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DAMAGE IDENTIFICATION OF TRUSSES USING LIMITED MODAL FEATURES AND ENSEMBLE LEARNING

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Abstract

A damage diagnosis method for trusses based on incomplete free vibration properties utilizing ensemble learning, e.g. Extreme gradient boosting (XGBoost), is presented in this work. Owing to the lack of measurement sensors, modal features are only measured at master degrees of freedom (DOFs) of a few first models instead of all DOFs of a structural system. Accordingly, a modal strain energy-based index (*MSEBI*) is employed to determine the most potentially damaged candidates. Then, an XGBoost-driven ensemble learning model is constructed from a finite element method (FEM)-simulated dataset. In which, inputs are eigenvectors corresponding to master DOFs, whilst outputs are damage ratios of suspected members. The accuracy of such a model is continuously enhanced by removing low-risk members via a damage threshold. As a consequence, the present paradigm can reliably detect damage to trusses. All test examples are programmed in Python to illustrate the reliability and efficiency of the proposed methodology.

Keywords: damage diagnosis; free vibration; ensemble learning; extreme gradient boosting (XGBoost); modal strain energy-based index (*MSEBI*); Python.

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1. Introduction

In reality, there are many civil engineering structures to be partially damaged, or even completely collapsed that the major reasons can come from various impacts such as overloading, aging material, harsh environmental influence and poor maintenance, etc. Consequently, in order to detect the timely damage for extending the lifespan of those structures, the structural health monitoring (SHM) field has emerged and become one of the hot research topics attracted the remarkable attention of numerous scholars in the academic community owing to its practical applications. Following this trend, there have been a variety of approaches to be developed. Notably, it can be mentioned that the model updating-based method resolved based on inverse optimization problems via optimization algorithms. These approaches discover both the location and extent of damaged members by minimizing the difference between the mechanical features of a real damaged structure and those numerically modeled by FEM. For example, Seyedpoor [1] used particle swarm optimization (PSO) as an optimizer to find out damaged truss members based on free vibration properties. Lieu et al. [2, 3] employed an adaptive hybrid evolutionary firefly algorithm (AHEFA) for the truss damage detection using modal modes. Recently, Dang et al. [4] also applied the AHEFA for the damage diagnosis of truss structures utilizing time-series acceleration, etc.

Furthermore, along with the very fast development of computer and data science in recent decades, artificial intelligence (AI) has rapidly emerged and been widely applied to various areas, including

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computational mechanics for structural problems [5–12]. Among them, the AI applications to the SHM have made breakthroughs. For instance, Lee et al. [13] used deep neural network (DNN) as a surrogate model to predict the damage to truss structures based on free vibration features. Torzoni et al. [14] suggested a method as a combination of a fully convolutional network (FCN) with model order reduction (MOR) for SHM under environmental changes. Truong et al. [15] employed autoencoder-convolutional gated recurrent unit (A-CGRU) for detecting the damage to structural connection joints, and so on. In most of those studies, it can be found that a large dataset is required to build deep learning (DL) models, and the computational effort is therefore necessary.

From the above discussions, it seems apparent that there have been no reports concerning constructing DL models based on small or moderate data for the truss damage detection, especially using Extreme gradient boosting (XGBoost) [16] associated with the second-order Neumann series expansion-driven MOR technique [17]. In this study, unmeasured data due to limited sensors are inferred by the MOR strategy. Then, the *MSEBI* [1] is applied to remove low-risk members, aiming to reduce the number of outputs for building XGBoost models in the next step. The learning model is trained based on a small dataset created by FEM. In which, inputs are eigenvectors of the first five modal modes at DOFs recorded by sensors, while outputs are damage ratios of potential members. The accuracy of the XGBoost is improved by eliminating low-risk members by a threshold. After several iterations, the XGBoost-driven method can diagnose damaged members. The reliability of the present paradigm is verified through two test examples.

2. Theoretical formulation

2.1. Model order reduction for free vibration analysis

The finite element model for analyzing the free vibration behavior of a structure can be expressed as follows

$$\left(\mathbf{K} - \omega^2\mathbf{M}\right)\mathbf{\Phi} = \mathbf{0} \tag{1}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the global stiffness and lumped mass matrices, respectively; ω^2 is the diagonal matrix including the eigenvalues $\omega_1^2, \ \ldots, \omega_i^2, \ldots, \omega_{ndof}^2$, in which ω_i the ith natural frequency of the structure, and $ndof$ is the total of DOFs. $\mathbf{\Phi}$ stands for the structural eigenvectors or free vibration modes.

The above model can be rewritten as follows

$$\begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{ms} \\ \mathbf{K}_{sm} & \mathbf{K}_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{\Phi}_m \\ \mathbf{\Phi}_s \end{Bmatrix} = \omega^2 \begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{ms} \\ \mathbf{M}_{sm} & \mathbf{M}_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{\Phi}_m \\ \mathbf{\Phi}_s \end{Bmatrix} \tag{2}$$

where the subscript "m" is devoted to the master DOFs which are recorded by limited sensors, while "s" is dedicated to the slave DOFs which are not measured.

For reduced order models (ROMs), Eq. (2) is given as

$$\mathbf{K}_{MOR}\mathbf{\Phi}_m = \omega_{MOR}^2\mathbf{M}_{MOR}\mathbf{\Phi}_m \tag{3}$$

where ω_{MOR}^2 is the diagonal matrix including the eigenvalues squared in ROMs; $\mathbf{K}_{MOR}$ and $\mathbf{M}_{MOR}$ are the global stiffness and lumped mass matrices which are established by ROMs, respectively. More concretely, $\mathbf{K}_{MOR}$ and $\mathbf{M}_{MOR}$ are computed as follows

$$\mathbf{K}_{MOR} = \mathbf{R}^T\mathbf{K}\mathbf{R}, \quad \mathbf{M}_{MOR} = \mathbf{R}^T\mathbf{M}\mathbf{R} \tag{4}$$

where $\mathbf{R}$ stands for the transform matrix which is utilized to condense a structural system from a larger space to a smaller one. In the previously reported author's works, the second-order Neumann series expansion suggested by Yang [17] has proven its accuracy and reliability to free vibration and transient analyses of truss structures, it is thus adopted herein. According to this technique, **R** is computed as follows

$$\mathbf{R} = \begin{bmatrix} \mathbf{I}_{mm} \\ -\left[\mathbf{B}_1 + \mathbf{K}_{ss}^{-1}\mathbf{M}_{ss}\left(\mathbf{A}_1\mathbf{A}_4 + \mathbf{A}_1\mathbf{A}_5\right)\right]^{-1}\left[\mathbf{B}_2 + \mathbf{K}_{ss}^{-1}\mathbf{M}_{ss}\left(\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_1\mathbf{A}_3\right)\right] \end{bmatrix} \tag{5}$$

in which

$$\begin{aligned} \mathbf{A}_1 &= \mathbf{K}_{ss}^{-1}\mathbf{M}_{ss}\mathbf{K}_{ss}^{-1}\mathbf{K}_{sm}\mathbf{M}_{mm}^{-1} \\ \mathbf{A}_2 &= \mathbf{K}_{mm}\mathbf{M}_{mm}^{-1}\mathbf{K}_{mm} \\ \mathbf{A}_3 &= \mathbf{K}_{ms}\mathbf{M}_{ss}^{-1}\mathbf{K}_{sm} \\ \mathbf{A}_4 &= \mathbf{K}_{mm}\mathbf{M}_{mm}^{-1}\mathbf{K}_{ms} \\ \mathbf{A}_5 &= \mathbf{K}_{ms}\mathbf{M}_{ss}^{-1}\mathbf{K}_{ss} \end{aligned} \tag{6}$$

and

$$\begin{aligned} \mathbf{B}_1 &= \mathbf{I}_{ss} + \mathbf{A}_1\mathbf{K}_{ms} \\ \mathbf{B}_2 &= \mathbf{K}_{ss}^{-1}\mathbf{K}_{sm} + \mathbf{A}_1\mathbf{K}_{mm} \end{aligned} \tag{7}$$

Ultimately, the ith eigenvector with regard to the slave DOFs is computed as

$$\mathbf{\Phi}_{s,i} = -\left[\mathbf{B}_1 + \mathbf{K}_{ss}^{-1}\mathbf{M}_{ss}\left(\mathbf{A}_1\mathbf{A}_4 + \mathbf{A}_1\mathbf{A}_5\right)\right]^{-1}\left[\mathbf{B}_2 + \mathbf{K}_{ss}^{-1}\mathbf{M}_{ss}\left(\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_1\mathbf{A}_3\right)\right]\mathbf{\Phi}_{m,i} \tag{8}$$

2.2. Modal strain energy-based index

The *MSEBI* was originally suggested by Seyedpoor [1] to preliminarily recognize the location of high-risk members of a damaged truss. This indicator is computed based on the difference between the element strain energy of a healthy truss and a damaged one. Nonetheless, the displacement field utilized comes from free vibration modes instead of static analysis. That is why the so-called modal strain energy (*MSE*) is named. From the above discussion, the eth *MSE* for the ith modal mode can be calculated as follows

$$MSE_i^e = \frac{1}{2}\left(\mathbf{\Phi}_i^e\right)^T \mathbf{K}^e \mathbf{\Phi}_i^e \tag{9}$$

where $\mathbf{K}^e$ denotes the eth global stiffness matrix, and $\mathbf{\Phi}_i^e$ is the eth global displacement vector with regard to the ith modal mode.

Then, the above term is normalized as follows

$$\overline{MSE}_i^e = \frac{MSE_i^e}{\sum_{e=1}^{ne} MSE_i^e} \tag{10}$$

where ne is the total number of elements defined in a truss structure.

Note that this index only shows its efficiency when the modal strain energy accumulated is large enough to recognize the difference between the energy of a healthy truss and a damaged one. In order to deal with this issue, the first nm modes need to be considered to compute the accumulative energy as defined in Eq. (10). Then, it is redefined as

$$\overline{MSE}^e = \frac{\sum_{i=1}^{nm} \overline{MSE}_i^e}{nm} \tag{11}$$

Now, the eth element is suspected to be damaged based on the following criterion

$$MSEBI^e = \begin{cases} \dfrac{\overline{MSE}^{e,d} - \overline{MSE}^{e,h}}{\overline{MSE}^{e,h}} > 0, & \text{damaged element,} \\ \\ \dfrac{\overline{MSE}^{e,d} - \overline{MSE}^{e,h}}{\overline{MSE}^{e,h}} \leq 0, & \text{healthy element.} \end{cases} \tag{12}$$

2.3. Damage model for truss structures

In this study, hypothesize that each truss member is modeled in FEM by one element. The damage is caused by a reduction in Young's modulus. Accordingly, the damage ratio of the eth member is defined by

$$\xi^e = \frac{E^{e,h} - E^{e,d}}{E^{e,h}} \tag{13}$$

where $E^{e,h}$ and $E^{e,d}$ are the eth Young's moduli of the healthy member and the damaged one, respectively.

3. Ensemble learning model

XGBoost is a scalable end-to-end tree boosting system originally developed by Chen and Guestrin [16]. XGBoost is also known as an ensemble learning model that is constructed based on multiple weaker ones, aiming to result in a stronger one. More concretely, for a given task, this learning model includes a set of decision trees. In which, each tree is iteratively added to an ensemble to fix the errors of the previous tree by computing the gradient and the second-order derivative of a loss function. The XGBoost has proven to be effective and accurate in many fields. And especially in a recently published work [12], this learning model has shown its competitive accuracy and reliability against other models such as random forest regression (RFR), adaptive boosting (AdaBoost), gradient boosting machine (GBM), light gradient boosting machine (LightGBM), and categorical gradient Boosting (CatBoost) for predicting plastic hinges of steel frames. However, it has not still been applied to damage detection of truss structures using modal features incompletely recorded by limited sensors until now. Consequently, this work is carried out as the first experiment. For a more comprehensive presentation, interesting readers are suggested to refer to the above-cited articles.

4. Test examples

In order to demonstrate the ability and reliability of the XGBoost-driven damage detection approach for trusses using reduced modal features recorded by limited sensors, this Section tests two benchmark examples including a 21-bar truss and a 31-bar truss. For all examined cases, a Python-programmed code source is implemented on version 3.7 via a laptop computer with Intel® Core™ i7-2670QM CPU @ 2.20GHz, 12.0GB RAM of memory, and Windows 7® Professional with 64-bit operating system.

4.1. 21-bar truss

As the first investigation, a 21-bar truss plotted in Fig. 1 is tested. The material parameters of this truss were provided in a formerly reported author's work [2]. In which, the cross-sectional area of truss members is classified into three kinds: (i) $A_1 - A_6 = 15e^{-4}$ m^2; (ii) $A_7 - A_{17} = 9e^{-4}$ m^2, and (ii) $A_{18} - A_{21} = 12e^{-4}$ m^2. Moreover, the material density and Young's modulus of all members are $\rho = 7800$ kg/m^3 and $E = 2.0\times10^{11}$ N/m^2. There are two damage scenarios to be examined as assumed in Table 1. The only vertical displacements at free nodes are recorded. Accordingly, only ten sensors are required for measurement. And this measurement can save 50% sensor number provided that all DOFs are recorded. Note that the structural eigenvectors of free vibration modes can be obtained from measured displacements by using a technique which is the so-called Experimental Modal Analysis (EMA) [18]. The current work only focuses on developing a numerical method based on the XGBoost for damage detection of truss structures using modal features incompletely recorded at master DOFs. Therefore, measured data are assumed to be provided. Interesting readers are suggested to consult the above work for a more comprehensive review.

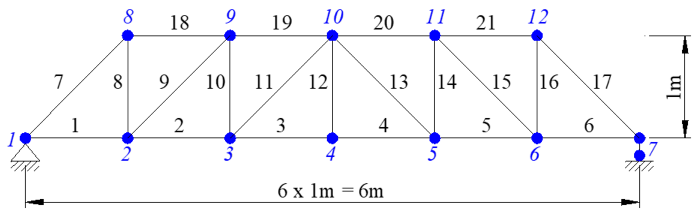
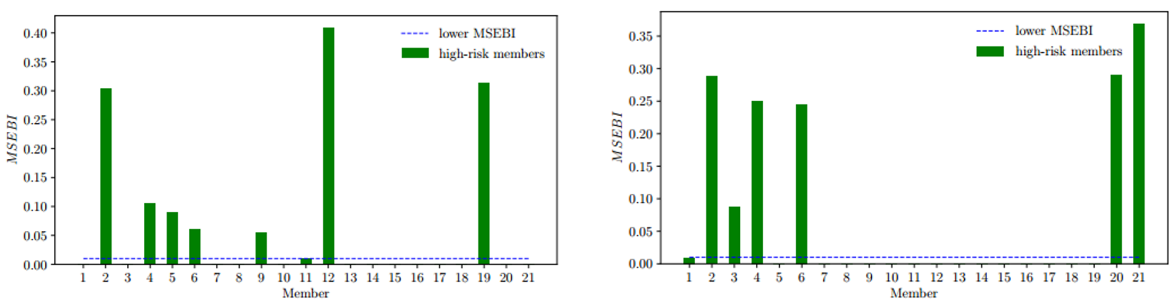


Figure 1. A 21-bar truss

Table 1. Two damage scenarios tested for the 21-bar truss

Case	1			2				
Member	2	12	19	2	4	6	20	21
Damage ratio	0.40	0.30	0.35	0.30	0.20	0.35	0.35	0.40

First, the *MSEBI* of each of all truss members is computed. In order to handle this issue, the displacement field recorded by the above-installed sensors including the first five modal modes is utilized. For the unmeasured data, they are inferred by the second-order Neumann series expansion as presented in Section 2.1. Herein, if the index of a certain member exceeds 0.01, it is treated as a high-risk one. And it is held to serve as an output, aiming to create the reduced data for constructing the XGBoost models.

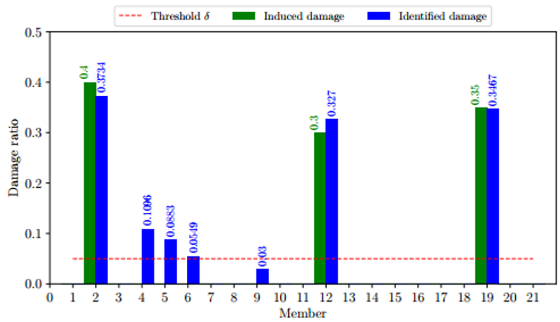


(a) Case 1

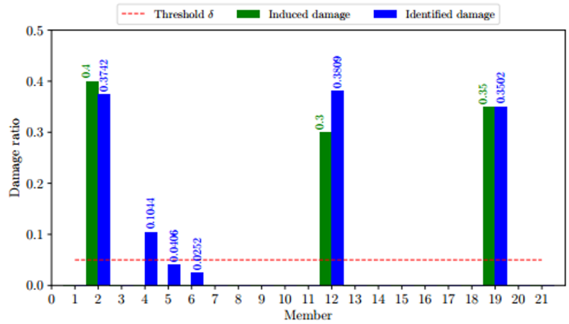
(b) Case 2

Figure 2. High-risk members determined by the *MSEBI* for two damage cases of the 21-bar truss

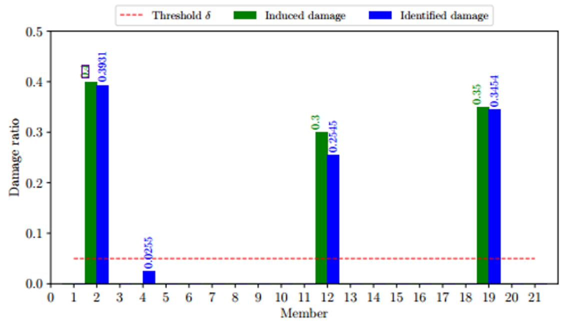
It can be easily seen from Fig. 2(a), the only most suspectedly flawed members are determined, while there is no diagnosis regarding their damage extent. Therefore, to evaluate the exact severity of damaged members, the XGBoost is applied. A dataset including 2000 samples for each iteration is generated by FEM. In which, inputs are the displacement field corresponding to measurement sensors of the first five modes. Outputs are the damage ratios of the above-indicated high-risk members. According to the sensors installed to record the data of the first five mode shapes at master DOFs, the



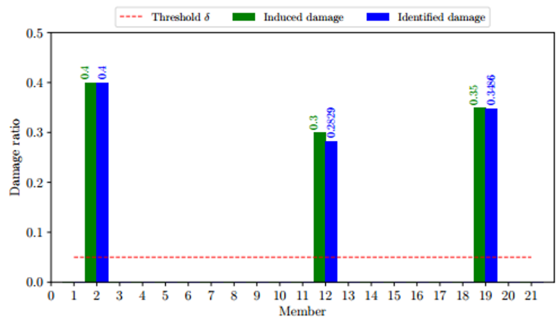
(a) The 1st XGBoost



(b) The 2nd XGBoost



(c) The 3rd XGBoost



(d) The 4th XGBoost

Figure 3. Damage detection outcomes of case 1 for the 21-bar truss obtained by the XGBoost

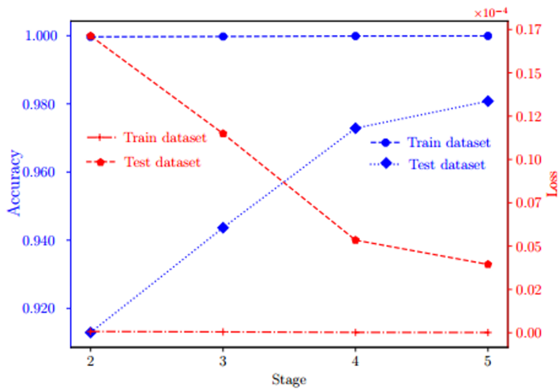
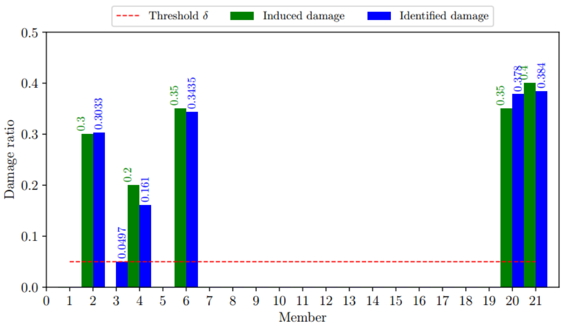
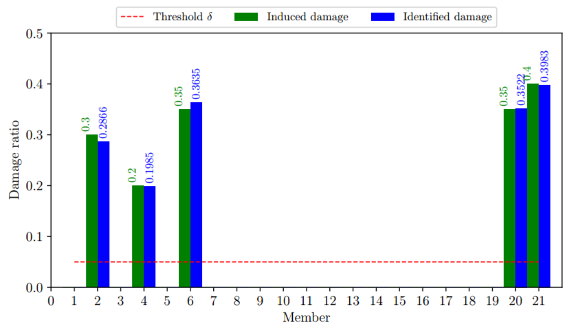


Figure 4. The convergence history of case 1 for the 21-bar truss obtained by the XGBoost

number of units of the input layer for both cases is 50. Meanwhile, the number of units of the output layer for cases 1 and 2 is 7 and 6, respectively, which correspond to high-risk members recognized by the MSEBI as shown in Fig. 2(a) and Fig. 2(b). Note that the output unit number will reduce after iteration owing to the elimination of low-risk members. The XGBoost model is trained with a maximum depth of 10, a learning rate of 0.1, and estimators of 100. Note that the parameters of the XGBoost used in this work are currently turned by the trial and error method. These parameters can be automatically selected via optimization algorithms. Nonetheless, this is out of the scope of this study. Now, to estimate the extent of damaged members, the free vibration data measured by limited sensors are employed to feed into the trained and tested XGBoost model. Fig. 3(a) shows the damage ratio of members predicted by the current ensemble learning model for case 1. It is clear that the accuracy, in this case, is low. Nevertheless, by removing low-risk members via a damage threshold of 0.05. And repeatedly doing such a procedure, the XGBoost becomes more precise in later iterations. This can be easily found via the convergence history of cases 1 and 2 as plotted in Fig. 4 and Fig. 6, respectively. Finally, the proposed method can reliably and accurately recognize both the location and extent of damaged members of case 1 as indicated in Fig. 3. Analogously, damage identification results of case 2 attained by the suggested paradigm are reported in Fig. 5.



(a) The 1st XGBoost



(b) The 2nd XGBoost

Figure 5. Damage detection outcomes of case 2 for the 21-bar truss obtained by the XGBoost

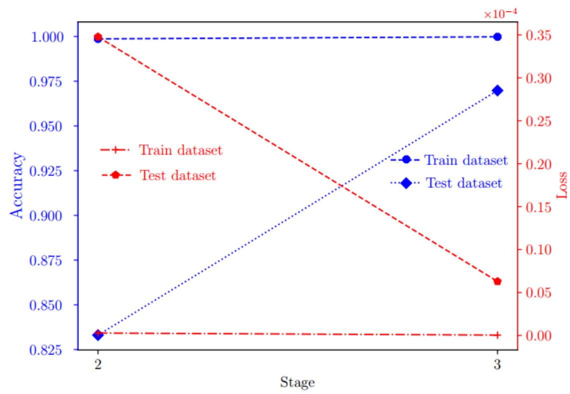
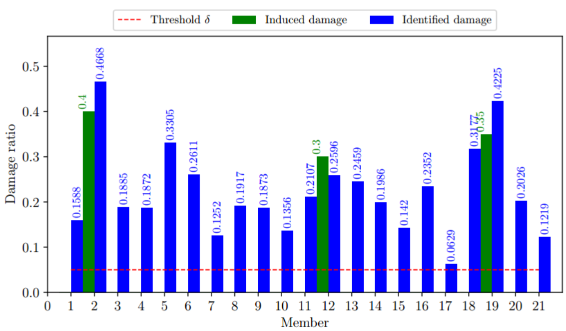
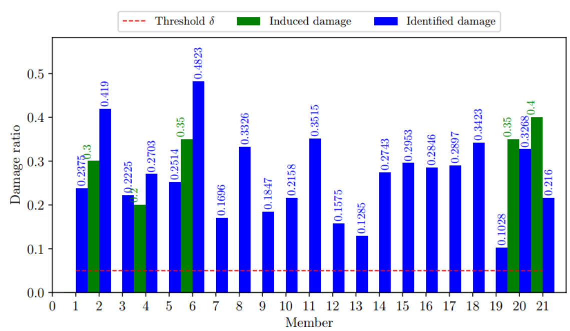


Figure 6. The convergence history of case 2 for the 21-bar truss obtained by the XGBoost



(a) Case 1



(b) Case 2

Figure 7. Damage detection outcomes for the 21-bar truss obtained by the XGBoost without applying the *MSEBI*

Nonetheless, in order to compare with the detection results given by the XGBoost without employing the *MSEBI*. In this case, for the same size of a dataset, the XGBoost always fails to determine both the position and severity of damaged members as shown in Fig. 7. This confirms that the present approach is more effective, especially for a small dataset.

Table 2. Computational time for cases of the 21-bar truss

Case	Time (s)
1 with *MSEBI*	122.4879
2 with *MSEBI*	69.2522
1 without *MSEBI*	227.1190
2 without *MSEBI*	231.0611

To close this example, Table 2 reports the computational time of the whole damage detection process including data generation for investigated cases. It can be found that if the *MSEBI* is considered, the computational cost is saved.

4.2. 31-bar truss

Now, in order to further show the capability of the current method to deal with damage scenarios, this example investigates a 31-bar planar truss as sketched in Fig. 8. This truss was studied in a previously published author's work [3]. All truss members are of the same material density ρ = 2770 kg/m^3 and Young's modulus E = 70 GPa. In addition, their cross-sectional area is 0.01 m^2. Two damage scenarios are assumed as reported in Table 3. In order to compute the *MSEBI*, the first five free vibration modes are taken into account. Similarly to the former example, the lower *MSEBI* utilized to curtail low-risk members in the first stage is 0.01. Note that both vertical and horizontal displacements at nodes 3, 4, 5, 9, 11 and 13 are installed by twelve sensors for measurement If all DOFs are measured, 24 sensors are required. In this case, 50% sensor quantity is saved. All parameters for constructing the XGBoost models are kept intact as those of the previous example. Furthermore, the same 2000 samples are created by FEM for each iteration.

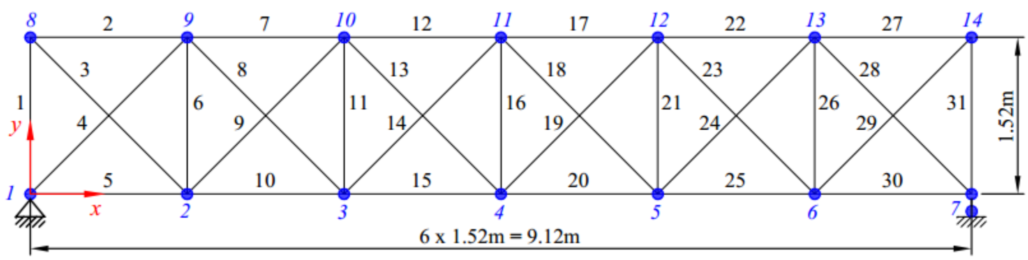
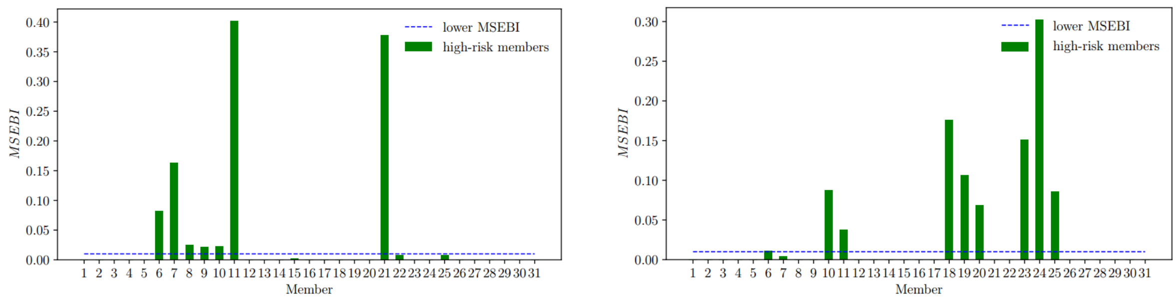


Figure 8. A 31-bar truss

Table 3. Two damage scenarios tested for the 31-bar truss

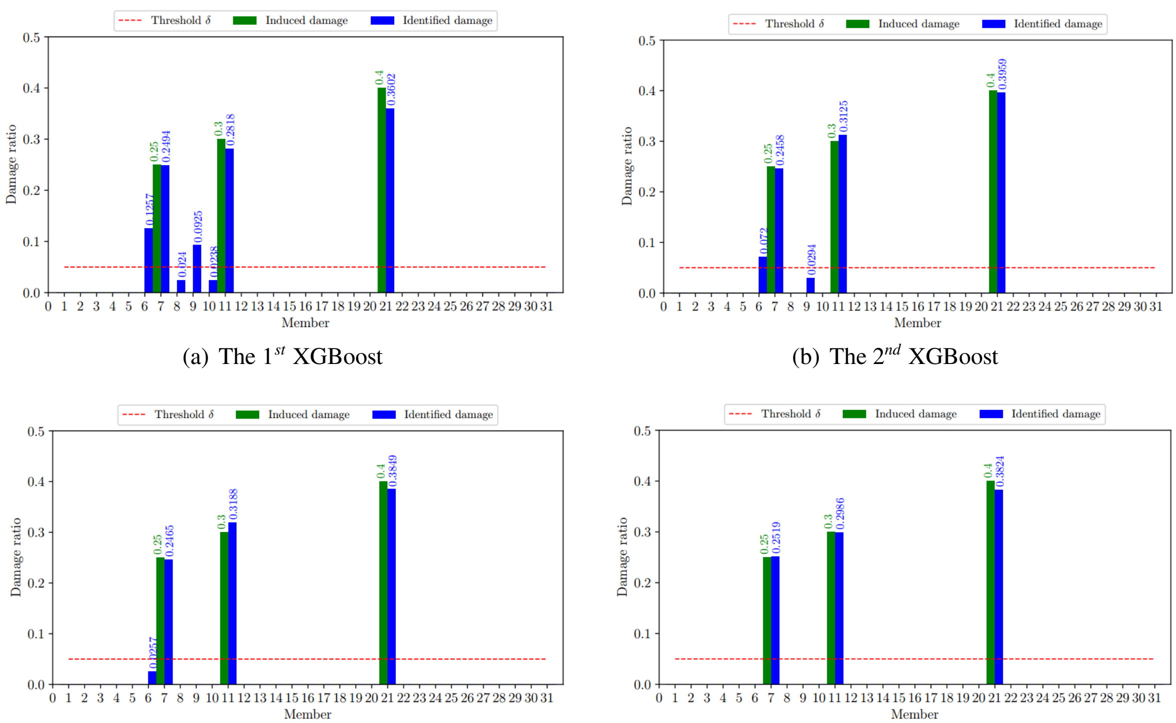
Case	1			2		
Member	8	12	22	10	19	23
Damage ratio	0.25	0.30	0.40	0.25	0.30	0.40

Fig. 9 shows the most potentially damaged candidates determined by the *MSEBI* for two cases. With the above-established parameters including the installed sensors and high-risk members discovered in Fig. 9, the number of units of the input layer for both cases is 60. Meanwhile, the number of units of the output layer for scenarios 1 and 2 is 7 and 9, respectively. The output unit numbers will lessen after iteration since only high-risk members are kept. Fig. 10 and Fig. 12 show the damage detection outcomes obtained by the XGBoost for cases 1 and 2, respectively. Fig. 11 and Fig. 13 display their convergence history. It is seen that the accuracy of the XGBoost in both investigated damage scenarios is continuously enhanced. Accordingly, the final XGBoost can accurately detect the damage to truss members. It should be noted that provided that the *MSEBI* is not applied, the XGBoost always fails when such a small dataset is utilized to establish its ensemble learning model as reported in Fig. 14.



(a) Case 1 (b) Case 2

Figure 9. High-risk members determined by the *MSEBI* for two damage cases of the 31-bar truss



(a) The 1st XGBoost (b) The 2nd XGBoost

(c) The 3rd XGBoost (d) The 4th XGBoost

Figure 10. Damage detection outcomes of case 1 for the 31-bar truss obtained by the XGBoost

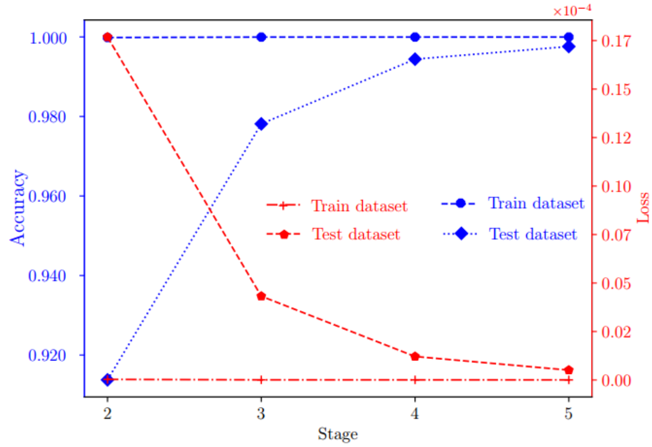
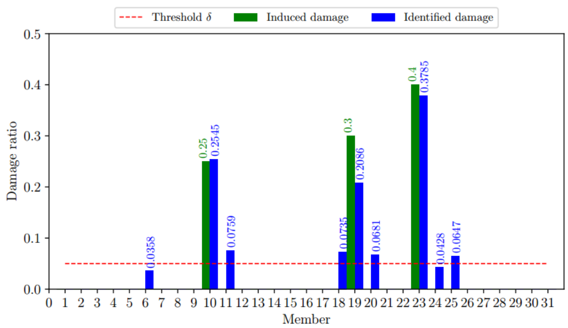
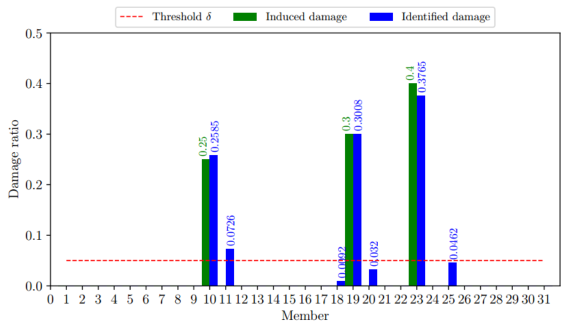


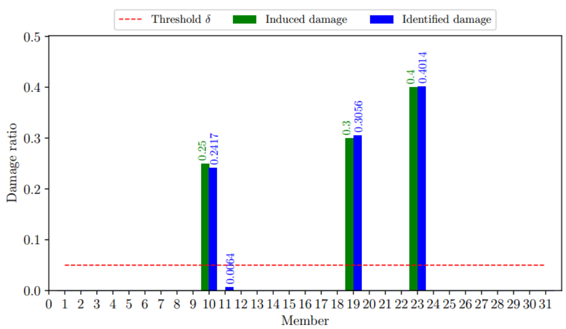
Figure 11. The convergence history of case 1 for the 31-bar truss obtained by the XGBoost



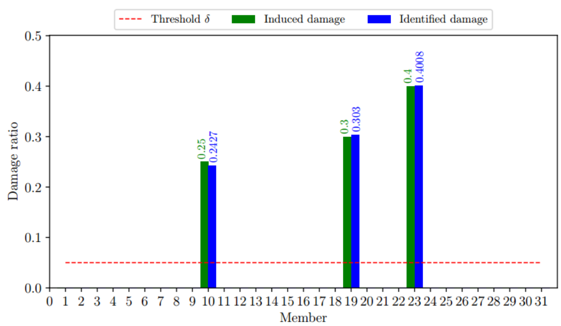
(a) The 1st XGBoost



(b) The 2nd XGBoost



(c) The 3rd XGBoost



(d) The 4th XGBoost

Figure 12. Damage detection outcomes of case 2 for the 31-bar truss obtained by the XGBoost

Table 4. Computational time for cases of the 31-bar truss

Case	Time (s)
1 with *MSEBI*	301.5691
2 with *MSEBI*	315.3340
1 without *MSEBI*	1185.9765
2 without *MSEBI*	441.1386

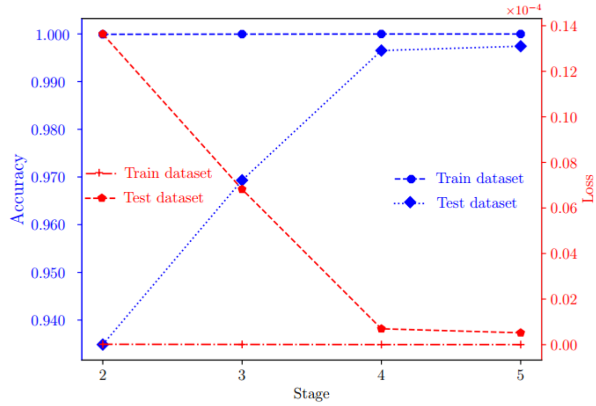
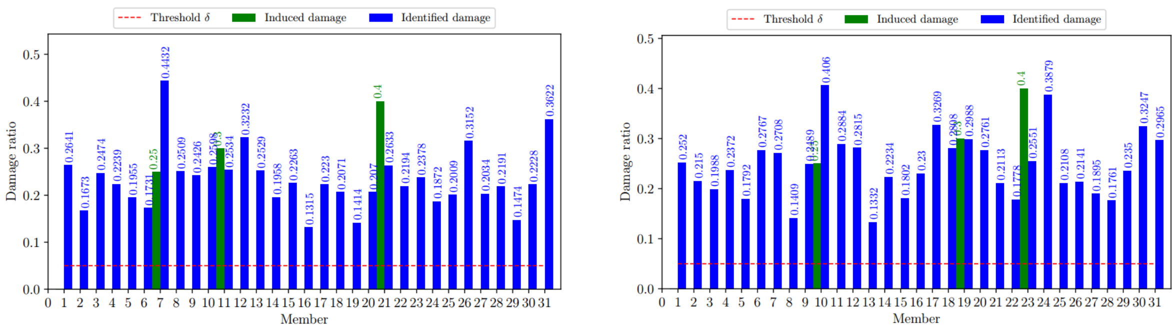


Figure 13. The convergence history of case 2 for the 31-bar truss obtained by the XGBoost



(a) Case 1 (b) Case 2

Figure 14. Damage detection outcomes for the 31-bar truss obtained by the XGBoost without applying the *MSEBI*

Finally, the computational time of the whole damage detection process including data generation for investigated cases in this case is summarized in Table 4. It can be found that the proposed methodology can detect damage members of cases 1 and 2 with only 301.5691 s and 315.3340 s, respectively.

5. Conclusions

In this paper, an XGBoost-driven damage detection approach for truss structures using modal features incompletely recorded at master DOFs is developed. For that purpose, the second-order Neumann series expansion is utilized to infer unknown data from measured ones owing to limited sensors, aiming to calculate the *MSEBI* of each of all members. Accordingly, the number of outputs for building ensemble learning models can be considerably reduced since only high-risk candidates are held. Then, the accuracy of XGBoost is continuously improved by curtailing low-risk members via a damage threshold. As a result, the present paradigm can reliably detect damage to truss structures. Outcomes obtained in numerical examples have also pointed out that the XGBoost model always fails to determine the damage to trusses for small data if the *MSEBI* is not applied. An application extension of the proposed approach to framed structures [19] is promising and will be soon carried out in the next studies.

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