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FORMULAS FOR DETERMINING THE CRITICAL BUCKLING STRESS OF I-SHAPED MEMBERS UNDER PURE BENDING

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Abstract

Steel structures are widely used in construction, and the stability conditions for these structures are of greater concern due to their long and slender characteristics. When a plate element is subjected to axial compression, bending, shear, or a combination of these forces in its plane, the plate may buckle locally before the member as a whole becomes unstable or before the yield stress of the material is reached. This local buckling behavior causes the plates in the cross-section of the steel member to interact with each other. Therefore, it is necessary to consider this interaction when calculating or checking for stability conditions. In this research, the proposed formulas determine the buckling coefficient as well as the local critical stress for I-shaped steel beams, accounting for the flange-web interaction when the flange-thickness to web-thickness ratio changes. Additionally, the buckling analysis results indicate that local buckling stress does not depend on the length-to-height ratio but is impacted by the height-to-width and thickness-to-width ratios. Comparisons between the proposed formulas and numerical results show that the suggested formulas have high reliability when the coefficient of variation is small and the coefficient of determination is very high.

Keywords: local buckling; local critical stress; buckling coefficient; flange-web interaction; I-shaped sections.

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1. Introduction

I-shaped steel members are widely used in construction, such as beams and columns. These members are composed of steel plates, and when subjected to bending or compression, they can become unstable due to the formation of compression regions in the cross-section. To ensure the load-bearing capacity of the I-section steel beams, the stability conditions must be checked. Recently, many authors have researched the local buckling of thin-walled steel members. Bhowmick and Grondin [1] investigated the local buckling of I-shaped members bent about their weak axis. Han and Lee [2] studied the effect of web slenderness on the elastic flange local buckling of I-beams. Shi et al. [3] examined the local buckling behavior of I-section beams fabricated from high-strength steel. Kuwamura [4] estimated the local buckling behavior of thin-walled stainless steel stub columns. Shi et al. [5, 6] conducted several experimental investigations on the local buckling behavior of high-strength steel welded-section stub columns. Cao et al. [7] investigated local buckling behavior of high-strength welded I-section columns under axial compression. Shi and Xu [8] experimented on I-beams under different loading conditions to study the local buckling behavior. Zhang et al. [9] studied local buckling behavior of steel faceplates and their influence on the compressive strength of steel-plate composite walls. Deepak and Anathi [10] evaluated the local buckling behavior of built-up cold-formed steel homogeneous and hybrid double I-box column sections under axial compression

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through experimental testing. Some authors [11, 12] researched the local buckling of high-strength steel plates at elevated temperatures. It can be observed that when the steel structures experience local buckling, the component plates of the cross-section interact with each other. In Eurocode 3, part 1-3 [13] and part 1-5 [14], with the Effective Width Method, the interaction between the flange and the web is neglected, and the connection between them is treated as simply supported. This leads to the conservative calculation and material consuming of local buckling for cold-formed structures; therefore, it is necessary to consider the interaction between the plates in the cross-section. Trahair [15] presented expressions for calculating the elastic buckling stress of rectangular hollow and I-section steel members under compression and bending, taking into account the interaction between the flange and web. Seif and Schafer [16] used the semi-analytical finite strip method to propose formulas for determining the plate buckling coefficient for thin-walled members, considering the interaction between plates in the cross-section subjected to axial force, major axis bending, and minor axis bending. Vieira et al. [17] carried out a parametric study concerning the evaluation of the local buckling coefficient for rectangular hollow section members under combined axial compression and biaxial bending, accounting for web-flange interaction. Ragheb [18] researched the influence of the interaction between the flange and web on the local buckling of welded steel I-sections subjected to bending. Bedair [19] considered the influence of the flange/web geometric proportions on the stability of web plates in W-shaped columns under uniform compression. Zhang et al. [20] investigated the local buckling of I-section columns, accounting for the interaction between the web and flanges. Szymczak and Kujawa [21] addressed the local buckling of the compressed flanges of cold-formed channel beams subjected to pure bending or axial compression, taking into account the web-flange interactions. Mitsui et al. [22] presented a novel formula for the local buckling coefficient of cold-formed open sections under uniform compression, taking into account the plate elements interaction of the cross-section. Gardner et al. [23] developed expressions for determining the elastic local buckling stress of structural steel profiles under comprehensive loading conditions, accounting for the interaction between individual plate elements via an interaction coefficient based on the local buckling stress of the isolated plate. Lapira et al. [24] provided formulas for calculating the elastic local buckling stresses of doubly-symmetric thin-walled I-section girders subjected to shear stress, accounting for the interaction between the plate elements.

In Ref. [16], expressions for determining the buckling coefficient were developed to apply to the sections in the AISC shape database. Therefore, it is necessary to establish calculation formulas for the local buckling coefficient of the cross-sections beyond the scope of this source data. This paper suggests approximate formulas for determining the elastic buckling stress of I-shaped sections under pure bending, accounting for the interaction between the component plates of the cross-section when the ratio of flange thickness to web thickness changes. These equations are based on the parametric study analyzed by the semi-analytical finite strip method in the CUFSM program. This method was pioneered by Cheung [25] using classical plate theory to establish the finite strip. Unlike the finite element method, the semi-analytical finite strip method uses trigonometric functions in the longitudinal direction and simple polynomial functions in the transverse direction. This method is very useful for analyzing members with constant thickness along the axis. A number of researchers have applied the semi-analytical finite strip method to analyze thin-walled structures. Bui [26] used this method to examine the buckling behavior of thin-walled circular hollow sections under pure bending. Bui [27] analyzed cold-formed sections with curved corners using the finite strip method based on Marguerre's shallow shell theory. Uy and Bradford [28] employed a finite strip model for elastic buckling to study the behavior of steel plates in composite steel-concrete members. Several authors developed the CUFSM [29] and THIN-WALL [30] programs to calculate geometric characteristics and analyze the buckling of thin-walled structures.

2. Formulas for determining the critical buckling stress of I-shaped steel beams

2.1. Formula for determining elastic buckling stress

The local buckling of the I-beams bent about their major axis can be approximated by assuming that the plate elements are hinged along their common edges. However, to accurately reflect the behavior of I-section steel members, it is necessary to consider the simultaneous buckling of the flange and web elements in I-beams under bending. Trahair [15] used the classical formula to calculate the local critical stress for I-shaped members subjected to bending, which is given by

$$\sigma_{crl} = k_\sigma \frac{\pi^2 E}{12\,(1-\nu^2)} \left(\frac{t_f}{b_f}\right)^2 \tag{1}$$

where k_σ is the elastic buckling coefficient, E is Young's modulus, ν is Poisson's ratio, b_f is the flange width and t_f is its thickness. It is clear that the local buckling stress depends on the variation of the buckling coefficient and the thickness-to-width ratio. Although Trahair provided this formula for determining the critical buckling stress, the method for calculating the buckling coefficient relied on graphical techniques.

These days, there are many tools available to easily calculate critical buckling stress, such as software using the finite element method (ABAQUS, ANSYS, SAP2000, ...), and the finite strip method (CUFSM, THIN-WALL, ...). However, determining the elastic buckling stress using these methods is significantly complex and inconvenient for design engineers, especially when multiple iterations are needed to achieve a suitable cross-section. Therefore, proposing approximate formulas to manually calculate the local buckling coefficient in Eq. (1) to simplify the determination of local buckling stress for steel structures. This approach allows engineers to shorten the time required to determine the input data for calculating the local buckling strength of steel structures.

2.2. Relationship between length-to-height ratio and buckling stress

When the I-shaped steel beams experience local buckling, the flange and web plates exhibit waves along their length. Therefore, when the beams are sufficiently long (in practical cases), their length does not impact the value of the local critical buckling stress. The relationship between local critical stress and the length-to-height ratio is presented in this section. Stability analyses for three groups are performed using the CUFSM program. The geometry of the specimens is illustrated in Table 1, and the analysis results are shown in Fig. 1.

Table 1. Dimension of specimens

	Specimens	b_f (mm)	h_w (mm)	t_f (mm)	t_w (mm)
Group 1	S1 – 1	100	200	6.0	6.0
	S1 – 2	200	400	12	12
Group 2	S2 – 1	200	300	8.0	6.0
	S2 – 2	400	600	16	12
Group 3	S3 – 1	200	200	10	6.0
	S3 – 2	400	400	20	12

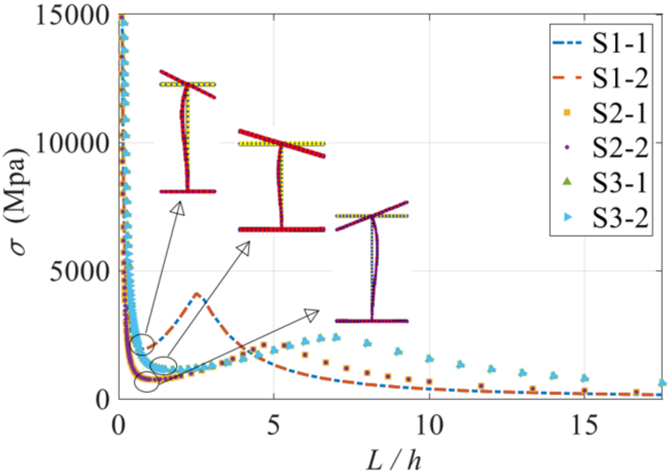


Figure 1. Relationship between buckling stress and length-to-height ratio

As shown in Fig. 1, it can be observed that when the flange-thickness to web-thickness ratio and height-to-width ratio remain unchanged, the local buckling stresses are the same if the length-to-height ratio is identical. This means that the critical buckling stress (the minimum point of the curves shown in Fig. 1) does not depend on the length-to-height ratio but rather on the thickness-to-width ratio, flange-thickness to web-thickness ratio, and height-to-width ratio, as shown in Eq. (1) and Fig. 1.

2.3. Proposed formula for determining the buckling coefficient of I-shaped beams with equal flange and web thickness

For I-section beams subjected to bending, the suggested expression for calculating the elastic buckling coefficient, based on Ref. [16], is as follows:

$$\frac{1}{k_w} = \frac{1.5}{\left((h_w/t_w)\left(2t_f/b_f\right)\right)^2} + 0.015 \tag{2}$$

$$k_f = k_w\left(\frac{t_w}{h_w}\right)^2\left(\frac{b_f}{t_f}\right)^2 \tag{3}$$

where k_w denotes the buckling coefficient of the web and k_f is the buckling coefficient of the flange, h_w and t_w are the height and thickness of the web, respectively.

In this section, the CUFSM software is used to calculate the elastic buckling stress for I-shaped members subjected to pure bending. The analysis results shown in Fig. 1 indicate that the value of local buckling stress may be influenced by the height-to-width ratio. Thus, the elastic buckling analyses are performed for I-shaped steel beams with height-to-width ratios ranging from 1.0 to 5.0, with a corresponding increment step of 0.1. Based on the obtained results, parameter study and statistical probability processing are used to give a predictive curve for determining the local buckling coefficient. Therefore, the proposed equation for calculating the buckling coefficient of I-beams under pure bending is as follows:

$$\begin{cases} k_\sigma = -a_1\left(\dfrac{h_w}{b_f}\right)^2 + b_1\left(\dfrac{h_w}{b_f}\right) + c_1 & \text{if} \quad 1.0 \le h_w/b_f < 3.0 \\ k_\sigma = a_2\left(h_w/b_f\right)^{-b_2} & \text{if} \quad 3.0 \le h_w/b_f \le 5.0 \end{cases} \tag{4}$$

The coefficients a_1, b_1, c_1, a_2, b_2 in Eq. (4) are determined using the regression method, with $a_1 = 0.12, b_1 = 0.08, c_1 = 3.2, a_2 = 13$, and $b_2 = 1.55$, corresponding to a coefficient of variation (CoV) of 0.015 and a coefficient of determination $\left(R^2\right)$ of 0.9982. Then, the formula for calculation the local buckling coefficient of the beam flange subjected to bending is as follows:

$$\begin{cases} k_\sigma = -0.12\left(\dfrac{h_w}{b_f}\right)^2 + 0.08\left(\dfrac{h_w}{b_f}\right) + 3.2 & \text{if} \quad 1.0 \leq h_w/b_f < 3.0 \\ k_\sigma = 13\left(h_w/b_f\right)^{-1.55} & \text{if} \quad 3.0 \leq h_w/b_f \leq 5.0 \end{cases} \tag{5}$$

The results shown in Fig. 2 point out that the predicted curve from the proposed formula and the simulation solution in the CUFSM program provides a comparable prediction for the elastic buckling coefficient, while the predicted curve from Ref. [16] is not consistent with the results from the numerical method.

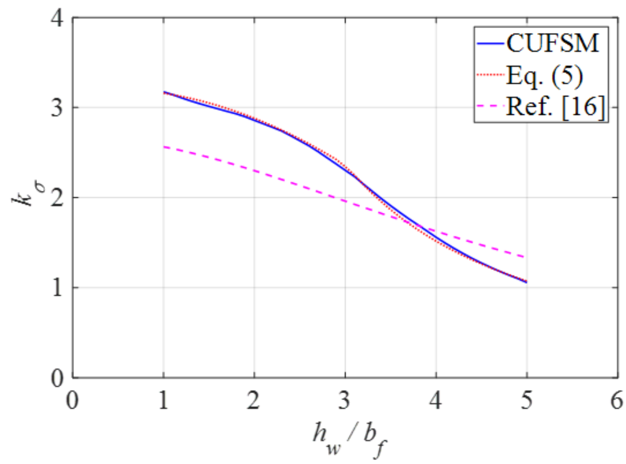


Figure 2. Influence of height-to-width ratio on buckling coefficient with equal flange and web thickness

As shown in Table 2, it is evident that the predictive results from Eq. (5) give good predictions with a mean value (μ) of 0.998 and a coefficient of variation (CoV) of 0.015. Meanwhile, Ref. [16] provides predictions for determining the local buckling coefficient with an average value of 0.933 and a CoV of 0.162. Furthermore, the coefficient of determination for the proposed formula is 0.9982, which is better than that in Ref. [16] with 0.6646.

Table 2. Comparisons between calculation results and simulation results

Calculating methods	μ	CoV	R^2
Ref. [16]	0.933	0.162	0.6646
Eq. (5)	0.998	0.015	0.9982

2.4. *Proposed formulas for determining buckling coefficient of I-shaped beam with unequal flange and web thickness*

The proposed formulas for determining the local buckling coefficient for I-section beams with the flange-thickness to web-thickness ratios ranging from 1.25 to 3.0 are established similarly to the case of equal flange and web thickness. This section presents only the obtained results, such as the relationship between the buckling stress and height-to-width ratio (as shown in Fig. 4), the proposed formulas (as shown in Table 3), and comparisons of the elastic buckling coefficients based on the proposed formulas with the results from the CUFSM program (as shown in Fig. 5 and Table 5).

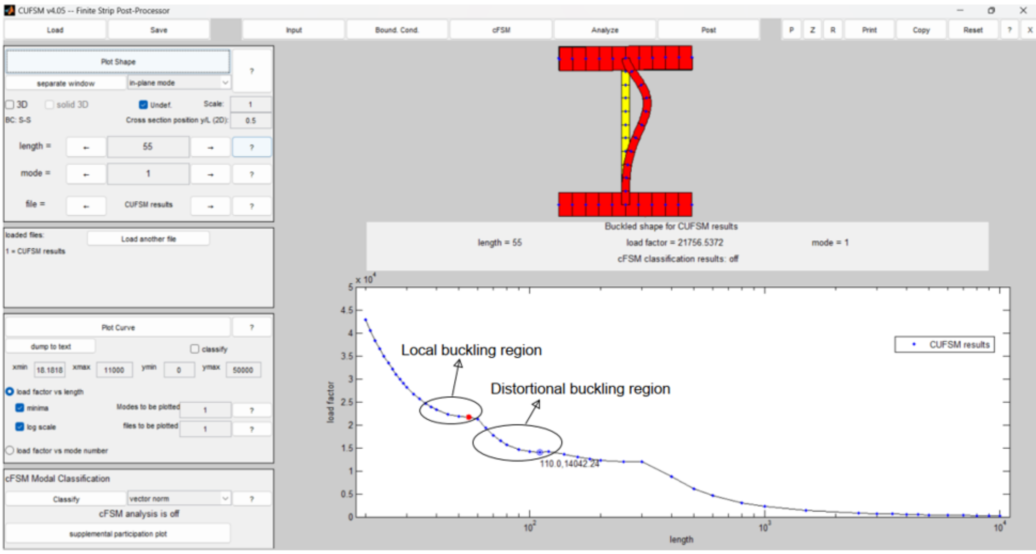


Figure 3. Analysis result in CUFSM program

Note: The signature curve of a steel beam with large flange thickness and web thickness is shown in Fig. 3. In this curve, there is no minimum point in the local buckling region; nevertheless, it appears in the distortional buckling region. This deduces that this cross-section does not have a local buckling stress. Therefore, this work does not consider this case but focuses only on members that exhibit a minimum point in the local buckling zone and have an elastic buckling stress value.

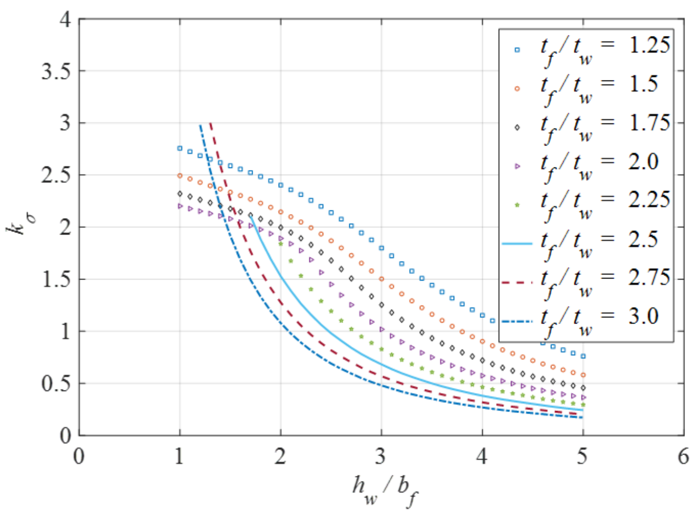


Figure 4. Effect of height/width ratio on buckling stress with unequal flange and web thickness

As shown in Fig. 4, the flange-thickness to web-thickness ratios ranging from 1.25 to 2.0 have similar curves to the height-to-width ratio equal to 1.0. Thus, the proposed equations for these ratios are have the same shape as in Eq. (5). The remaining ratios (from 2.25 to 3.0) have proposed formulas in the form of exponential expressions as follows:

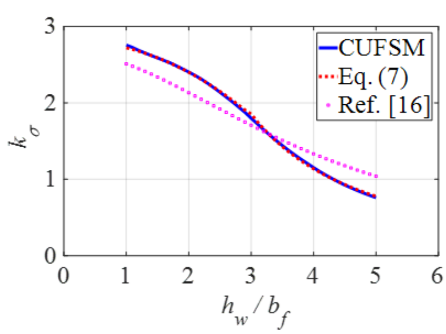
$$k_\sigma = a\left(h_w/b_f\right)^{-2} \tag{6}$$

Table 3. The proposed formulas for determining local buckling coefficient

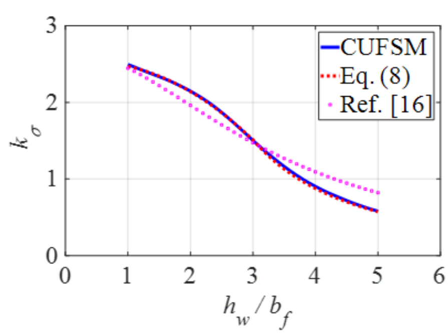
Ratios	Proposed formulas	
$t_f/t_w = 1.25$	$\begin{cases} k_\sigma = -0.12\left(\dfrac{h_w}{b_f}\right)^2 + 0.04\left(\dfrac{h_w}{b_f}\right) + 2.8 & \text{if} \quad 1.0 \le h_w/b_f < 3.0 \\ k_\sigma = 12\left(h_w/b_f\right)^{-1.7} & \text{if} \quad 3.0 \le h_w/b_f \le 5.0 \end{cases}$	(7)
$t_f/t_w = 1.5$	$\begin{cases} k_\sigma = -0.15\left(\dfrac{h_w}{b_f}\right)^2 + 0.12\left(\dfrac{h_w}{b_f}\right) + 2.5 & \text{if} \quad 1.0 \le h_w/b_f < 3.0 \\ k_\sigma = 12.2\left(h_w/b_f\right)^{-1.9} & \text{if} \quad 3.0 \le h_w/b_f \le 5.0 \end{cases}$	(8)
$t_f/t_w = 1.75$	$\begin{cases} k_\sigma = -0.2\left(\dfrac{h_w}{b_f}\right)^2 + 0.34\left(\dfrac{h_w}{b_f}\right) + 2.1 & \text{if} \quad 1.0 \le h_w/b_f < 3.0 \\ k_\sigma = 11.1\left(h_w/b_f\right)^{-2} & \text{if} \quad 3.0 \le h_w/b_f \le 5.0 \end{cases}$	(9)
$t_f/t_w = 2.0$	$\begin{cases} k_\sigma = -0.3\left(\dfrac{h_w}{b_f}\right)^2 + 0.57\left(\dfrac{h_w}{b_f}\right) + 1.9 & \text{if} \quad 1.0 \le h_w/b_f < 3.0 \\ k_\sigma = 9.3\left(h_w/b_f\right)^{-2} & \text{if} \quad 3.0 \le h_w/b_f \le 5.0 \end{cases}$	(10)
$t_f/t_w = 2.25$	$k_\sigma = 7.4\left(h_w/b_f\right)^{-2}$ if $2.0 \le h_w/b_f \le 5.0$	(11)
$t_f/t_w = 2.5$	$k_\sigma = 6.1\left(h_w/b_f\right)^{-2}$ if $1.7 \le h_w/b_f \le 5.0$	(12)
$t_f/t_w = 2.75$	$k_\sigma = 5.1\left(h_w/b_f\right)^{-2}$ if $1.3 \le h_w/b_f \le 5.0$	(13)
$t_f/t_w = 3.0$	$k_\sigma = 4.3\left(h_w/b_f\right)^{-2}$ if $1.2 \le h_w/b_f \le 5.0$	(14)

Note: to determine the local buckling coefficients for the I-section steel beams when the t_f/t_w ratios are not listed in Table 3, interpolation can be used.

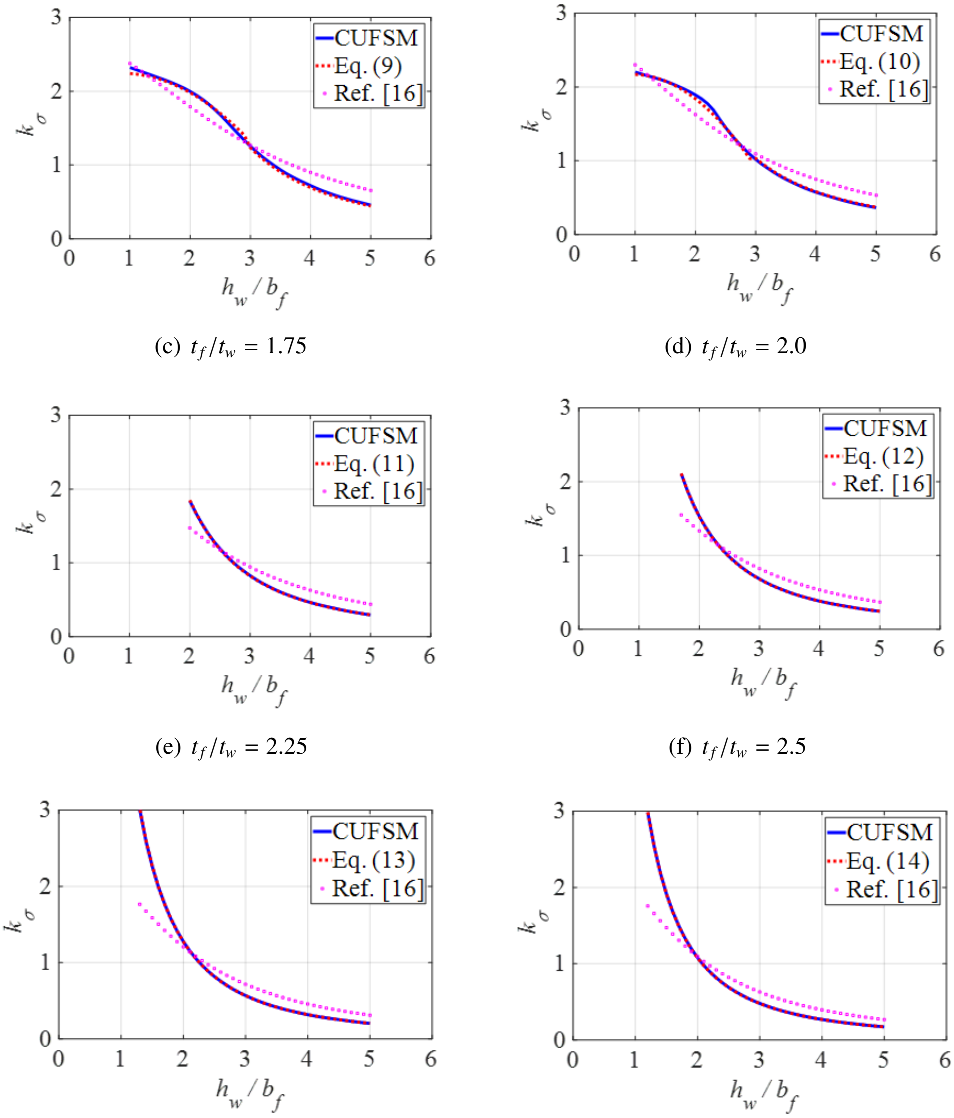
The results shown in Fig. 5 indicate that the predicted curves from the proposed formulas are close to those from the numerical method for all ratios. In contrast, the predicted curve from Ref. [16] is not consistent with the results from the CUFSM software for the ratios shown in Fig. 5.



(a) $t_f/t_w = 1.25$



(b) $t_f/t_w = 1.5$



(c) t_f/t_w = 1.75 (d) t_f/t_w = 2.0

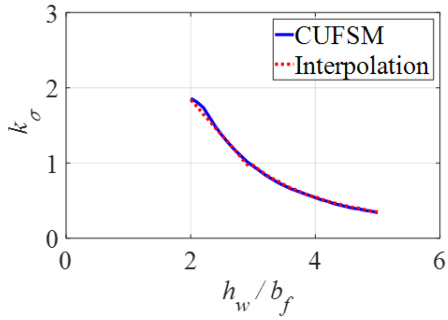
(e) t_f/t_w = 2.25 (f) t_f/t_w = 2.5

(g) t_f/t_w = 2.75 (h) t_f/t_w = 3.0

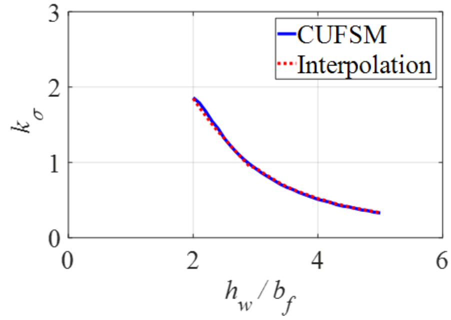
Figure 5. Influence of height-to-width ratio on buckling coefficient of I-shaped beams with unequal flange and web thickness

As shown in Table 3, Eq. (10) differs from Eq. (11) when the t_f/t_w ratio is between 2.0 and 2.25. Therefore, to demonstrate the use of interpolation, a study is conducted on three cases within the t_f/t_w ratio ranging from 2.0 to 2.25.

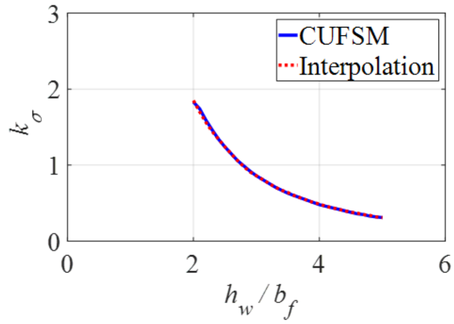
The local buckling coefficients shown in Fig. 6 and Table 4 indicate that the predicted curve for the buckling coefficient using the interpolation method is very close to the results of the CUFSM program. The coefficients in Table 4, such as the average value (μ), coefficient of variation (CoV), and the coefficient of determination $\left(R^2\right)$, are highly accurate, with the μ approximately equal to 1.0, the CoV less than 0.020, and the R^2 greater than 0.9970 for all ratios. These results show that the interpolation method can be applied to t_f/t_w ratios not listed in Table 3.



(a) t_f/t_w = 2.075



(b) t_f/t_w = 2.125



(c) t_f/t_w = 2.2

Figure 6. Buckling coefficient of CUFSM and interpolation

Table 4. Comparisons between interpolation results and numerical solution

Ratios	Interpolation		
	μ	CoV	R^2
t_f/t_w = 2.075	1.006	0.020	0.9972
t_f/t_w = 2.125	1.005	0.017	0.9979
t_f/t_w = 2.2	1.002	0.009	0.9994

As shown in Table 5, it can be seen that the results from the proposed formulas give good predictions with a mean value of around 1.0 and coefficient of variation less than 0.03. Whereas, Ref. [16] provides prediction results that clearly deviate from the simulation results in the CUFSM software, especially the coefficients of variation for all results in the Table 5, which are greater than 0.150. In addition, the coefficients of determination from the proposed equations for all ratios is very high, exceeding 0.998.

Table 5. Comparisons between calculation results and numerical solutions

Ratios	Proposed formulas			Ref. [16]		
	μ	CoV	R^2	μ	CoV	R^2
t_f/t_w = 1.25	1.001	0.011	0.9993	1.033	0.156	0.8940
t_f/t_w = 1.5	0.988	0.013	0.9992	1.073	0.155	0.9380
t_f/t_w = 1.75	0.984	0.023	0.9976	1.095	0.166	0.9431

Ratios	Proposed formulas			Ref. [16]		
	μ	CoV	R^2	μ	CoV	R^2
$t_f/t_w = 2.0$	1.000	0.019	0.9985	1.119	0.181	0.9389
$t_f/t_w = 2.25$	0.996	0.004	0.9999	1.219	0.169	0.8639
$t_f/t_w = 2.5$	0.998	0.003	1.0000	1.228	0.187	0.8669
$t_f/t_w = 2.75$	1.000	0.003	1.0000	1.214	0.227	0.7998
$t_f/t_w = 3.0$	0.997	0.002	1.0000	1.242	0.224	0.8125

3. Numerical applications

This section illustrates calculation examples for determining the critical buckling stress of I-shaped members with pinned ends under pure bending with variations in height-to-width ratios. The dimensions of the segments are shown in Table 6, including the width, height, flange thickness, and web thickness. The steel material has a modulus of elasticity $E = 210000$ (MPa) and a Poisson's ratio $\nu = 0.3$.

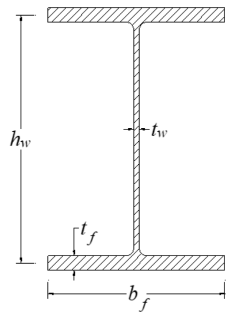


Figure 7. Cross-section

Table 6. Dimensions of the I-beams

Specimens	b_f (mm)	h_w (mm)	t_f (mm)	t_w (mm)
R1-1	150	250	6.0	6.0
R1-2	150	400	6.0	6.0
R1-3	150	600	6.0	6.0
R1-4	150	750	6.0	6.0
R2-1	150	250	7.5	6.0
R2-2	150	400	7.5	6.0
R2-3	150	600	7.5	6.0
R2-4	150	750	7.5	6.0
R3-1	150	250	9.0	6.0
R3-2	150	400	9.0	6.0
R3-3	150	600	9.0	6.0
R3-4	150	750	9.0	6.0
R4-1	150	250	14	8.0
R4-2	150	400	14	8.0
R4-3	150	600	14	8.0
R4-4	150	750	14	8.0
R5-1	150	250	16	8.0
R5-2	150	400	16	8.0
R5-3	150	600	16	8.0
R5-4	150	750	16	8.0
R6-1	150	400	18	8.0
R6-2	150	600	18	8.0
R6-3	150	750	18	8.0
R7-1	150	400	25	10

Specimens	b_f (mm)	h_w (mm)	t_f (mm)	t_w (mm)
R7-2	150	600	25	10
R7-3	150	750	25	10
R8-1	150	250	27.5	10
R8-2	150	400	27.5	10
R8-3	150	600	27.5	10
R8-4	150	750	27.5	10
R9-1	150	250	30	10
R9-2	150	400	30	10
R9-3	150	600	30	10
R9-4	150	750	30	10

The I-section steel beam R1-1, having dimensions $b_f = 150$ (mm), $h_w = 250$ (mm), and $t_f = t_w = 6.0$ (mm), is calculated. Since the height-to-width ratio is less than 3.0 and t_f/t_w ratio is 1.0, the local buckling coefficient is determined by the following steps:

Eq. (5): $k_\sigma = -0.12\left(\dfrac{h_w}{b_f}\right)^2 + 0.08\left(\dfrac{h_w}{b_f}\right) + 3.2 = 3.0.$

Then, using (1) calculates the local buckling stress:

Eq. (1): $\sigma_{crl} = k_\sigma \dfrac{\pi^2 E}{12\,(1-\nu^2)}\left(\dfrac{t_f}{b_f}\right)^2 = 911.0\,(\text{MPa}).$

The remaining specimens in Table 6 are calculated similarly. The obtained results are presented and compared in Table 7.

Table 7. Comparisons of the calculation methods

Specimens	$\sigma_{crl,CUFSM}$ (MPa)	$\sigma_{crl,THIN\text{-}WALL}$ (MPa)	σ_{crl} (MPa)	$\sigma_{Seif\,\&\,Schafer}$ [16] (MPa)
R1-1	900.5	900.7	911.0 (1.2%)	728.8 (-19.1%)
R1-2	770.0	770.5	777.4 (1.0%)	630.5 (-18.1%)
R1-3	477.1	477.1	460.4 (-3.5%)	493.8 (-18.1%)
R1-4	321.3	312.3	325.8 (1.4%)	404.9 (26.0%)
R2-1	1201.7	1201.9	1202.1 (0.0%)	1078.2 (-10.2%)
R2-2	967.4	967.4	974.3 (0.7%)	876.0 (-9.4%)
R2-3	550.2	550.8	539.4 (-2.0%)	632.7 (15.0%)
R2-4	357.6	361.4	369.1 (3.2%)	493.8 (38.1%)
R3-1	1557.4	1558.0	1560.2 (0.2%)	1457.7 (-6.4%)
R3-2	1202.5	1202.6	1198.0 (-0.4%)	1111.0 (-7.6%)
R3-3	622.7	623.9	598.5 (-3.9%)	746.8 (19.9%)
R3-4	399.8	401.6	391.7 (-2.0%)	560.6 (40.2%)
R4-1	3514.0	3517.4	3490.4 (-0.7%)	3289.6 (-6.4%)
R4-2	2552.6	2548.5	2619.7 (2.6%)	2356.3 (-7.7%)
R4-3	1189.2	1192.3	1147.0 (-3.5%)	1489.5 (25.3%)
R4-4	758.4	758.7	734.1 (-3.2%)	1085.3 (43.1%)
R5-1	4365.3	4370.8	4355.0 (-0.2%)	3986.8 (-8.7%)
R5-2	2775.6	2781.7	2778.6 (0.1%)	2693.8 (-2.9%)
R5-3	1242.1	1246.6	1255.2 (1.1%)	1617.6 (30.2%)

Specimens	$\sigma_{crl,CUFSM}$ (MPa)	$\sigma_{crl,THIN\text{-}WALL}$ (MPa)	σ_{crl} (MPa)	$\sigma_{Seif\,\&\,Schafer}$ [16] (MPa)
R5-4	788.2	788.6	803.3 (1.9%)	1151.7 (46.1%)
R6-1	2865.3	2868.1	2844.2 (-0.7%)	2987.0 (4.2%)
R6-2	1269.5	1274.5	1264.1 (-0.4%)	1718.9 (35.4%)
R6-3	808.6	809.0	809.0 (0.1%)	1202.2 (48.7%)
R7-1	4530.7	4537.5	4522.6 (-0.2%)	5061.3 (11.7%)
R7-2	2005.8	2014.6	2010.0 (0.2%)	2811.9 (40.2%)
R7-3	1280.6	1281.5	1286.4 (0.5%)	1939.2 (51.4%)
R8-1	11689.9	11731.0	11712.6 (0.2%)	9244.1 (-20.9%)
R8-2	4573.6	4581.7	4575.2 (0.0%)	5398.6 (18%)
R8-3	2025.2	2034.4	2033.4 (0.4%)	2913.0 (43.8%)
R8-4	1295.0	1296.1	1301.4 (0.5%)	1986.8 (53.4%)
R9-1	11762.8	11813.3	11752.4 (-0.1%)	10122.7 (-13.9%)
R9-2	4602.0	4611.5	4590.8 (-0.2%)	5686.9 (23.6%)
R9-3	2038.7	2048.4	2040.4 (0.1%)	2994.9 (46.9%)
R9-4	1303.1	1304.4	1305.8 (0.2%)	2024.5 (55.4%)

As shown in Table 7, it can be observed that the calculation results from the proposed formulas converge with the simulation results from the CUFSM and THIN-WALL software, with the error for all cases in Table 7 being less than 4.0%. Meanwhile, the results from Ref. [16] show a significant error, with the maximum error being approximately 55% for the segment R9-4. This indicates that the proposed formulas can be applied to the cases in this work.

4. Conclusion

The buckling analysis for I-beams under pure bending in the CUFSM program shows that the elastic local critical stress does not depend on the length-to-height ratio, but only on the h_w/b_f and t_f/b_f ratios. Therefore, the proposed formulas for determining the local buckling coefficient, as well as the local buckling stress for pure bending steel beams, must consider the interaction between the web and flange. The suggested formulas apply to flange thickness-to-web thickness ratios ranging from 1.0 to 3.0. Additionally, the calculation results from the proposed formulas agree well with the results from the CUFSM and THIN-WALL software, showing coefficients of variation less than 0.03 and very high coefficients of determination greater than 0.998 for all cases. Thus, the proposed formulas ensure reliability for application in the local buckling design of I-beams under pure bending.

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