CALIBRATING RESISTANCE FACTORS UNDER LOAD AND RESISTANCE FACTOR DESIGN METHOD (LRFD) USING MONTE - CARLO SIMULATION

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Summary: *Calibrating resistance factors is to ensure safety degrees for foundation structures with high load effects from a superstructure system with target reliability levels. There have been some reliability methods used to calibrate resistance factors under the Load and Resistance Factor Design method (LRFD), such as First Order Second Moment (FOSM) and First Order Reliability Method (FORM). In this paper, a Monte-Carlo simulation - based resistance factor calibration is proposed. These three reliability methods are used to calibrate resistance factors for 16 cases of calculation, which are based on a bored pile database covering various types of soil, resistance prediction methods and construction methods. Correlation analyses between calibrated resistance factors according to three reliability methods are also considered herein. Through obtained results, some extra findings are illustrated in this paper.*

Keywords: *LRFD; resistance factor calibration; monte-Carlo simulation; target reliability index.*

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1. Introduction

The allowable Stress Design (ASD), also called the Working Stress Design (WSD) method, has been used in civil engineering since the early 1800s. Under ASD, a design load, which consists of the actual forces applied to the piles, has to be less than the resistance divided by a single factor of safety. This method has several shortcomings, the most significant of which is that it does not provide a consistent framework for incorporating the individual sources of risk into the design. In fact, each component of the load and the resistance has a different level of variability and uncertainty.

In the 1950s, the demand for the more economical design of piles brought about the use of the Limit State Design (LSD) method. Two types of limit states are usually considered, Ultimate Limit State (ULS) and Serviceability Limit State (SLS). ULS pertains to structural safety and involves structural collapse or, in relation to piles, the ultimate bearing capacity of soils. SLS pertains to conditions, such as excessive deformations and settlements or deterioration of the structure that would affect the performance of the structure under expected working loads. The format of limit state design equations involves the application of partial factors to increase the loads and to decrease the resistances. This approach represents a fundamental improvement over the single factor of safety in ASD, because the partial factors are applied directly to the uncertain quantities of loads and resistances.

The partial factors were determined subjectively based on two criteria: (i) A larger partial factor should be applied to a more uncertain quantity; (ii) the partial factors should result in approximately the same dimensions as those from traditional practice. This approach did not satisfy one of the basic requirements of LSD because it is impossible to demonstrate the occurrence of each limit state [7]. The next logical step in LSD has been to apply probabilistic reliability analysis to establish the partial factors, in order to account for the uncertainty and variability for loads and resistances. One of the advantages of this approach is that all components of the structure, including the foundations, can be designed to a uniform level of safety. The LSD method based on the probabilistic reliability analysis has been used increasingly with a new name as the Load and Resistance Factor Design (LRFD) method, in which the partial factors applied to loads are termed load factors and those applied to resistances are resistance factors. Each resistance factor is the product of a calibration study in which a Limit State Function (LSF) is evaluated to predict a specific component of resistance to a specified target reliability level.

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As described by Withiam et al. [14], calibrating load factors and resistance factors can be carried out by the use of: (i) Judgment; (ii) fitting to other codes or past practice; (iii) reliability-based analysis; or (iv) a combination of approaches. Only the third approach, reliability-based analysis, satisfies the objective of LSD and establishes load and resistance factors to achieve a defined target reliability level. Several comprehensive works based on the reliability analysis were conducted for the resistance factor calibration, for example, FOSM was used by Barker et al. [6], FORM was used by Paikowsky et al. [13] and Monte-Carlo Simulation (MCS) was utilized in the works of Abu-Farsakh and Yu [1], Allen et al. [2] and Bach [5].

In this paper, resistance factors are calibrated according to different reliability-based methods, in which MCS is proposed. A database, which involves sixteen cases of calculation of axially loaded bored piles, is collected and computed under the ULS. Each case is represented by a soil type, a resistance prediction method and a construction method. The resistance factors are calibrated with specified target reliability levels. The correlations between resistance factors calibrated by proposed MCS with FOSM and FORM are presented and discussed.

2. Reliability - based design methods

According to Ayyub et al. [4], there are two primary approaches for the reliability-based design: (i) Load and resistance factor design (LRFD); and (ii) direct reliability-based design. The LRFD approach is called the level I reliability method. Level I reliability methods use Partial Safety Factors (PSF's) that are reliability based; but the methods do not require explicit use of the probabilistic description of the random variables. The direct reliability-based design approach can include level II and/or level III reliability methods. Level II reliability methods (e.g., FOSM, FORM) are based on the moments (mean and variance) of random variables and sometimes, with a linear approximation of non-linear limit state functions. Level III reliability methods such as numerical integration, MCS, use the complete probabilistic characteristics of the random variables.

3. Load and Resistance Factor Design (LRFD)

The LRFD states that a factored (reduced) resistance of a structural component is larger than a linear combination of factored (magnified) load effects as given by a following general format:

$$\phi R_n \geq \sum_{i=1}^{n} \gamma_i Q_{ni} \tag{1}$$

where ϕ is the resistance factor, R_n is the nominal (or prediction) resistance, γ_i is the load factor for the ith load component, and Q_{ni} is the nominal (or design) value for the ith load component. Generally, the higher the uncertainty associated with a load, the higher the corresponding load factor; and the higher the uncertainty associated with the resistance, the lower the corresponding resistance factor. These factors are calibrated using reliability methods based on the probabilistic characteristics of basic random variables for load effects and the resistance including statistical and prediction (or modelling) uncertainties.

4. Resistance factor calibration

4.1 Resistance factor calibration based on FOSM

Based on FOSM and assumed log-normal distributions for the resistance, Barker et al. [6] determined the resistance factor as:

$$\phi = \frac{\bar{\lambda}_R\left(\sum \gamma_i Q_{ni}\right)\sqrt{\dfrac{1+COV_Q^2}{1+COV_R^2}}}{\sum Q_{ni}\exp\{\beta_T\sqrt{\ln[(1+COV_R^2)(1+COV_Q^2)]}\}} \tag{2}$$

in which $\bar{\lambda}_R$ is the mean of resistance bias factor; COV_Q and COV_R are the coefficient of variation of the load and resistance bias factors, respectively; β_T is the target reliability index. When just dead and live loads are considered, Eq. 2 can be rewritten as:

$$\phi = \frac{\bar{\lambda}_R\left(\dfrac{\gamma_D Q_D}{Q_L}+\gamma_L\right)\sqrt{\dfrac{1+COV_{QD}^2+COV_{QL}^2}{1+COV_R^2}}}{\left(\dfrac{\bar{\lambda}_{QD} Q_D}{Q_L}+\bar{\lambda}_{QL}\right)\exp\{\beta_T\sqrt{\ln[(1+COV_R^2)(1+COV_{QD}^2+COV_{QL}^2)]}\}} \tag{3}$$

where γ_D and γ_L are the dead load and live load factors, respectively; Q_D/Q_L is the dead to live load ratio; $\overline{\lambda}_{QD}$ and $\overline{\lambda}_{QL}$ are the mean of dead load and live load bias factors, respectively. The dead to live load ratio varies with the span length of bridges. Hansell and Viest [9] determined these ratios indicated in Table 1 for the LRFD approach.

Table 1. *Relationship between dead load to live load ratio and span length*

Span length (m)	9	18	27	36	45	60	75
Ratio Q_D/Q_L	0.52	1.04	1.56	2.07	2.59	3.46	4.32

The actual loads transferred from the superstructure to the foundations are, by and large, unknown. The load uncertainties are taken, therefore, as those used for the superstructure analysis. The probabilistic characteristics of the dead load, Q_D, and live load, Q_L, are assumed to be those used by Nowak [11] with the following load factors and normal distributions shown in Table 2.

Table 2. *Load factors and probabilistic characteristics for dead and live load bias factors*

Type of load	Load factor, γ	Mean of bias factor, $\overline{\lambda}_Q$	Coefficient of variation, *COV*	Distribution
Dead load	1.25	1.05	0.10	Normal
Live load	1.75	1.15	0.20	Normal

Both $\overline{\lambda}_R$ and COV_R are computed through the theoretically predicted resistance, R_P and the measured nominal resistance R_M. The measured nominal resistance was defined as the load corresponding to a displacement that is equal to 5% diameter of bored piles or the plunging load in static load tests (O'Neill and Reese [12]), whichever comes first. Paikowsky et al. [13] evaluated that this criterion provides a reliable and simple failure interpretation. For the mean of resistance bias factor,

$$\overline{\lambda}_R = \frac{\sum_{i=1}^{N} \lambda_{Ri}}{N} \tag{4}$$

here $\lambda_{Ri} = R_{Mi}/R_{Pi}$, and N is the number of considered bored piles. The standard deviation of resistance bias factor is determined as:

$$\sigma_R = \sqrt{\frac{\sum_{i=1}^{N}(\lambda_{Ri} - \overline{\lambda}_R)^2}{N-1}} \tag{5}$$

Finally, the coefficient of variation of resistance bias factor is given:

$$COV_R = \frac{\sigma_R}{\overline{\lambda}_R} \tag{6}$$

4.2 Resistance factor calibration based on FORM

Based on the Hasofer and Lind [10] approach, the present studies using the FORM provide a means for calculating the partial safety factors, ϕ and γ_i, as indicated in Eq. 1 for a target reliability index β_T. In design practice, there are usually two types of limit state, which are ULS and SLS. Both types can be represented generally by the following limit state function:

$$g(X) = g(X_1, X_2, \ldots, X_n) \tag{7}$$

in which X is a vector of basic random variables, X_i, for the resistance and loads. The limit state is defined when g(X) = 0 and therefore, failure occurs as g(X) < 0. The target reliability index, β_T, is defined as the shortest distance from the origin of the reduced coordinate system to the failure surface at the design point, x_i^* as indicated in Fig. 1.

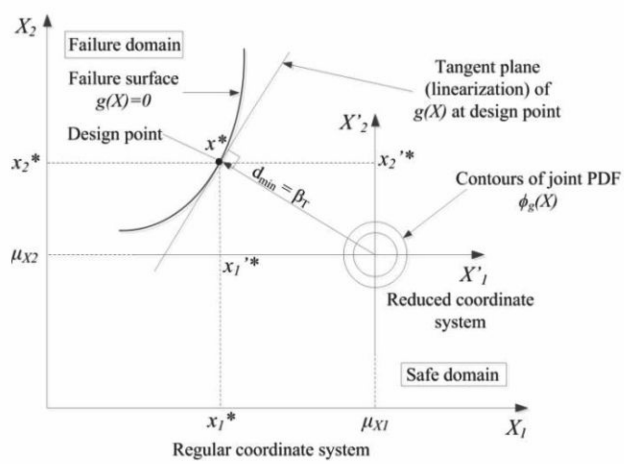

Figure 1. *Determination scheme of partial safety factors*

The most general format of the calibration process is to apply the partial safety factors, $\overline{\gamma}_i$, to the basic variables, these factors may be apply to the respective mean values of the basic variables (Ang and Tang [3]); thus,

$$g(\overline{\gamma_1}\mu_{X_1}, \overline{\gamma_2}\mu_{X_2}, \ldots, \overline{\gamma_n}\mu_{X_n}) = 0 \tag{8}$$

From Eq. 8, $\overline{\gamma}_i\mu_{Xi}$ should be on the failure surface; in particular, it may be at the design point. Hence, the required partial safety factors are:

$$\overline{\gamma_i} = \frac{x_i^*}{\mu_{X_i}} \tag{9}$$

Therefore, the determination of the required safety factors is also a problem of determining the design poin x_i^*. The computational steps to determine the safety factors for a specified target reliability index, β_T, are as follows:

- Step 1: In the regular coordinate system, assume a design point, x_i^*, and in the reduced coordinate system, obtain corresponding point, $x_i^{'*}$, using a transformation:

$$x_i^{'*} = \frac{x_i^* - \mu_{X_i}}{\sigma_{X_i}} \tag{10}$$

where μ_{Xi} and σ_{Xi} are the mean value and the standard deviation of basic random variable X_i, respectively. The mean value of the vector of basic random variables is often used as an initial value for the design point.

- Step 2: If the distribution of basic random variables is non-normal, approximate this distribution with an equivalent normal distribution at the design point, having the same tail area and ordinate of the probability density function, that is an equivalent mean,

$$\mu_{X_i}^N = x_i^* - \Phi^{-1}[F_{X_i}(x_i^*)]\sigma_{X_i}^N \tag{11}$$

and an equivalent standard deviation,

$$\sigma_{X_i}^N = \frac{\phi\{\Phi^{-1}[F_{X_i}(x_i^*)]\}}{f_{X_i}(x_i^*)} \tag{12}$$

where $\mu_{X_i}^N$ and $\sigma_{X_i}^N$ are the mean and standard deviation of the equivalent normal distribution for variable X_i, respectively. $F_{xi}(x_i^*)$ and $f_{xi}(x_i^*)$ are the original cumulative distribution function (CDF) and original probability density function (PDF) of variable, X_i, evaluated at the design point x_i^*, respectively. Φ and ϕ are the CDF and PDF of the standard normal distribution, respectively.

-Step 3: Set $x_i^{'*} = -\alpha_i^* \beta_i$ in which α_i^* is the direction cosine determined as:

$$\alpha_i^* = \frac{\left(\frac{\partial g}{\partial X_i'}\right)_*}{\sqrt{\Sigma\left(\frac{\partial g}{\partial X_i'}\right)_*^2}} \tag{13}$$

Where,

$$\left(\frac{\partial g}{\partial X_i'}\right)_* = \left(\frac{\partial g}{\partial X_i}\right)_* \sigma_{X_i}^N \tag{14}$$

- Step 4: A new design point obtained as

$$x_i^* = \mu_{X_i}^N - \alpha_i^* \beta_T \sigma_{X_i}^N \tag{15}$$

In general, the determination of x_i^* requires an iterative solution. Steps 1 to 3 are repeated until convergence of α_i^* is achieved. Then, through Eq. 9, the resistance factor, $\overline{\phi}$, and the load factors, $\overline{\gamma}_i$, are obtained:

$$\overline{\phi} = \frac{r^*}{\mu_R} \tag{16}$$

and,

$$\overline{\gamma}_i = \frac{q_i^*}{\mu_{Q_i}} \tag{17}$$

here r* and q_i^* are the design points of resistance and loads, respectively. μ_R and μ_{Qi} are the mean values of resistance and loads, respectively. The resistance factor is generally less than one, whereas the load factors are greater than one.

As specified by Paikowsky et al. [13], for a given target reliability index and probability distributions for resistance and loads, the partial safety factors determined by the FORM approach may differ with failure mode. For this reason, the calibration of the partial safety factors is to maintain the same values for all loads at different failure modes. In the case of geotechnical codes, the resistance factor calibration is performed for a set of load factors already specified in the structural code. Thus, the load factors are fixed, the following algorithm is used to determine the resistance factor only:

- For a given target reliability index, probability distributions and statistical parameters of load and resistance variables, compute mean resistance using FORM.

- With the mean value of resistance computed above, the resistance factor, Φ can be revised for a given set of load factors:

$$\phi = \frac{\sum_{i=1}^{n} \frac{\gamma_i q_i^*}{\bar{\lambda}_{Qi}}}{\frac{r^*}{\bar{\lambda}_R}} \tag{18}$$

here $\bar{\lambda}_{Qi}$ is the mean of load bias factor for the ith load.

4.3 Resistance factor calibration based on MCS (proposed)

This paper follows the calibration procedure based on MCS as recommended by Allen et al. [2] to determine the resistance factor of bored piles. The dead and live loads are considered abiding the ultimate limit state. Thus, the limit state function can be written as:

$$g = R_M - Q_{MD} - Q_{ML} \tag{19}$$

in which R_M, Q_{MD}, and Q_{ML} are the measured nominal resistance, the measured dead load and the measured live load, respectively. If all measured terms in Eq. 19 are converted to predicted terms using bias factors, λ_R, λ_{QD} and λ_{QL} for the resistance, dead load and live load, respectively, Eq. 19 can be rewritten:

$$g = R_P\lambda_R - Q_D\lambda_{QD} - Q_L\lambda_{QL} \tag{20}$$

Combine Eq. 1 into Eq. 20, after several transformations, the limit state function can be given:

$$g = Q_L\left(\frac{\gamma_D \frac{Q_D}{Q_L} + \gamma_L}{\phi}\lambda_R - \lambda_{QD}\frac{Q_D}{Q_L} - \lambda_{QL}\right) \tag{21}$$

where all terms in Eq. 21 are the same as those aforementioned in Section 4.2.

There are three random variables which are resistance, dead load and live load bias factors. The computation steps are as follows:

- Step 1: Assign a target reliability index, β_T

- Step 2: Select a trial resistance factor, ϕ

- Step 3: Generate random numbers for each set of bias factors λ_R, λ_{QD} and λ_{QL}

- Step 4: define the limit state function g as described in Eq. 21. Find the number of cases in which $g \le 0$. The probability of failure is then computed as:

$$P_f = \frac{count(g \le 0)}{N_S} \tag{22}$$

where N_S is the number of simulations. The corresponding calculated reliability index, β, is then defined:

$$\beta = -\Phi^{-1}(P_f) \tag{23}$$

where Φ^{-1} is the inverse CDF of the standard normal distribution. If the calculated reliability index, β, is different from the specified target reliability index, β_T, the trial resistance factor, ϕ, in Step 2 should be changed and a new iteration needs to be repeated until $|\beta-\beta_T| \le$ tolerance.

5. A case study

5.1 Database for calibration

A database for axially loaded bored piles was collected from Report NCHRP 507 [13]. Sixteen cases of calculation consisting of the number of considered pile cases, soil types, calculation methods and construction methods are categorized in Table 3.

In order to calibrate resistance factors, statistical parameters and probability distributions for the resistance, dead load and live load bias factors have to be determined. As mentioned in Section 4.1, the statistical parameters and probability distributions for the dead load and live load bias factors were already estimated by Nowak [11] and shown in Table 2. Therefore, the next section will focus on the determination of probabilistic characteristics for the resistance bias factors. To avoid redundancy hereafter, cases of calculation will be denoted briefly by a group of words. For example, "Sand-RW-Slurry", that is, the soil type is the sand, the resistance prediction method is the Reese and Wright method, and the construction method is the slurry method.

Table 3. *Calculation cases for bored piles*

No.	No. of piles	Soil type	Prediction method	Construction method
1	12	Sand	FHWA	Casing
2	9	-	-	Slurry
3	12	-	Reese & Wright (1977)	Casing
4	9	-	-	Slurry
5	13	Clay	FHWA	Casing
6	36	-	-	Dry
7	21	Sand+Clay	FHWA	Casing
8	11	-	-	Dry
9	9	-	-	Slurry
10	21	-	Reese & Wright (1977)	Casing
11	11	-	-	Dry
12	9	-	-	Slurry
13	46	Rock	Carter & Kulhawy (1988)	Mixed
14	30	-	-	Dry
15	46	-	IGM	Mixed
16	30	-	-	Dry

5.2 Probabilistic characteristics for resistance bias factors

Based on the database, apply Eqs. 4, 5 and 6 to calculate the mean, standard deviation and coefficient of variation of resistance bias factors, respectively. Assume that the probability distribution of the resistance bias factors is log-normal. The Kolmogorov-Smirnov test (K-S test) will be used to verify the fitness of the empirical cumulative distribution function against the fitted cumulative distribution function. The K-S test will be satisfactory as the adjusted K-S test statistic is less than the critical value:

$$\left(\sqrt{N} + 0.12 + \frac{0.11}{\sqrt{N}}\right) D_N \leq CV \tag{24}$$

where N is the number of considered resistance bias factors; it is also the number of considered piles in each calibration case. D_N is the K-S test statistic. CV is the critical value which has a value of 1.358, corresponding to a significance level of 5%. For a case of Rock-IGM-Mixed the determined adjusted K-S test statistic is 0.562, much less than 1.358, and satisfies the requirement of the K-S test. Hence, the log-normal distribution attributed to the resistance bias factors is acceptable. The results for the remaining fifteen cases of calculation are also satisfactory with the K-S test.

5.3 Resistance factor calibration

The calibration is performed for sixteen cases of calculation according to three reliability methods as FOSM, FORM and MCS. In order to evaluate the variability (sensitivity) of the resistance factor, target reliability indices are assigned as 2.0, 2.5, 3.0 and 3.5; which correspond to the target probabilities of failure as 0.0228, 0.0062, 0.0013, and 0.0002; a range of the dead load to live load ratios, Q_D/Q_L, is taken from 1.04 to 4.32; which correspond to the bridge span lengths varying from 18 to 75 m (see Table 1). The statistical parameters for the dead load and live load bias factors are shown in Table 2, the probability distribution for both is normal. The probability distribution for the resistance bias factors is log-normal. For the purpose of demonstration and comparison, the case of Sand+Clay-RW-Slurry is selected. The results are shown in Figs. 2, 3 and 4 according to the used reliability methods as FOSM, FORM, and MCS, respectively.

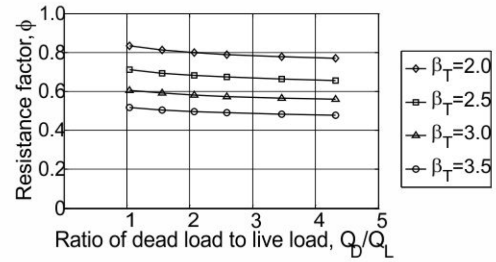

Figure 2. *Calibrated resistance factors using FOSM*

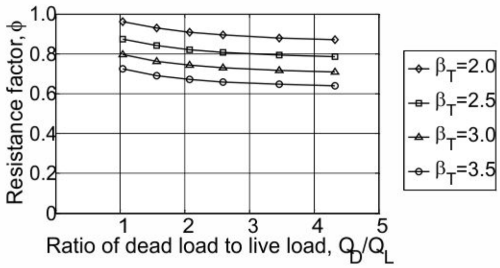

Figure 3. *Calibrated resistance factors using FORM*

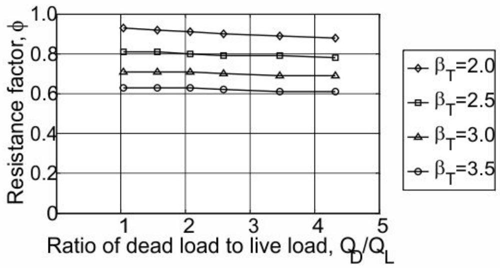

Figure 4. *Calibrated resistance factors using MCS*

Table 4. *Calibrated resistance factors for target reliability indices, β_T = 2.5 and 3.0*

		FOSM		FORM		MCS	
No.	**Calculation case**	β_T=2.5	β_T=3.0	β_T=2.5	β_T=3.0	β_T=2.5	β_T=3.0
1	Sand-FHWA-Casing	0.82	0.65	0.93	0.76	0.89	0.71
2	Sand-FHWA-Slurry	0.33	0.23	0.36	0.26	0.34	0.25
3	Sand-RW-Casing	0.44	0.33	0.49	0.38	0.47	0.36
4	Sand-RW-Slurry	0.21	0.15	0.23	0.17	0.22	0.16
5	Clay-FHWA-Casing	0.28	0.22	0.31	0.25	0.30	0.24
6	Clay-FHWA-Dry	0.34	0.27	0.39	0.33	0.38	0.31
7	Sand+Clay-FHWA-Casing	0.53	0.45	0.62	0.55	0.61	0.52
8	Sand+Clay-FHWA-Dry	0.70	0.59	0.82	0.73	0.79	0.69
9	Sand+Clay-FHWA-Slurry	0.70	0.59	0.83	0.74	0.80	0.70
10	Sand+Clay-RW-Casing	0.43	0.35	0.50	0.42	0.48	0.40
11	Sand+Clay-RW-Dry	0.60	0.50	0.69	0.60	0.67	0.58
12	Sand+Clay-RW-Slurry	0.66	0.57	0.79	0.72	0.78	0.68
13	Rock-CK-Mixed	0.48	0.38	0.54	0.45	0.53	0.43
14	Rock-CK-Dry	0.50	0.39	0.56	0.46	0.54	0.44
15	Rock-IGM-Mixed	0.60	0.49	0.69	0.59	0.67	0.56
16	Rock-IGM-Dry	0.64	0.52	0.73	0.63	0.71	0.60

Based on the graphs, several general observations can be given:

- The greater the target reliability index, the smaller the resistance factor.

- For a target reliability index, the value of resistance factors gradually decreases with the increase of the dead load to live load ratio and reaches a stable value when this ratio is larger than 3.0.

One issue arising herein is, which target reliability index will be selected for the practical designs. Based on the review of the studies, the survey of common practice and the evaluation of several authors, Paikowsky et al. [13] recommended the use of the target reliability indices for single pile designs as follows:

- For redundant piles, defined as 5 or more piles per pile cap, the recommended probability of failure is 0.0099, corresponding to a target reliability index of 2.33.

- For non-redundant piles, defined as 4 or fewer piles per pile cap, the recommended probability of failure is 0.0013, corresponding to a target reliability index of 3.0.

Hence, the values of the calibrated resistance factors for all sixteen cases of calculation shown in Table 4 are taken for the target reliability indices as 2.5 and 3.0; and the dead load to live load ratio is selected herein as 3.46 (i.e., greater than 3.0).

6. Correlation analyses

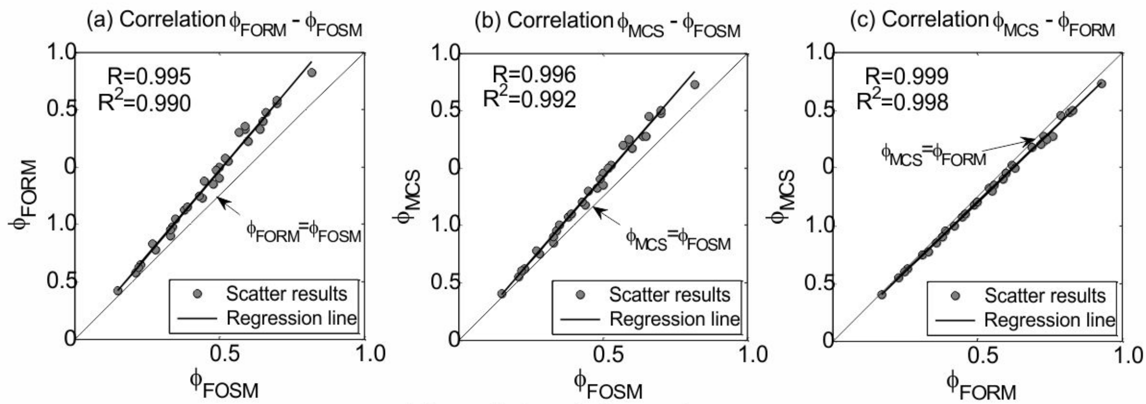

Figure 5. *Correlation analyses*

The correlations between the resistance factors obtained by the different reliability methods need to be checked. These correlations are quantified and expressed in terms of correlation coefficients. The values of ϕ using FOSM, FORM and MCS are taken from Table 4 with the target reliability indices of 2.5 and 3.0. The correlation analysis results are shown in Fig. 5. Based on the results described in Fig. 5, the correlation coefficients between the resistance factors for couples of FORM-FOSM, MCS-FOSM and MCS-FORM are R=0.995, 0.996, and 0.999, respectively. These three reliability methods produce the resistance factors which have good correlations. However, the resistance factors obtained from MCS and FORM have a somewhat better correlation than those between FORM and FOSM or between MCS and FOSM.

7. Conclusions

Some conclusions are drawn as follows:

- A greater target reliability index will result in a smaller resistance factor.

- The value of the resistance factor decreases gradually with the increase of the dead to live load ratio and reaches a stable value when this ratio is larger than 3.0.

- The resistance factors obtained from MCS and FORM have a somewhat better correlation than those between FORM and FOSM, or between MCS and FOSM.

- MCS as proposed in this paper is a good and effective tool to calibrate resistance factors under the LRFD approach.

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