FLEXURAL-TORSIONAL BUCKLING OF MONO-SYMMETRICAL I-SECTION BEAM WITH HOLLOW FLANGE

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***Summary**: This paper presents formulae and procedure to determine geometrical properties of a mono-symmetrical I-section beam with hollow flange; this is an opened-closed section. Then the expressions of flexural-torsional buckling can be applied to obtain critical moments. The comparison of critical moments between the symmetrical I-section and the mono-symmetrical I-section shows the efficiency of the hollow flange.*

***Keywords**: Flexural-torsional buckling; opened-closed section; shear center; sectorial coordinates; warping constant; torsion constant; uniform and non-uniform torsion.*

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1. Introduction

The history of the theories of flexural-torsional buckling is already more than 100 years. The first authors are probably Prandtl and Michell who published in 1899 the research on the flexural-torsional buckling of simply supported beam of narrow rectangular section, which is subjected to the uniform moment. After that, in 1905, Timoshenko established the governing equation and solved the buckling problem of the I-section beam subjected to the uniform moment. Timoshenko continued to develop the theory of elastic stability; he presented his research in a book written together with Gere [1]. In 1930s, Wagner developed the theory of elastic stability of the mono-symmetrical I-section beam and introduced a formula to calculate the factor which presents the mono-symmetry of the section. In 1940s, Vlasov [2] developed a general theory of thin-walled members in which there is the theory of flexural-torsional buckling of the beam. Vlasov is the first author presenting the concept of the warping torsion, sectorial coordinates and warping constant of the thin-walled section. Bleich [3] used the energy method in which the total potential energy of the beam transversally loaded is equal to the sum of the internal energy and the external work done by the load on the buckling displacements. Timoshenko and Gere [1] established governing equations of the buckling when they considered the balance of an infinitesimal element in conjunction with the balance of a part of the beam.

After the above authors, many researchers investigated the buckling of the beam on both sides theoretical and experimental. Anderson and Trahair [4], Attard and Bradford [5] did experiences on the cantilever of mono-symmetrical I-section. Anderson and Trahair [4], Assadi and Roeder [6], Ings and Trahair [7] studied the influence of the loading position from experiences on the beam of doubly symmetrical I-section. Trahair [8] established a formula calculating the normal strain which includes linear and nonlinear components. From that, the theory of buckling beam is established based on the total potential energy which is equal to the sum of the internal linear energy, the internal nonlinear energy done by normal stresses and the work done by the external load on second order displacements when the beam is buckled. Zhang and Tong [9] constructed a new theory based on the variational principle and the theory of thin shell. However, Bui [10] proved that the theory of buckling beam proposed by Zhang and Tong coincides to the classical theory in almost cases. Among others, Djalaly [11] proposed successfully a general formula to calculate the critical moment of flexural-torsional buckling. This formula is adopted by Eurocode 3 and written in Annex F [12] as follow:

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$$M_{cr} = C_1 \frac{\pi^2 EI_y}{(kL)^2}\left[\sqrt{\left(\frac{k}{k_\omega}\right)^2 \frac{I_\omega}{I_y} + \frac{(kL)^2 GI_t}{\pi^2 EI_y} + (C_2 y_g - C_3 y_j)^2} - (C_2 y_g - C_3 y_j)\right] \quad (1)$$

where C_1, C_2, C_3 are factors depending of loading and restraint conditions; k_ω and k are factors of effective length concerning flexural and torsional restraints at the ends of the beam; y_g is height of load which is the distance between the applied point of load and the shear center;

$$y_g = y_a - y_s \quad (2)$$

where y_a is coordinate of the applied point of transversal load; y_s is coordinate of the shear center with respect to the gravity center.

$$y_j = y_s - \frac{1}{2I_x}\int_F y\left(x^2 + y^2\right)dA \quad (3)$$

The coordinates y are positive when they are measured from the shear center to the compressed flange.

We approached the question of buckling member when studying the theory of thin-walled structure. A more complete expression of total potential energy in the buckling analysis of beam is presented in [10]. Then, a beam-type finite element program was implemented [13]. This program can analyze the flexural-torsional buckling of isostatic and hyperstatic I-section beams. The beam section can be mono-symmetrical and we can consider a sophisticated static loading, that is, the arbitrary position of the load along the beam and the height of applied point of the load in comparison with the shear center of the section. Based on the procedure and formulae developed by Murray [14], a sub-program is written to calculate automatically geometrical properties of the section [15], the sub-program can determine the gravity and shear centers, the first and second moments, the sectorial coordinates, the warping and uniform torsion constants of an opened mono-symmetrical I-section. Recently, we have studied the uniform and non-uniform torsions of closed and opened-closed sections. The way to determine geometrical properties of closed and opened-closed sections was presented through examples of the study [16]. We would like to apply the results of the previous research to a case study, namely: flexural-torsional buckling of mono-symmetrical I-section beam with hollow flange. If the beam is designed in symmetrical I-section (Figure 1a), it is unconservative under the verification of flexural-torsional buckling. We can weld two inclined stiffeners on the compressed part of the beam to form a hollow flange as indicated in Figure 1b. This solution aims to increase the flexural and torsional rigidities of the compressed part and of the section as suggested in some books of steel structures [17,18].

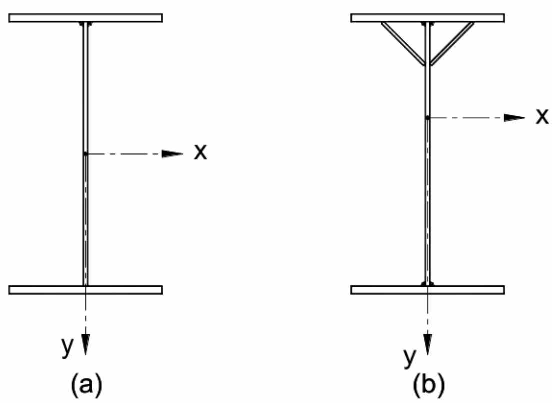


Figure 1. *I-section with hollow flange*

2. Application of the theories of torsion in the calculation of section properties

a) Theory of uniform torsion

Saint Venant's theory of uniform torsion was presented in [14] for the determination of the torsion constant I_t of the section.

For a section with some closed parts in Figure 2, we can establish an equation for each closed part:

$$\frac{1}{2A_i}\left[-\psi_{i-1}\int_A^D \frac{ds}{t} + \psi_i \oint \frac{ds}{t} - \psi_{i+1}\int_B^C \frac{ds}{t}\right] = 1 \quad (4)$$

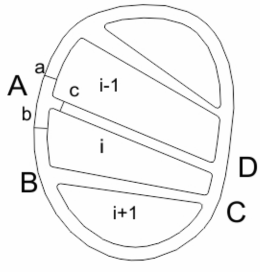


Figure 2. *Section of closed parts*

in which A_i is the area enclosed by the part i, t is the thickness of the considered segment.

Hence, a system of equations is obtained for all closed parts of the section. Resolving these equations, we obtain the value of ψ_i and the torsion constant is calculated: $I_t = 2\sum \psi_i A_i$ (5)

When the section has only a closed part: $I_t = \dfrac{4A^2}{\oint \frac{ds}{t}}$ (6)

The demonstration of Equations (4)÷(6) can be found in [14,16].

If the section includes closed and opened parts, we make a sum:

$$I_t = \left(\sum I_t\right)_{closed} + \left(\sum I_t\right)_{opemed} = 2\sum \psi_i A_i + \frac{1}{3}\sum l_k t_k^3 \quad (7)$$

b) Theories of non-uniform torsion

Vlasov's theory is applied for opened section while Karman and Christensen's theory is used to closed section. These theories were presented in [14] for the determination of sectorial coordinates ω.

For opened parts of the section: $\omega_B(s) = \int_0^s \rho_B(s) ds$ (8)

For closed parts of the section: $\omega_B(s) = \int_0^s \left[\rho_B(s) - \frac{\psi_i}{t}\right] ds$ (9)

where $\rho_B(s)$ is the sectorial radius corresponding with the pole B.

The derivation of Equations (8) and (9) can be found in [14,16].

Once having the sectorial coordinates, the sectorial properties of the section can be calculated, namely: the shear center and the warping constant I_ω.

3. Geometrical properties of thin-walled opened-closed section

In a study [16], we based on the formulae developed by Murray [14] to present a procedure determining geometrical properties of a thin-walled section. This procedure can be utilized for both opened and closed sections.

Considering an opened-closed section as indicated in Figure 3, there are 3 systems of coordinates in use:

i) Initial axes $\bar{x}, \bar{y}$ with the origin A; the sectorial pole B and starting point V are arbitrarily chosen. Of course, we have:

$\int_F \bar{\omega}_B \, dF = S_{\bar{\omega}_B} \neq 0$.

ii) Intermediate axes x*, y* with the origin C being gravity center of the section; the x*, y* are parallel to the initial $\bar{x}, \bar{y}$, respectively. The sectorial pole B is preserved the initial location but the starting point V is chosen so that: $\int_F \omega_B^* dF = S_{\omega_B^*} = 0$.

iii) Principal axes x, y with the origin C; the axes x, y are rotated an angle ψ from the x*, y* and the sectorial pole B is moved to the location of the principal sectorial pole M.

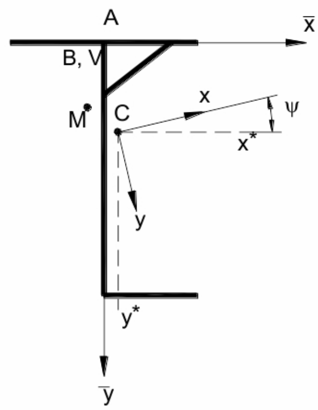


***Figure 3.** An opened-closed section*

The steps of calculation can be performed as follow:

a) In the initial axes $\bar{x}, \bar{y}$, calculating respectively geometrical properties of the section:

$$S_{\bar{x}} = \int_F \bar{y} dF \quad S_{\bar{y}} = \int_F \bar{x} dF \quad I_{\bar{x}} = \int_F \bar{y}^2 dF \quad I_{\bar{y}} = \int_F \bar{x}^2 dF \quad I_{\bar{x}\bar{y}} = \int_F \bar{x}\bar{y} dF \quad (10)$$

With the sectorial pole B and starting point V, drawing the diagram of sectorial coordinates $\bar{\omega}$ and determining sectorial properties of the section:

$$S_{\bar{\omega}_B} = S_{\bar{\omega}} = \int_F \bar{\omega} dF \quad I_{\bar{\omega}_B \bar{x}} = I_{\bar{\omega}\bar{x}} = \int_F \bar{\omega}\bar{y} dF \quad I_{\bar{\omega}_B \bar{y}} = I_{\bar{\omega}\bar{y}} = \int_F \bar{\omega}\bar{x} dF \quad I_{\bar{\omega}_B \bar{\omega}_B} = I_{\bar{\omega}\bar{\omega}} = \int_F \bar{\omega}\bar{\omega} dF \quad (11)$$

The coordinates of gravity center C in the axes $\bar{x}, \bar{y}$ are specified according to the condition:

$$S_{x^*} = S_{y^*} = 0 \quad (12)$$

From that: $\bar{x}_C = \dfrac{S_{\bar{y}}}{F} \quad \bar{y}_C = \dfrac{S_{\bar{x}}}{F}$ (13)

These are well-known formulae determining the gravity center C of any section.

b) The intermediate axe x*, y* are obtained by translating the initial axes $\bar{x}, \bar{y}$ from the origin A to the origin C. The coordinates of any point in the axes x*, y* can be specified:

$$x^* = \bar{x} - \bar{x}_C \qquad y^* = \bar{y} - \bar{y}_C \tag{14}$$

The initial location of the sectorial pole B is preserved but the starting point V is moved to V* so that:

$S_{\omega_B^*} = \int_F \omega^* dF = 0$, which can be realized if we chose: $\bar{\omega}_o = \dfrac{S_{\bar{\omega}_B}}{F}$ (15)

And the sectorial coordinates are calculated according to the pole B and starting point V*:

$$\omega_B^* = \bar{\omega}_B - \bar{\omega}_o = \bar{\omega}_B - \frac{S_{\bar{\omega}_B}}{F} \tag{16}$$

Evidently: $S_{\omega_B^*} = \int_F \omega_B^* dF = \int_F \left(\bar{\omega}_B - \frac{S_{\bar{\omega}_B}}{F}\right) dF = 0$

The formula (15) is of the same meaning with the formulae (13).

The geometrical properties in the axes x*, y* are now determined:

$$I_{x^*y^*} = \int_F (\bar{x} - \bar{x}_C)(\bar{y} - \bar{y}_C) dF = I_{\bar{x}\bar{y}} - \bar{x}_C(S_{\bar{x}} - \bar{y}_C F) - \bar{y}_C(S_{\bar{y}} - \bar{x}_C F) - \bar{x}_C \bar{y}_C F = I_{\bar{x}\bar{y}} - \frac{S_{\bar{x}} S_{\bar{y}}}{F} \tag{17}$$

Identically, we have:

$$I_{\omega^*x^*} = I_{\bar{\omega}\bar{x}} - \frac{S_{\bar{\omega}} S_{\bar{x}}}{F} \quad I_{\omega^*y^*} = I_{\bar{\omega}\bar{y}} - \frac{S_{\bar{\omega}} S_{\bar{y}}}{F} \quad I_{x^*} = I_{\bar{x}} - \frac{S_{\bar{x}}^2}{F} \quad I_{y^*} = I_{\bar{y}} - \frac{S_{\bar{y}}^2}{F} \quad I_{\omega^*} = I_{\bar{\omega}} - \frac{S_{\bar{\omega}}^2}{F} \tag{18}$$

c) The principal axes x, y are received by rotating the intermediate axes x*, y* an angle ψ so that:

$$I_{xy} = \int_F xy dF = 0 \tag{19}$$

The relation between the coordinates x, y and x*, y* of any point is expressed:

$$x = x^* \cos\psi - y^* \sin\psi \qquad y = -x^* \sin\psi + y^* \cos\psi \tag{20}$$

Using the condition (19) to get the angle : $\tan 2\psi = \dfrac{2I_{x^*y^*}}{-I_{x^*} + I_{y^*}}$ (21)

The angle ψ is positive when the rotation is clockwise.

Now, the shear center M can be located by annulling sectorial products of area:

$$I_{\omega_M x} = \int_F \omega_M y dF = 0 \qquad I_{\omega_M y} = \int_F \omega_M x dF = 0 \tag{22}$$

In use of (22), the coordinates of the shear center M can be calculated from the initial pole B in the x*, y* system:

$$x_M^* - x_B^* = \frac{I_{\varpi_B^* x^*} I_{y^*} - I_{\varpi_B^* y^*} I_{x^*y^*}}{I_{x^*} I_{y^*} - I_{x^*y^*}^2} \qquad y_M^* - y_B^* = \frac{I_{\varpi_B^* x^*} I_{x^*y^*} - I_{\varpi_B^* y^*} I_{x^*}}{I_{x^*} I_{y^*} - I_{x^*y^*}^2} \tag{23}$$

When transforming from the pole B to the principal pole M, the sectorial coordinate changes:

$$\omega_M = \omega_B^* - (x_M^* - x_B^*) y^* + (y_M^* - y_B^*) x^* \tag{24}$$

Coordinating with (22), it can be seen that the sectorial coordinate ω_M above is principal because:

$$S_{\omega_M} = \int_F \omega_M dF = \int_F \left[\omega_B^* - (x_M^* - x_B^*) y^* + (y_M^* - y_B^*) x^*\right] dF = S_{\omega_B^*} - (x_M^* - x_B^*) S_{x^*} + (y_M^* - y_B^*) S_{y^*} = 0 \tag{25}$$

Finally, in the principal axes, the geometrical properties can be yielded:

$$F = \sum_k l_k t_k \tag{26}$$

$$I_x = \int_F y^2 dF = \int_F (-x^* \sin\psi + y^* \cos\psi)^2 dF = I_{y^*} \sin^2\psi + I_x \cos^2\psi - I_{x^*y^*} \sin 2\psi \tag{27}$$

$$I_y = \int_F x^2 dF = \int_F (x^* \cos\psi + y^* \sin\psi)^2 dF = I_{y^*} \cos^2\psi + I_x \sin^2\psi + I_{x^*y^*} \sin 2\psi \tag{28}$$

$$I_{\omega_M} = \int_F \omega_M^2 dF = \int_F \left[\omega_B^* - (x_M^* - x_B^*) y^* + (y_M^* - y_B^*) x^*\right]^2 dF = I_{\varpi_B^*} - (x_M^* - x_B^*) I_{\omega_B^* x^*} + (y_M^* - y_B^*) I_{\omega_B^* y^*} \tag{29}$$

4. Flexural-torsional buckling of mono-symmetrical I-section beam with hollow flange

In this section, we consider a case study: a beam of I-section which has the dimensions and section properties shown in Figure 4a. When the beam is designed in symmetrical I-section, it is unconservative under the verification of flexural-torsional buckling. We can weld two inclined stiffeners on the compressed part of the beam to form hollow flange as indicated in Figure 4b. This solution aims to increase the flexural and torsional rigidities of the compressed part and of the section. All steps of the procedure presented in Sections 2 and 3 are carried-out to receive section properties. Then the expression of flexural-torsional buckling of beam (1) can be applied to obtain critical moments.

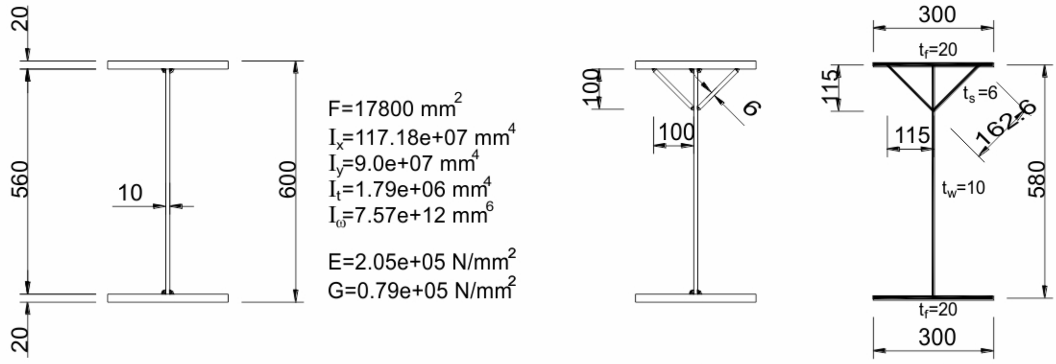


Figure 4. *Sections of the beam before and after stiffening*

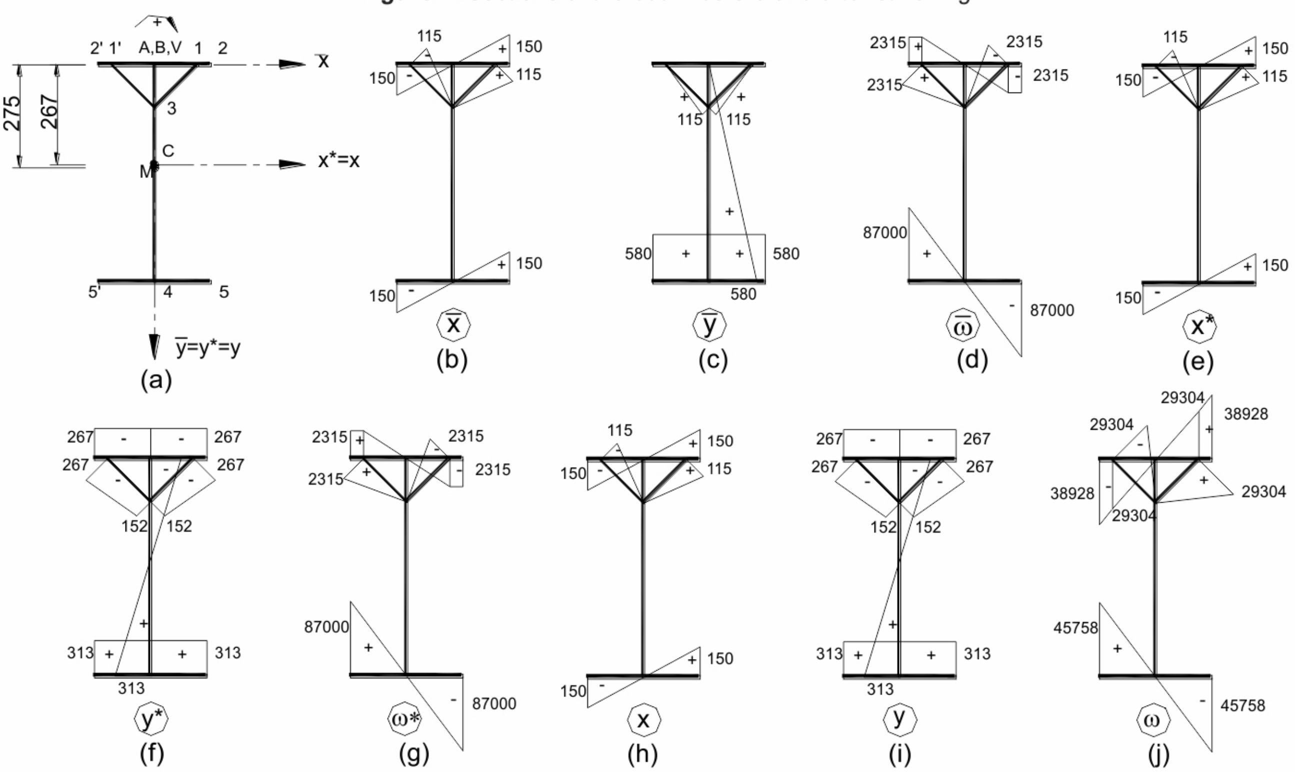


Figure 5. *Gravity center, shear center, Cartesian and sectorial coordinates of the section*

a) Section properties

The I-section in Figure 4b can be considered as a thin-walled section, that is, the section may be interpreted by median lines of segments (Figure 4c). The calculation of section properties is performed on this interpretation. It is noted that for the uniform torsion, Saint-Venant's theory is used while concerning the non-uniform torsion, Vlasov's theory is applied for the opened parts and Karman and Christensen's theory is used for the closed parts of the section. The presentation of these theories for the determination of sectorial properties can be found in [14,16].

The initial axes $\bar{x}, \bar{y}$ with the origin A, the sectorial pole B and the initial starting point V are chosen as indicated in Figure 5a. Firstly, we need to draw diagrams of Cartesian coordinates $\bar{x}, \bar{y}$ and sectorial coordinate $\bar{\omega}$. The diagram of $\bar{x}$ and $\bar{y}$ coordinates are shown in Figures 5b and 5c. The sectorial coordinates $\bar{\omega}$ are calculated below:

For each triangle hollow part of the section: $\oint\frac{ds}{t}=\frac{115}{20}+\frac{115}{10}+\frac{162.6}{6}=44.35$

For the segment between two hollow parts : $\int\frac{ds}{t}=\frac{115}{10}=11.5$

In use of Equation (4), equation for each hollow part is established.

For the hollow part in the left : $\frac{1}{2\times\frac{1}{2}\times115\times115}\left[44.35\psi_1-11.5\psi_2\right]=1$

For the hollow part in the right : $\frac{1}{2\times\frac{1}{2}\times115\times115}\left[-11.5\psi_1+44.35\psi_2\right]=1$

Resolving these equations, we have : $\psi_1=\psi_2=402.588$

From Equation (9), sectorial coordinates of closed parts are obtained:

From V to 1 : $\bar{\omega}_1=\int_0^{115}\left(0-\frac{402.588}{20}\right)ds=-2315$

From 1 to 3 : $\bar{\omega}_3=-2315+\int_0^{162.6}\left(81.32-\frac{402.588}{6}\right)ds=0$

From 3 to V: $\psi=\psi2-\psi1=0$ $\qquad \bar{\omega}_V=0+\int_0^{115}\left(0-\frac{0}{10}\right)ds=0$

From Equation (8), sectorial coordinates of opened parts are obtained:

From 1 to 2 : $\bar{\omega}_2=-2315+\int_0^{35}0ds=-2315$ $\qquad$ From 3 to 4 : $\bar{\omega}_4=0+\int_0^{465}0ds=0$

From 4 to 5 : $\bar{\omega}_5=0-\int_0^{150}580ds=-87000$

The $\bar{\omega}$ diagram Figure 5d is antisymmetrical about the vertical centerline. The formulae of numerical integrals presented in [14] can be used to calculate the section properties (Equations (10) and (11)).

$$F=19751.2mm^2 \quad S_{\bar{x}}=5274194mm^3 \quad S_{\bar{y}}=0mm^3 \quad S_{\bar{\omega}}=0mm^4 \quad I_{\bar{x}\bar{y}}=0mm^4$$

$$I_{\bar{x}}=267.737487\times10^7mm^4 \quad I_{\bar{y}}=9.860154\times10^7mm^4 \quad I_{\bar{\omega}\bar{x}}=0mm^5$$

$$I_{\bar{\omega}\bar{y}}=-27.110797\times10^9mm^5 \quad I_{\bar{\omega}}=15.157206\times10^{12}mm^6$$

The coordinates of the gravity center are specified by Equation (13):

$$\bar{x}_C=\frac{0}{19751.2}=0mm \qquad \bar{y}_C=\frac{5274194}{19751.2}=267mm$$

From Equation (15): $\bar{\omega}_o=\frac{0}{19751.2}=0$

It signifies that the starting point V* coincides to the initial V: $\omega_B^*=\bar{\omega}_B-\bar{\omega}_o=\bar{\omega}_B$

From the transformation formulae (17) and (18):

$$I_{x^*y^*}=0-\frac{5274194\times0}{19751.2}=0 \qquad I_{\omega^*x^*}=0-\frac{0\times5274194}{19751.2}=0 \qquad I_{\omega^*y^*}=-27.110797\times10^9mm^5$$

$$I_{x^*}=126.899855\times10^7mm^4 \qquad I_{y^*}=9.860154\times10^7mm^4 \qquad I_{\omega^*}=15.157206\times10^{12}mm^6$$

The coordinates of the shear center M are found from Equation (23):

$$x_M^*-x_B^*=\frac{0\times9.860154\times10^7-(-27.110797\times10^9)\times0}{126.899855\times10^7\times9.860154\times10^7-0^2}=0$$

$$y_M^*-y_B^*=\frac{0\times0-(-27.110797\times10^9)\times126.899855\times10^7}{126.899855\times10^7\times9.860154\times10^7-0^2}=275mm$$

ω_B^* $(=\overline{\omega}_B)$ can be transformed to ω_M by using Equation (24): $\omega_M = \omega_B^* + 275x^*$

And this is plotted in Figure 5j.

The product section properties are found from Equations (27)÷(29), whence:

$$I_x = I_{x^*} = 126.899855\times10^7\,mm^4 \qquad I_y = I_{y^*} = 9.860154\times10^7\,mm^4 \qquad I_{\omega_M} = 7.703092\times10^{12}\,mm^6$$

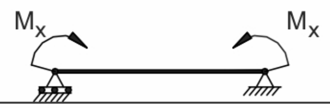
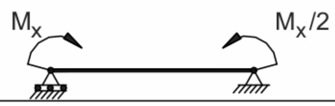
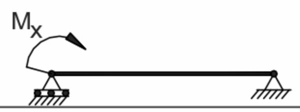
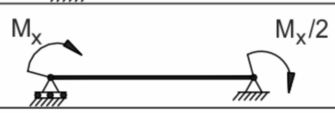
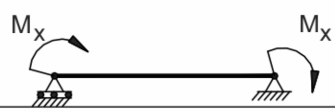
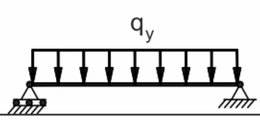
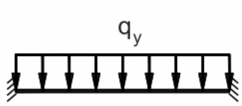
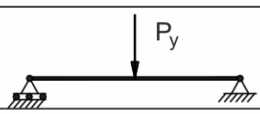
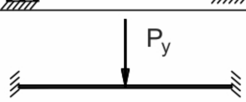
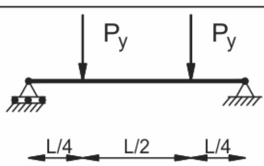
The torsion constant I_t is obtained from Equation (7):

$$I_t = 2\times2\times402.588\times\frac{1}{2}\times115\times115+\frac{1}{3}\times\left[2\times35\times20^3+465\times10^3+2\times150\times20^3\right]$$

$$=10.648453\times10^6+1.141667\times10^6=11.79012\times10^6\,mm^6$$

The last calculation shows that the torsion constant of closed parts is much more important than of opened parts (10.648453x10^6 mm^6 vs 1.141667x10^6 mm^6). The formation of closed parts improves much the torsion rigidity of the section. Comparing to the initial symmetrical I-section, the torsion constant I_t of the stiffened section increases to $\dfrac{11.79012\times10^6}{1.79\times10^6}=6.6$

Table 1. *Critical moment (k=1.0, k_w=1.0)*

Restraint and loading conditions	C_1	C_2	C_3	M_{cr1} Tm	M_{cr2} Tm	M_{cr2}/M_{cr1}
				Initial section	Section with hollow flange	
	1.0	-	1.0	73.2	141.5	1.93
	1.32	-	0.99	96.6	186.9	1.94
	1.88	-	0.94	137.6	266.7	1.94
	2.70	-	0.68	197.7	386.9	1.96
	2.75	-	0.00	201.3	404.7	2.01
	1.13	0.46	0.53	59.7	137.4	2.30
	1.28	1.56	0.75	35.6	105.7	2.97
	1.36	0.55	1.73	67.6	152.9	2.26
	1.56	1.27	2.64	50.3	132.6	2.64
	1.05	0.43	1.12	56.6	126.2	2.23

b) Critical moment

Using the formula (1), we can compare the critical moment of the beams of symmetrical section (Figure 4a) and mono-symmetrical section with hollow flange (Figure 4b) in Table 1. The length of the beam is 10m. It is noted that for the symmetrical section y_s=0, y_j=0, y_g=290mm while for the mono-symmetrical section y_s=267.03-274.95=-7.92mm, y_j=-28.8mm, y_g=275mm.

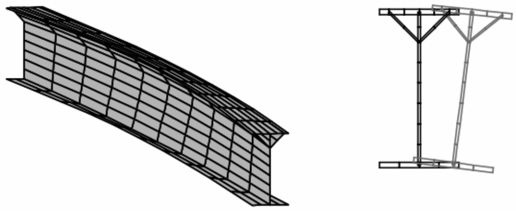


Figure 6. *Flexural-torsional buckling shape of the beam in uniform bending*

We can see in Table 1 that the critical moment of the mono-symmetrical section with hollow flange is greatly increased, from 1.93 to 2.97 in comparison to the initial symmetrical section while the area is only increased of $\frac{19751.2}{17800}=1.11$. The flexural-torsional buckling shape of the beam in uniform bending is presented in Figure 6.

5. Conclusions

The article presents concretely the formulae and procedure to determine the section properties of a mono-symmetrical I-section with hollow flange. The calculation of critical moment of the beam with various loading types shows the much more rigidity of the mono-symmetrical section with hollow flange. Moreover, the two inclined stiffeners welded on the compressed part of the I-section enhance also the local stability of the flange and web of the section. So, this technical solution is efficient and the formulation implemented in the article can be practically applied to the design of the beam subjected to the flexural-torsional buckling condition.

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