parameter variations. The analysis assumes maximum variations of −5% for the modulus of elasticity E and yield strength f, ±10% for the dead load q_0, ±30% for the live load p_0, and ±20% for the wind load w_0. The results are summarized in Table 5, which compares four different approaches: a deterministic assessment and three fuzzy methods (Wang et al. [18], Shi and Lu [19], and the current study). The membership function of the limit state function is shown in Fig. 5. Some key observations and discussion are given below:

Deterministic assessment: The deterministic approach based on nominal values of the input parameters in Table 4 yields a safety possibility of 1 and a failure possibility of 0. This indicates a binary outcome where the structure is considered entirely safe with no possibility of failure. However, this approach does not account for uncertainties or variations in the input parameters, making it overly simplistic and potentially unrealistic for real-world applications with inherent variability.

Fuzzy method by Wang et al.: Wang et al.'s fuzzy method provides a safety possibility of 0.8823 and a failure possibility of 0.1177, summing to 1. This suggests a more refined assessment, acknowledging a small but non-zero probability of failure (11.77%). The reduction in safety possibility

Table 5. Results of safety and failure possibilities for the portal steel frame

Possibility	Deterministic assessment	Fuzzy method		
		Wang et al.	Shi and Lu	This study
Safety possibility	1.0000	0.8823	1.1364	1.3936
Failure possibility	0.0000	0.1177	−0.1364	−0.3936
Sum	1.0000	1.0000	1.0000	1.0000

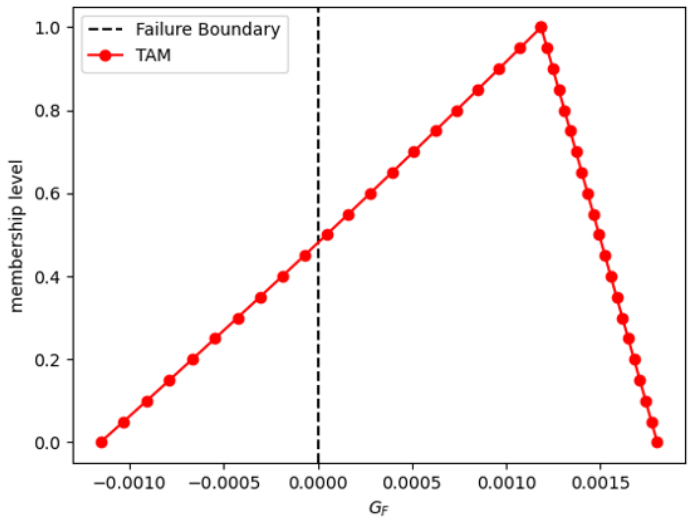


Figure 5. Membership function of G_F for considered portal steel frame

compared to the deterministic model reflects the incorporation of parameter uncertainties, offering a more realistic evaluation. This approach adheres to the traditional possibility measure interval of $(0, 1)$.

Fuzzy method by Shi and Lu: The Shi and Lu model reports a safety possibility of 1.1364 and a failure possibility of −0.1364, also summing to 1. Unlike the traditional possibility measure, which is confined to the interval $(0, 1)$, Shi and Lu's fuzzy possibility measure operates within the interval $(-1, 2)$.

Current study method: The proposed model in this study yields a safety possibility of 1.3936 and a failure possibility of −0.3936, summing to 1. Similar to Shi and Lu's model, the current study adopts a fuzzy possibility measure within the interval $(-1, 2)$, rather than the traditional $(0, 1)$ range. The negative failure possibility is smaller than, and the safety possibility is greater than those of Shi and Lu method, indicating a tailored approach to incorporating the importance of the information in the portal steel frame analysis. This suggests the model is able to capture broader or more complex uncertainty distributions, but its results must be interpreted within the context of this non-traditional possibility measure.

5. Conclusions

This study discusses and investigates the fuzzy possibility-based safety assessment for structures, specialized for the portal steel frame. It also introduces a practical procedure for fuzzy possibility analysis. A new model is proposed to estimate the fuzzy safety and fuzzy failure possibility degrees.

Some remarks drawn from the numerical results are:

- The deterministic model's binary outcome contrasts sharply with the fuzzy methods, which account for parameter uncertainties and provide a range of possibilities. This highlights the advantage of fuzzy logic in handling real-world variability.

- Traditional possibility model, adhering to the $(0, 1)$ interval, appears more conservative and conventional, making it easier to interpret and compare with standard probability assessments. However, it may not capture the full range of uncertainties and cannot differentiate the absolute safety or absolute failure cases.

- The current study's models, operating within the $(-1, 2)$ interval as that of Shi and Lu [19], produce results outside the conventional $(0, 1)$ range, which is intentional and reflects a modified fuzzy possibilistic approach. The negative failure possibilities and safety possibilities greater than 1 are designed to handle specific uncertainty scenarios. Additionally, the proposed fuzzy possibility measures offer more reasonable results compared to existing models (as implied in the broader context of the paper), allowing for a broader representation of safety and failure possibilities.

Implications and recommendations:

- The safety evaluation of the portal steel frame, as per TCVN 5575:2012, in the presence of uncertainties in input parameters, necessitates a thorough and meticulous assessment using fuzzy approach.

- The fuzzy model and its possibility measure significantly impact the assessment of the portal steel frame's safety. Traditional approach, with its $(0, 1)$ range, offers a straightforward and interpretable outcome suitable for standard engineering applications. In contrast, the current study's use of the $(-1, 2)$ interval provides a more flexible but less conventional framework, potentially better suited for complex uncertainty scenarios.

- Practitioners should be cautious when interpreting negative failure possibilities or safety possibilities exceeding 1 in the current study's models. These values are valid within the $(-1, 2)$ interval but may be misinterpreted if viewed through the lens of traditional fuzzy logic.

- Future research could focus on validating the $(-1, 2)$ interval models against empirical data, conducting sensitivity analyses to assess the robustness of the results under varying parameter uncertainties, and exploring the physical or engineering implications of negative failure possibilities and safety possibilities greater than 1.

- To bridge the gap between models, a normalization or transformation method could be developed to map results from the $(-1, 2)$ interval to the traditional $(0, 1)$ interval, facilitating comparison and communication across different fuzzy approaches.

In summary, while the fuzzy methods provide a more comprehensive assessment of the portal steel frame than the deterministic approach, the differences in possibility measure intervals $(0, 1)$ for traditional method versus $(-1, 2)$ for the current study highlight the importance of understanding the underlying framework. The extended $(-1, 2)$ interval used by the current study offers an alternative way to assess safety and failure, but its results require careful interpretation and validation to ensure reliable and practical safety evaluations for the portal steel frame.

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