OPTIMAL VOLUME DESIGN OF A WORM-GEAR DRIVE WITH A BRANCH AND BOUND METHOD

Le Hong Chuong[1*], Nguyen Quoc Dung[2]

***Summary**: In mechanical design, an important aspect is to keep the volume of design objects to a minimum. It also means reduced the cost of production. So that, using optimization design method in modern designs is necessary. In this paper, a nonlinear optimization method is presented to design a worm-gear driver. The chosen objective function was the volume bounded by the inner surface of the worm and worm-gear as well as obtained an optimum minimum distance between them. It can be observed that the proposed optimal design has the potential to yield considerably better solutions than the traditional heuristics.*

***Keywords**: Optimal design; worm and worm-gear drive; nonlinear method; Branch and Bound algorithm.*

Received: September 12th, 2016, revised: September 26th, 2016, accepted: October 13th, 2016

1. Introduction

Designing a new mechanical product, there are a lot parameters and phases that a designer need to care, for example the depth of design, input data, design strategy, procedures and results. Depending on the design requirements, they prioritized to choose certain objectives for optimization such as strength, deflection, weight, wear, and corrosion. However, design optimization for a complete mechanical assembly leads to a complicated objective function with a large number of design variables. Therefore, designers normally chose optimization techniques for individual components or intermediate assemblies than a complete assembly.

A gearbox design involves computations based on tooth bending strength, tooth surface durability, tooth surface fatigue, interference, efficiency… It also involves empirical formulas, different graphs and tables, which lead to a complicated design. Manual design is very difficult considering the relationship of the above facts for having the optimal parameters. With the aid of computers, design can be carried out iteratively and the design variables, which satisfy the given conditions, can be determined. The design so obtained may not be the optimum one, because in the above process the design variables so obtained satisfy only one condition at a time; for example, if the module is calculated based on bending strength, the same module is substituted to calculate the surface durability. It is accepted if it is within the strength limit of surface durability; otherwise it is changed accordingly. So optimization methods are required to determine design variables, which simultaneously satisfy the given conditions. As the optimization problem involves the objective function and constraints that are not stated as explicit functions of the design variables, it is hard to solve it by classical optimization methods. Moreover, increasing demand for compact, efficient and reliable gear forces the designer to use optimal design methodology. Many researchers have reported solutions to optimal gearbox designs. One of the first who dealt with this problem was Golinski [1, 2]. An optimal mass design for a single stage spur gear box by mean of nonlinear programming and random method was presented. Huang et al. [3] developed interactive physical programming approach of the optimization model of three-stage spur-gear reduction unit with minimum volume, maximum surface fatigue life and maximum load-carrying capacity as design objectives and core hardness, module, face width of gear, tooth numbers of pinion, tooth numbers of gear and diameter of shat as design variables. In this model, tooth bending fatigue failure, shaft torsional stress, face width, interference and tooth number are considered as constraints. The MATLAB constrained optimization package is used to solve this nonlinear programming problem. Jhalani and Chaudhary [4] discussed the various parameters, which can affect

[1]*Dr, Faculty of Mechanical Engineering. National University of Civil Engineering (NUCE).*
[2]*MSc, Faculty of Mechanical Engineering. National University of Civil Engineering (NUCE).*
** Corresponding author. E-mail: chuong40m@gmail.com.*

the design of the gearbox for knee mounted energy harvester device and later it frames the optimization problem of the mass function based on the dimensions of gearbox for the problem. The problem is solved using multi start approach of the MATLAB global optimization toolbox and value of global optimum function is obtained considering all the local optimal solutions of the problem. Padmanabhan et al. [5] investigated that in many real-life problems, objectives under consideration conflict with each other, and optimizing a particular solution with respect to a single objective can result in unacceptable results with respect to the other objectives. Multi-objective formulations are realistic models for many complex engineering optimization problems. Ant Colony Optimization was developed specifically for a worm gear drive problem with multiple objectives. Sanghvi et al. [6] presented optimal methods for two-stage helical gear train. In the research, the volume and load carrying capacity are optimized. For the optimization purpose, face width, module and number of teeth are taken as design variables. Constraints are imposed on bending strength, surface fatigue strength and interference. It is apparent from the comparison of results that the result obtained by NSGA-II is more superior than the results obtained by other methods in terms of both objectives. In Vietnam, V.N. Pi [7] presented optimal calculation of two-stage worm-gear reducer. Pi and V.Q. Dac [8] investigated optimal calculation of partial transmission ratios of worm-helical gear reducers for minimal gearbox length by using theoretical analysis methods.

In this paper, a worm gear drive is considered. The optimization model formulated here includes these factors in constraints. The optimization is carried out using Brand and Bound Algorithm of Mixed integer Non-linear programming in GAMS software. This method is applied to minimize the volume. The results obtained by this method are compared with hand calculation.

2. Mixed Integer Nonlinear Programming

2.1 Introduction

Mixed-integer nonlinear programming (MINLP) problems combine the combinatorial difficulty of optimizing over discrete variable sets with the challenges of handling nonlinear functions. MINLP is one of the most general modeling paradigms in optimization and includes both nonlinear programming (NLP) and mixed-integer linear programming (MILP) as sub-problems. Following [9], the MINLPs are conveniently expressed as:

$$\begin{cases} \text{minimize} & f(x), \\ \text{subjectto} & c(x) \leq 0, \\ & x \in X \\ & x_i \in Z, \quad \forall i \in I \end{cases} \tag{1}$$

where f: $R^n \rightarrow R$ and c: $R^n \rightarrow R^m$ are twice continuously differentiable functions; $X \subset R^n$ is a bounded polyhedral set and $I \subseteq \{1, \ldots, n\}$ is the index set of integer variables. Note that it can readily include maximization and constraints, such as equality constraints or lower and upper bounds LB ≤ c(x) ≤ UB, where LB and UB are the lower and upper bounds. More general discrete constraints that are not integers can be modeled by using so-called special-ordered sets of type I [10], [11].

Problem (2.1) is an NP - hard combinatorial problem, because it includes MILP [12] and its solution typically requires searching enormous search trees; see Fig. 1. Worse, non-convex integer optimization problems are in general un-decidable by Jeroslow [13]. He provides an example of a quadratically constrained integer program and shows that no computing device exists that can compute the optimum for all problems in this class.

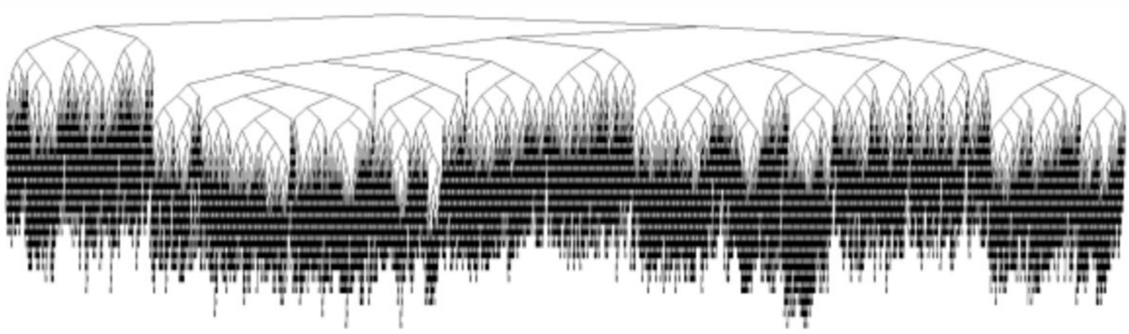

Figure 1. *Branch-and-bound tree without pre-solve after 360 s CPU time has more than 10,000 nodes.*

2.2 MINLP Algorithm

Some major algorithms for solving the MINLP problem include the following: Branch and bound; Generalized Benders Decomposition; Outer-Approximation.

In this section, present a branch-and-bound algorithm for MINLP models is implemented by Kalvelagen [14]. Branch-and-bound methods are used extensively for mixed-integer linear programming models and go back to [16]. The basic method is directly applicable to models with nonlinear functions in which case a nonlinear solver needs to evaluate the relaxed sub-problems (Fig.2).

The root node is the problem with all integer restrictions relaxed.

Branch-and-bound methods are based on the concept of relaxations: sub-problems with one or more of the discrete variable relaxed to continuous variables. The number of relaxations to explore is often very large for problems with many integer variables. Mixed-integer linear programming branch-and-bound solvers put much emphasis on solving relaxations very fast (e.g. by using dual methods). Modern solvers also do lots of work on preprocessing: resolving the model to make it smaller and adding cuts to make the feasible region smaller (although this makes the problem larger in terms of number of constraints).

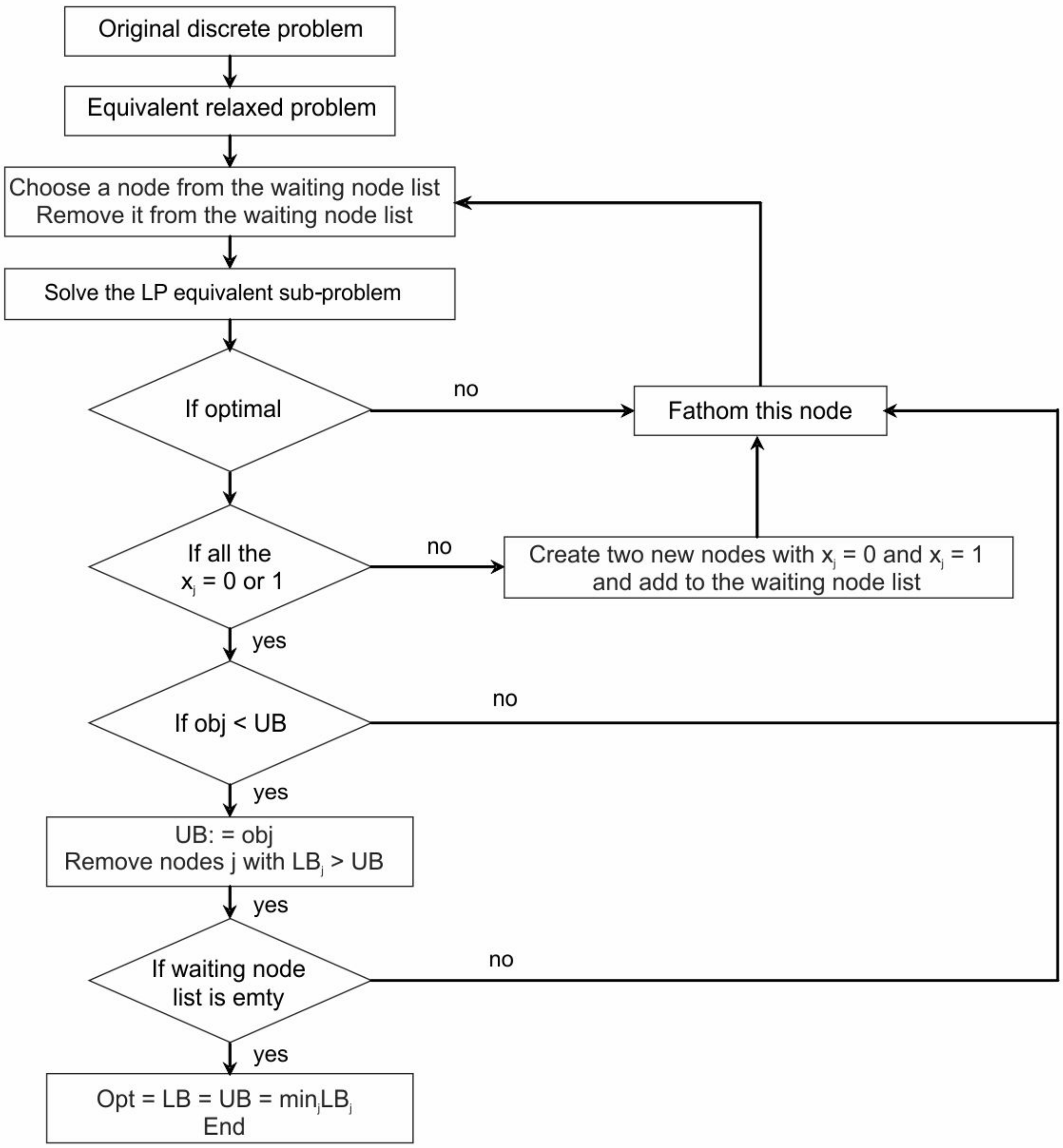

Figure 2. *Flow chart of Branch and Bound Algorithm*

3. Design problem formulation

Using an example was presented in [16]. The optimal model of one-way rotation worm gear drive is formulated in this section, with the minimum volume as a design object. The schematic illustration of it is shown in Fig. 3 with the following data:

Maximum torque on the worm-gear: T_2 = 670 Nm; Transmission ratio u = 19; Rotation speed of worm: n_1 = 930 rpm; Gear necessary life: L = 10000 hours.

The materials chosen for the worm and worm-gear are:

- Worm: 40 steel, quenched and tempered at HRC 45;

- Worm-gear: дрАшн 10-4-4; Allowable Hertzian stress: $[\sigma_H]$ = 206.4 MPa; Allowable bending stress: $[\sigma_F]$ = 129.5 MPa; Maximum allowable Hertzian stress: $[\sigma_H]_{max}$ = 1200 MPa; Maximum allowable bending stress: $[\sigma_F]_{max}$ = 480 MPa.

4. Optimization of worm-gear drive

In order to perform the optimal design, it is necessary to set up: the variables that uniquely describe the problem, the parameters, the objective function and the constraint functions. The following design equations and constraints are adopted from design book [16].

4.1 Design variables

The design problem variables are presented in Tab 1.

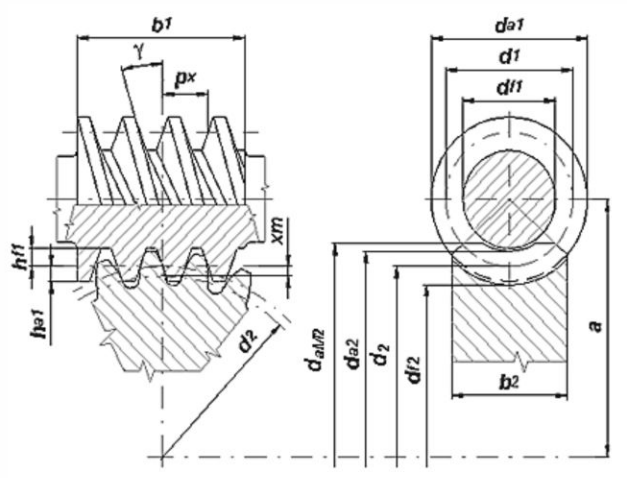

Figure 3. *Schematic illustration of worm and worm-gear*

Table 1. *Variables of optimization problem*

No	Variables	Range	No	Variables	Range
1	Number of worm threads (Z_1)	1, 2 or 4	11	Pitch diameter of worm-gear (d_2), mm	
2	Number of worm-gear (Z_2)	26…80	12	Outside diameter of worm-gear (d_{aM2}), mm	
3	Worm diameter ratio (q)	6.3…25	13	Throat diameter of worm (d_{a1}), mm	
4	Load factor (K)	1.1…1.3	14	Throat diameter of worm-gear (d_{a2}), mm	
5	Working centre distance (a), mm		15	Root diameter of worm (d_{f1}), mm	
6	Module (m), mm	2…20	16	Root diameter of worm-gear (d_{f2}), mm	
7	Normal addendum modification coefficient (x), mm	-0.7…0.7	17	Tooth form factor (Y)	1.24…1.98
8	Face width of worm (b_1), mm		18	Over load factor (K_0)	17…40
9	Face width of worm-gear (b_2), mm		19	Lead angle (γ), radian	
10	Pitch diameter of worm (d_1), mm				

4.2 Objective function

For the optimization, the volume of the worm-gear drive is minimized. Calculation formula of the volume is:

$$V = \frac{\pi}{4}(d_1^2 b_1 + d_2^2 b_2), \quad mm^3 \tag{2}$$

4.3 Constraint functions

The solutions of the optimization program have to satisfy the following constraints listed below according to [16].

- The number of worm-gear (Z_2) and the number of worm threads (Z_1) should be related:

$$Z_2 = uZ_1 \tag{3}$$

- The relationship between worm diameter ratio (q) with number of worm-gear (Z_2):

$$q \geq (0.25...0.3)Z_2 \tag{4}$$

- Value of working centre distance (a) was calculated following:

$$a = (Z_2 + q)\sqrt[3]{\left(\frac{170}{Z_2[\sigma_H]}\right)^3 \frac{T_2 K_H}{q}} \tag{5}$$

- The normal addendum modification coefficient (x) related to worm-gear parameters:

$$x=\frac{a}{m}-0.5(q+Z_2) \quad (6)$$

- Face width of worm-gear (b_2) should be less or equal to 0.75 times of the throat diameter of worm (d_{a1}):

$$b_2 \le 0.75 d_{a1} \quad (7)$$

- The outside diameter of worm-gear (d_{aM2}) has to satisfy the following condition:

$$d_{aM2} \le (d_{a2}+2m) \quad (8)$$

- The values of Hertz stress and bending stress should be less or equal to the corresponding allowable stresses.

$$\sigma_H \le [\sigma_H], \quad \sigma_F \le [\sigma_F] \quad (9)$$

- The values of over load of Hertz stress and bending stress should be less or equal to the corresponding maximum allowable stresses.

$$\sigma_{H\max} \le [\sigma_H]_{\max}, \quad \sigma_{F\max} \le [\sigma_F]_{\max} \quad (10)$$

4.4 Results and Discussion

The optimization problem is solved by using the Branch and Bound algorithm of MINLP method in Gams software. [17] shown that the General Algebraic Modeling System (GAMS) is specifically designed for modeling linear, nonlinear and mixed integer optimization problems. GAMS allows the user to concentrate on the modeling problem by making the setup simple. It is especially useful for handling large, complex, one-of-a-kind problems which may require many revisions to establish an accurate model. The user can change the formulation quickly and easily, can change from one solver to another and can even convert from linear to nonlinear with little trouble. Using GAMS, data are entered only once in familiar list and table form. Models are described in concise algebraic statements which are easy for both humans and machines to read. Whole sets of closely related constraints are entered in one statement. GAMS automatically generates each constraint equation and lets the user make exceptions in cases where generality is not desired.

The obtained values of all considered variables are presented in Tab. 2. To note that according to the design standards [16], values of Z_1, K_0 were selected following the initial conditions. Whereas, the face width of worm (b_1) was calculated following value of x.

Table 2. *Variable values were solved by B&B method and Traditional method*

No	Variable	Method		No	Variable	Method		No	Variable	Method	
		MINLP	Hand			MINLP	Hand			MINLP	Hand
1	Z_1	2	2	10	d_1, mm	125	78.75	19	K_0	21	21
2	Z_2	38	39	11	d_2, mm	190	245.7	20	Y	1.24	1.55
3	q	25	12.5	12	daM_2, mm	205	273.3	21	σ_H, Mpa	206.4	195.6
4	u	19	19.5	13	da_1, mm	135	91.35	22	σ_F , Mpa	22.75	16.96
5	a, mm	157.5	165	14	da_2, mm	200	263.85	23	σ_{Hm} , Mpa	1164	808.95
6	m, mm	5	6.3	15	df_1, mm	113	63.63	24	σ_{Fm} , Mpa	477.8	288.32
7	x, mm	0	0.44	16	df_2, mm	178	236.13	25	V, mm^3	2.9×10^6	3.7×10^6
8	b_1, mm	78	94	17	γ, degree	4.59	8.5				
9	b_2, mm	67	69	18	K	1.241	1.2				

Tab. 2 also shown side-by-side the corresponding values were calculated by hand in [16]. Comparative study of two solutions shown in this table leads to the following conclusions:

- The optimal design solution did not change the transmission ratio, while the traditional method changed it.

- The volume of worm-gear drive calculated with the classical method is 3.7×10^6 mm^3 while the optimal design solution offers a smaller volume, equal to 2.9×10^6 mm^3, i.e. a 21.6% reduction. It indicates that the geometric parameters of this transmission were calculated more rationally while ensuring the requirements of design standards.

- The stress values such as σ_H , σ_{Hm} , σ_{Fm} increases closer to the allowable stress values respectively. This more demonstrates that the drive was calculation designed to be better and it avoids wasting materials.

5. Conclusions

From the discussions above demonstrates that the optimal methods will be brought better results than the traditional calculation. Besides, it also helps the designer to easily adjust the parameters, variables to suit the design requirements; it also helps to reduce computation times. However, a requirement set out for the Designers must have knowledge about the optimal methods for selecting the constraints, the consistent parameters... in the calculation process.

The problem can be extended to more than minimization of volume, it could be combined maximum of load. Other recently developed evolutionary algorithms can also be tried to solve this problem. A similar approach can be followed in case of other mechanical applications, such as minimization of weight of crane tower and minimization of weight of pulley system.

References

1. Golinski, J., (1970), “Optimal synthesis problems solved by Means of Nonlinear Programming and Random Method”, *Journal of Mechanisms*, Vol. 5, pp. 287-309.

2. Golinski, J., (1973), “An Adaptive Optimization System Applied to Machine Synthesis”, *Mechanism and Machine Theory*, Vol. 8, pp. 419-436.

3. H.-Z.Huang, Z.-G.Tian and M.J.Zuo, (2005), “Multi-objective optimization of three-stage spur gear reduction units using interactive physical programming”, *Journal of Mechanical Science and Technology*, Vol.19, no.5, pp. 1080–1086.

4. D. Jhalani and H. Chaudhary, (2012), “Optimal design of gearbox for application in knee mounted biomechanical energy harvester”, *International Journal of Scientic & Engineering Research*, Vol. 3, no. 10, pp. 1071–1075.

5. S. Padmanabhan, M. Chandrasekaran, and V. Srinivasa, (2013), “Design optimization of worm Gear drive,” *International Journal of Mining,* Metallurgy and Mechanical Engineering, Vol. 1, pp.57–61.

6. R. C. Sanghvi, A. S. Vashi, H. P. Patolia and G. Jivani, (2004), “Multi-Objective Optimization of Two-Stage Helical Gear Train Using NSGA-II”, *Journal of Optimization*, Vol. 2014.

7. V. N. Pi, (2003), “Optimal calculation of two-stage worm-gear reducer”, *Ministerial research project*.

8. V.N. Pi and V.Q. Dac, (2007), “Optimal calculation of partial transmission ratios of worm-helical gear reducers for minimal gearbox length”, *Journal of Science & Technology of Technical Universities*, no. 61, pp. 73-77.

9. P. Belotti et al., (2012), “Mixed-Integer Nonlinear Optimization”, Argonnenational Laboratory.

10. Beale, E. and Tomlin, J. (1970), “Special facilities in a general mathematical programming system for non-convex problems using ordered sets of variables”, *Proceedings of the 5th International Conference on Operations Research*, In Lawrence, J., editor, Venice, Italy, pp. 447–454.

11. Beale, E. M. L. and Forrest, J. J. H., (1976), “Global optimization using special ordered sets”, *Mathematical Programming*, Vol.10, pp. 52–69.

12. Kannan, R. and Monma, C. (1978), “On the computational complexity of integer programming problems”, In Henn, R., Korte, B., and Oettli, W., editors, *Optimization and Operations Research*, Vol. 157 of Lecture Notes in Economics and Mathematical Systems, pages 161–172. Springer.

13. Jeroslow, R. G. (1973), “There cannot be any algorithm for integer programming with quadratic constraints”, *Operations Research*, Vol. 21, no. 1, pp. 221–224.

14. Erwin Kalvelage(2014), “Some MINLP solution algorithms”, http://www.amsterdamoptimization.com/pdf/minlp.pdf

15. A. H. Land and A. G. Doig, (1960), “An automatic method for solving discrete programming problems”, *Econometrical*, Vol. 28, no. 3, pp. 497-520.

16. T. Chat, L. V. Uyen, (2006), *Design calculation for mechanical drive systems*, Education publisher, Vol. 1, pp. 157-161.

17. https://www.gams.com/