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EFFECTS OF THE SIZE POLYDISPERSITY AND FRICTION COEFFICIENT ON THE COMPRESSIVE STRENGTH OF WET GRANULAR MATERIALS

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Abstract

We numerically study the effects of the size polydispersity and the friction coefficient on the compressive strength of wet granules under a diametrical compression test by using discrete element simulations. The numerical method is coupled with the capillary cohesion law which is enhanced by the cohesive forces between grains in the pendular regime. The wet granules are composed of primary spherical particles whose size polydispersity is varied from 1 to 10 and the friction coefficient between grains is varied from 0.1 to 1.0. By applying a constant compression velocity on the top plate in quasi-static regime, the granule is deformed but does not abruptly rupture due to the cohesive effects between grains and the rearrangement of primary particles. This no abrupt rupture is characterized by the appearance of the peak compressive stress in a long vertical strain before the onset failure of the granule. The compressive strength of the granule increases with increasing the friction coefficient for all cases of the size polydispersity but seemly declines for low values of the size polydispersity and almost level-off for high-size polydispersity with the friction coefficient larger than 0.5.

Keywords: capillary cohesion, compressive strength, discrete element method, diametrical compression, friction coefficient, size polydispersity.

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1. Introduction

Wet granular materials are omnipresent in nature, industrial processes, and engineering applications [1–3]. In which, wet granules or cohesive power mixtures are known as products of mixing water or adhesives and raw materials with different material properties such as size polydispersity, density, and roughness [4–8]. Basically, the increase of the size polydispersity leads to the enhancement of the granule strength [9, 10]. The grains roughness characterized by their friction coefficient also affects the mechanical properties of wet granular materials. Similar to the size polydispersity of grains, the friction coefficient of grains may help to increase the granule strength due to the enhancement of the frictional forces between grains [11]. However, the increase of the friction coefficient tends

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to reduce the solid fraction of wet granular materials as a consequence of increasing the pore space between grains [12, 13]. This effect may lead to weakening the density of the force network, leading to reducing the compressive strength of granules.

Over the last few decades, the effects of the size polydispersity and the friction coefficient on the behavior of granular materials have been investigated, especially the segregation, mechanical strength, and the rheological properties [11, 14–21]. The increase of the size polydispersity and friction coefficient leads to increase the size segregation of granular materials in different configurations [17, 21–24]. In contrast, the size polydispersity and the friction coefficient of grains represent different effects on the rheological properties of granular materials in the steady flowing state. Meanwhile, the size polydispersity of granular materials is nearly independent to the shear strength of granular flows [10, 25], the friction coefficient of grains lead to a significant increase in the shear strength of granular materials [11, 19, 20, 26]. Recently, Vo et al. numerically reported the mechanical strength of wet agglomerates under the effects of the low size polydispersity of grains and the liquid volume characterized by the rupture distance of capillary bonds between grains [18]. The results showed that the mechanical strength of agglomerates increases with increasing the size span and the rupture distance of the capillary bonds. In order to extend our previous work and also overcome the limitation of experimental investigations due to the difficulty of varying the material properties such as the size polydispersity and the friction coefficient, this numerical work is performed.

In this paper, we report in detail the compressive strength of wet granules under a diametrical compression test by means of the discrete element simulations. The numerical method is coupled with the capillary cohesion law which is enhanced by the cohesive forces between grains having a gap not exceeding a rupture distance. The wet granules are composed of primary particles with different size polydispersity and the friction coefficient between them. By applying a constant downward velocity compression on the top platen, such granules are deformed and their compressive stress changes significantly during the test depending on the values of the size ratio between the largest and smallest particle diameters and their roughness. Remarkably, by normalizing the compressive strength of wet granules by the maximum cohesive stress exerted on the mean particle diameter, it slightly decreases with increasing the size polydispersity of primary particles, but slightly increases with small values of the friction coefficient and almost level-off for the friction coefficient beyond 0.5.

The rest of our paper is arranged as follows. We first introduce shortly the discrete element method and the model preparation with different key parameters in Section 2. The paper then shows the diametrical compression test and the compressive strength of wet granules under the action of the constant downward velocity on the top platen by varying the broad range of values of the size polydispersity and the friction coefficient of grains in Section 3. The conclusion is set in Section 4 with the salient results and further research directions.

2. Numerical simulation

The simulations are performed by using an in-house 3D molecular dynamics DEM program, namely cFGd-3D++code, originally developed by Mutabaruka [27]. This coding program is then improved by the author in order to apply to previous works and also for this current simulation [28–30]. The primary particles are modeled by using spheres as rigid bodies and interacting with others by considering the contact forces law based on a linear spring-dashpot model. The interactions between two primary particles i and j involve the normal contact forces f_n, the tangential contact forces f_t, and capillary cohesion forces f_c [30], as shown in Fig. 1, where f_n is obtained by considering the normal elastic and normal damping components, f_t is determined by the Coulomb friction law [21], and the capillary cohesion force f_c is proportional to the cohesion pre-factor $\kappa = 2\pi\gamma_s$, where γ_s denotes the liquid-vapor surface tension [30].

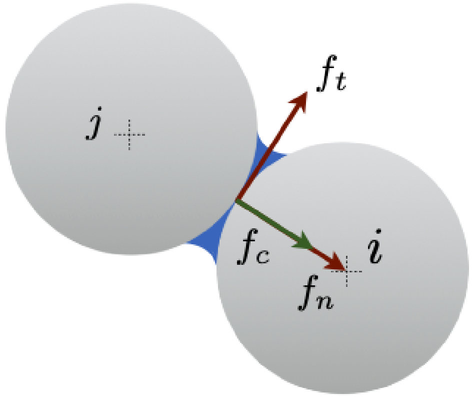
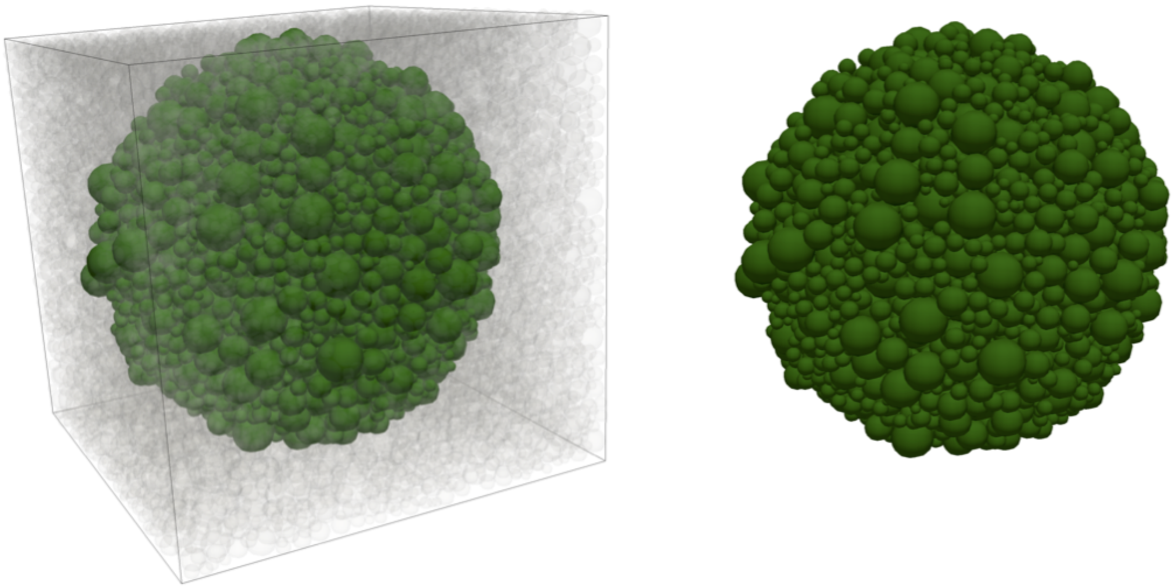


Figure 1. A schematic drawing represents the contact model between two wet particles i and j

The preparation and simulation of this current work involve three different stages: 1) creating assemblies of primary particles with different size polydispersities by using an isotropic compression; 2) extracting granules by applying a spherical probe into the packings of primary particles; 3) applying a constant downward velocity on the top platen to compress the granules. In particular, large samples of granular materials with 10 different size polydispersities ($\alpha = d_{max}/d_{min}$) ranging from 1 to 10 were first prepared in a box, then applied isotropic compression to all box walls into reaching a dense packing in the first stage, where d_{max} is the largest particle diameter, it is kept constant in all simulations, d_{min} is the smallest particle diameter. In each sample (except $\alpha = 1$), the diameter of primary particles is assumed to be distributed uniformly by particle volume fraction, this means that the particle volume of all size classes of the packing is the same [18, 31]. In the next stage, a spherical probe is then applied to the center of the packing and its radius is increased until exactly reaches 5000 primary particles inside the probe, as shown in Fig. 2(a). In all these steps, the particle gravity and cohesion between grains are absent. After extracting a granule from an assembly, as shown in Fig. 2(b), the cohesion forces between grains are activated. In all simulations, the liquid volume is characterized by the rupture distance, this is set equal 0.1 times to the largest particle diameter ($d_{rupt}/d_{max} = 0.1$) and kept constant in all simulations. After reaching a static equilibrium, all granules are compressed by applying a constant downward velocity on the top platen of the model in the third stage, as shown in Fig. 3. The particle gravity is absent in all simulations in order to avoid the local effects during the compression [32]. All values of the other key parameter are given in Table 1.

The wet granules are then subjected to the axial compression by applying a constant velocity v = 0.1 m/s on the top platen, this velocity is slow enough for considering the quasi-static diametrical compression in all simulations. During the compression, the wet granules are deformed, as illustrated in Fig. 3, and the granule stress is changed with different rates and magnitudes depending on the values of the size polydispersity and the friction coefficient between grains. In these simulations, the stress-strain curve for each value of the size polydispersity and friction coefficient between grains is considered. The cumulative vertical strain is determined as a ratio between the cumulative downward displacement Δh of the top platen and the initial diameter D_0 of the wet granules.

$$\varepsilon = \frac{\Delta h}{D_0} = \frac{v \times t}{D_0} \tag{1}$$



(a) Assembly of granular materials and the setting of a granule inside a box

(b) A granule extracted from the assembly

Figure 2. The numerical preparation of a wet granule with the size polydispersity α = 6

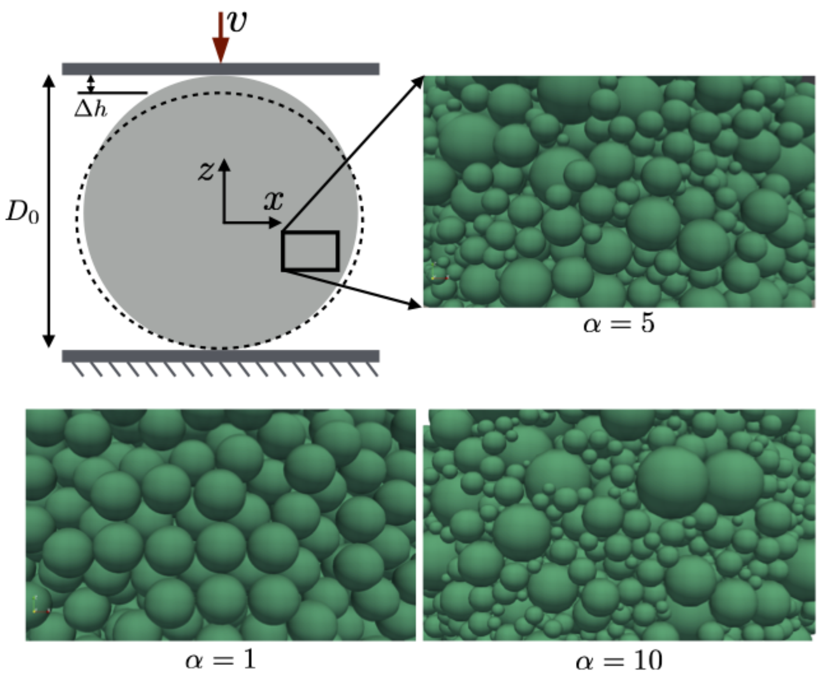


Figure 3. Schematic drawing represents the diametrical compression test of wet granules composed of primary particles with different size polydispersities and friction coefficient by applying a constant downward velocity on the top platen

Table 1. Main parameters and their values in all numerical simulations

Parameter	Symbol	Value	Unit
Largest particle diameter	$d_{\max}$	10	µm
Size polydispersity	α	[1, 10]	-
Density of particles	ρ	2600	kg/m^{-3}
Number of grains	N_p	5000	-
Coefficient of friction	μ	[0.1, 1.0]	-
Normal stiffness	k_n	10^3	N/m
Tangential stiffness	k_t	8×10^2	N/m
Normal damping	γ_n	5×10^{-2}	Ns/m
Tangential damping	γ_t	5×10^{-2}	Ns/m
Cohesion pre-factor	κ	0.47	-
Time step	Δt	1×10^{-9}	sec.
Compression velocity	v	0.1	m/s

In order to analyze the compressive strength of wet granules under the diametrical compression, we calculate the mean vertical stress σ_{zz}, determined by considering the stress tensor of the granules during the test. The mean vertical stress σ_{zz} is obtained by considering z-components of the force vectors that combining all normal contact forces, capillary cohesion forces, and the tangential contact forces and the branch vectors joining centers between two particles in contacts [33], as given by the following expression:

$$\sigma_{zz} = \frac{1}{V_0}\sum_{k=1}^{N_c} f_z^k l_z^k = n_c < f_z^k l_z^k >_k \tag{2}$$

where V_0 is the volume of granules, k is the interaction inside the granules, N_c denotes the number of capillary bonds during the compression, n_c is the density of number of capillary contacts, f_z^k and l_z^k denote the z-components of the force vector and branch vector at the contact k. The stress-strain curve and the compressive strength of wet granules are analyzed in detail below.

3. Results of simulations

Fig. 4 shows the evolution of the mean vertical stress σ_{zz} as a function of the cumulative vertical strain ε for 10 different values of the friction coefficient with the size polydispersity $\alpha = 1$. It is interesting to see that the mean vertical stress for all cases of wet grains first increases rapidly with the rate that increases with increasing the friction coefficient of primary particles. This stress then reaches the peak values in a long range of the cumulative vertical strain ε, implying the wet granules do not abruptly rupture after the stress reaches the peak. This behavior supplies evidence for the ductile properties of the wet granules before the onset failure of such granules. The ductile properties of wet granules may be due to the rearrangement of primary particles and the number of broken capillary bonds are not much with small cumulative vertical strain. The mean vertical stress then declines smoothly but with the rate that nearly increases with increasing the friction coefficient μ. The increase of the declination rate of σ_{zz} may be due to the increase of the number of broken capillary bonds.

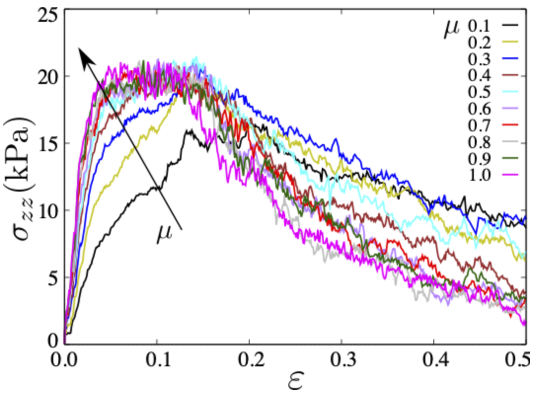


Figure 4. Mean vertical stress σ_{zz} of wet granules as a function of the cumulative vertical strain ε during the compression for different values of the friction coefficient μ of primary particles with the size polydispersity $\alpha = 1$

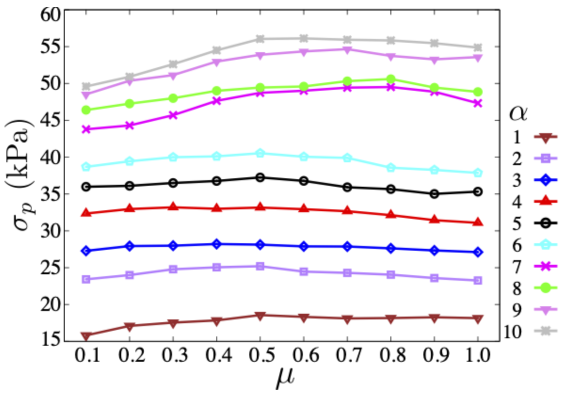
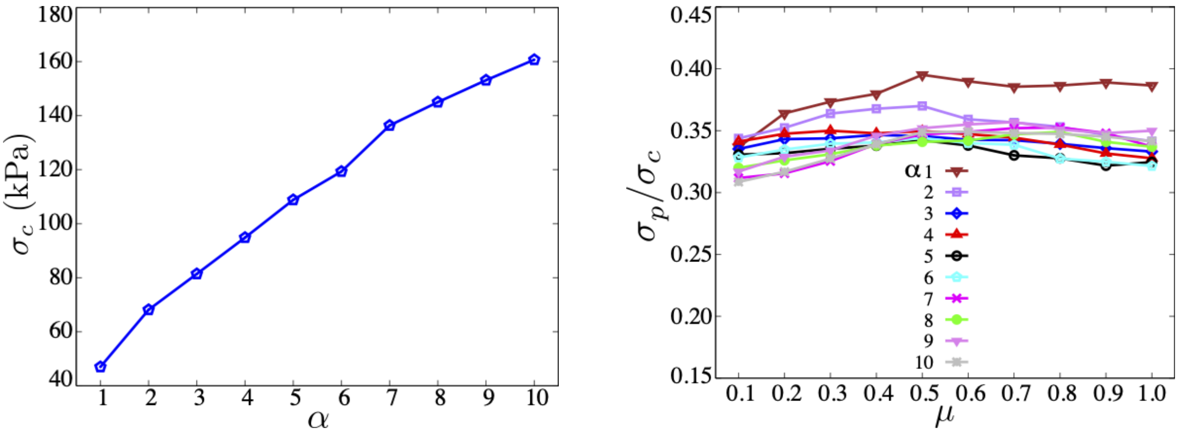


Figure 5. The peak stress σ_p of wet granules composed of primary particles as a function of the friction coefficient μ for 10 different values of the size polydispersity α

In order to highlight the effects of both the size polydispersity and the friction coefficient of primary particles on the compressive strength of wet granules under the diametrical compression test, the peak value σ_p of mean vertical stress σ_{zz} obtained in each simulation when varying α and μ is considered. Fig. 5 displays the evolution of the peak vertical stress σ_p as a function of the friction coefficient for 10 different values of the size polydispersity α. It is remarkable to note that the peak stress σ_p of wet granules increases significantly with increasing the size polydispersity α. In particular, σ_p increases from nearly 19 kPa to approximate 55 kPa when increases the size polydispersity α from 1 to 10 for the case of $\mu = 0.5$. However, the growth rate of the peak vertical stress σ_p is different when increases α in its small range. Furthermore, the friction coefficient also represents different effects on the peak stress of wet granules. However, the common trend of the peak stress is nearly similar for all values of α. Indeed, the peak stress of granules increases slightly when the friction coefficient changes from 0.1 to 0.5. Passing these values of the friction coefficient, σ_p seemly decreases for low size polydispersities ($\alpha \leq 6$) or seemly level-off for higher size spans when $\mu > 0.5$. These differences may be due to the saturated behavior of the friction forces in particle's interactions especially for granules with low size polydispersity and high friction coefficient. These analyses provide evidence for the effects of the size polydispersity and the friction coefficient on the mean vertical stress of wet granules.

To unify the representation of the compressive strength of wet granules under a diametrical compression test as the relationship between dimensionless parameters for all values of the size polydispersity α and the friction coefficient μ of primary particles, we consider the normalization of the peak stress σ_p and the cohesive stress $\sigma_c \sim \kappa/\langle d\rangle$, where κ is the cohesion pre-factor, $\langle d\rangle$ denotes the mean particle diameter which decreases when increasing the size polydispersity α. Fig. 6(a) shows the increase of the cohesive stress σ_c as a function of the size polydispersity α. The increase of σ_c represents the decrease of the mean particle diameter as a consequence of keeping a constant value of the cohesion pre-factor κ. By considering the ratio of the peak stress σ_p and the cohesive stress σ_c, it illustrates the different effects of the size polydispersity and the friction coefficient of grains on the compressive strength of granules. This ratio first increases for all cases of the size polydispersity with small values of the friction coefficient ($\mu \leq 0.5$), σ_p/σ_c then seemingly decreases for higher values of



(a) Cohesive stress exerted on particles having mean diameter

(b) Compressive strength of agglomerates

Figure 6. (a) The cohesive stress exerted on particles having mean diameter as a function of the size polydispersity, (b) the peak stress normalized by the cohesive stress as a function of the friction coefficient for different values of the size polydispersity

friction coefficient with low-size polydispersity, as shown in Fig. 6(b). Remarkably, the compressive strength of granules composed of monodisperse primary particles is significantly higher than that for the case of polydisperse spherical particles, especially for high-size polydispersity.

The different effects of the friction coefficient on the compressive strength of wet granules in Fig. 6(b) under the diametrical compression test represent the saturated behavior of the Coulomb frictional interactions. Indeed, the compressive strength of wet granules increases continuously for small values of the friction coefficient. This property provides evidence for the weakness of the frictional forces observed via the material properties as compared to the frictional threshold observed via the Coulomb friction law. Continuously increasing the friction coefficient of granular materials and keeping the external loading (as compression forces), the frictional force may exceed the frictional threshold, leading to reaching a constant compressive strength of granules. These findings observed in the current work show a good agreement with previous investigations on the effects of the friction coefficient of material on the shear strength of the granular samples [34]. The porosity of a granular sample increases with increasing the friction coefficient, meanwhile, its shear strength levels off for high values of the friction coefficient. These observations have important meanings in using raw materials in many engineering applications.

4. Conclusions

As reported in the paper, the mean compressive stress of wet granules evolves differently depending on the size polydispersity and the friction coefficient. In particular, this stress increases rapidly then reaches a peak in a long vertical cumulative strain, this property represents the ductile behavior of wet granules as a consequence of the rearrangement of the primary particles and the cohesive effects of the capillary bonds. Remarkably, by considering the ratio between the peak compressive stress and the cohesive stress exerted on the mean particle diameter, it decreases with increasing the size polydispersity for the whole range values of the friction coefficient. For each case of the size polydispersity, the compressive strength of wet granules increases slightly for low friction coefficient, this strength then seemly declines with higher friction coefficient for $\alpha \leq 6$ but nearly level-off for the high-size polydispersities. The numerical findings observed in this current work may provide a better understanding of the effects of the polydispersity and friction coefficient of primary particles on the compressive strength of wet granules under diametrical compression. These are clearly difficult to obtain in experimental works, especially for performing with changing the friction coefficient of grains.

Within the scope of the paper research and the limitation of real experiments, it is meaningfully found that the role of friction coefficient has different influences on the mechanical strength of granules or cohesive powder mixtures, leading to providing evidence for using appropriate raw materials in different engineering applications.

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