

AN ENHANCED SINGLE DAMAGE IDENTIFICATION IN BEAMS USING NATURAL FREQUENCY SHIFTS AND ANALYTIC MODAL CURVATURES

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Abstract

This article presents an improved procedure for detecting single damage in beam-like structures. Many damage detection methods are based on the analysis of beam natural frequency shifts in damaged and intact states and its analytical modal curvatures. They identify the damage by constructing curves of damage coefficients along the beam and visually determining the point of their mutual intersection. However, for some cases, the absence of an unambiguous point of intersection of these curves or the low accuracy of its localization makes it difficult to use these methods. To accurately determine the mutual intersection of damage coefficient curves, this work proposes to use a subset of modes without one having its curvature close to zero at the site of the alleged damage. This approach greatly simplifies the determination of the point of intersection of the curves of damage coefficients. An enhanced procedure for the numerical determination of this intersection point is also proposed, rather than the visual one used in existing methods. The effectiveness of the proposed technique was confirmed by numerical simulation followed by experimental verification.

Keywords: beams; damage detection; natural frequency shift; mode shape curvature.

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1. Introduction

Structural damage detection is essential, especially in the early damage state, to avoid sudden failures and improve the safety and longevity of structures [1]. Many of the vibration-based fault detection methods in mechanical systems are based on monitoring changes in the dynamic properties of these systems, such as natural frequencies, mode shapes, and damping ratios [2, 3]. These methods are well suited to real structures, especially when operational modal analysis is applied, due to their ability to continuously monitor structures without interrupting their normal functioning.

The great variety of these methods also results from the way in which the listed parameters are used. For example, the damping ratio is less frequently used than the system's natural frequencies and mode shape parameters because it is sensitive to environmental factors such as humidity and temperature. Also, from a technical point of view, the natural frequencies of a structure can be identified from signal measurements with only one sensor, however, multiple sensors are needed to determine the mode shapes.

Among other techniques that use the above parameters for damage identification, the following studies are of interest from the point of view of using and analyzing these parameters. Lee and Chung

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[4] identified a single damage for a cantilever beam in which the approximate crack location is estimated using Armon's rank ordering method, which uses the first four frequencies of the damaged structure. Patil and Maiti [5] developed a crack detection technique that uses a rotational spring model to simulate the crack effect in a beam, with the damage index being an indicator of how much strain energy is stored in the spring. Gillich et al. [6] proposed a method to compare the measured frequency changes with the values obtained analytically and then use the pattern recognition problem to determine the location of the damage and its severity. The damaged zone was identified according to the classification of the first four frequencies of structures in [7]. Sha et al. [8] developed a damage detection method that combines relative numerical natural frequency variation and measured values using Bayesian inference. Surace et al. [9] used the ratios of natural frequencies of different modes for a characteristic of damage.

Many researchers have focused on changes in modal shapes and/or modal curvatures. Changes in mode shapes can directly provide information about the location of a damage [10]. The difference in modal curvatures between intact and damaged states was used to determine the location of damage [11, 12]. Gorgin [13] developed a damage localization method based on the analysis of the first mode shape of the structure. In general, these methods are mainly based on the change of mode shapes between a healthy state and a damaged one. However, they require a large number of measurements at different locations and are almost exclusively limited to damage localization.

The use of mode shapes (or their derivatives) is limited to the localization level. Therefore, combined methods for damage quantification using both natural frequencies and mode shapes (or their derivatives) are proposed. Dahak et al. [14] developed a damage detection method using curves intersection based on the curvature of the intact mode shape and the measured frequencies. The use of the relationship between natural frequency variation and modal curvature for damage detection was discussed in [15].

In addition, significant improvements in computing power and advances in sensor technology have made it possible to use machine learning techniques in damage detection applications. Lee [16] constructed a set of training patterns of a neural network for damage detection in pipe-type beams using changes in natural frequencies. Other methods such as genetic algorithm [17], hybrid optimization [18], particle swarm optimization [19] have been used for damage detection. These methods are generally effective but require a considerable computational resource to achieve the convergence of the algorithm and are therefore time consuming.

This work focuses on rapid damage identification methods which are effective at the stage of early damage detection in beam-like structures. Among the discussed above studies, methods using natural frequency shifts and analytical modal curvatures seem to be simple and easy to detect single cracks in beams [6, 14, 15]. First, from the shifts of natural frequencies and modal curvatures, curves are generated along the entire length of the beam. The mutual intersection of all these curves is then used to localize the damage. However, the intersection is not always evident, which complicates the method. To overcome this limitation, an improved procedure is proposed that uses a subset of modal curvatures to highlight the intersection, thereby simplifying identification of its location. Numerical examples and an experimental test are used to validate the proposed procedure.

2. Theoretical background and existing method

The relationship between natural frequency shift and damage at location x_0 is obtained as [20]:

$$\frac{\Delta f_i}{f_i} = \frac{f_i - \bar{f}_i}{f_i} \approx \delta_K \frac{\phi_i''^2(x_0)}{\|\phi_i''\|_2^2} + \delta_M \frac{\phi_i^2(x_0)}{\|\phi_i\|_2^2} \quad (1)$$

where f_i and \bar{f}_i are the natural frequencies of the i^{th} mode of the intact and damaged states, respectively; $\delta_K = \frac{\Delta EI \Delta L}{2EI}$ and $\delta_M = -\frac{\Delta \mu \Delta L}{2\mu}$ are the relative local variation of bending stiffness and mass, respectively; ϕ_i and ϕ_i'' are the mode shape and the modal curvature of the i^{th} mode. The first five analytic mode shapes and the corresponding modal curvatures of a cantilever beam are presented in Fig. 1.

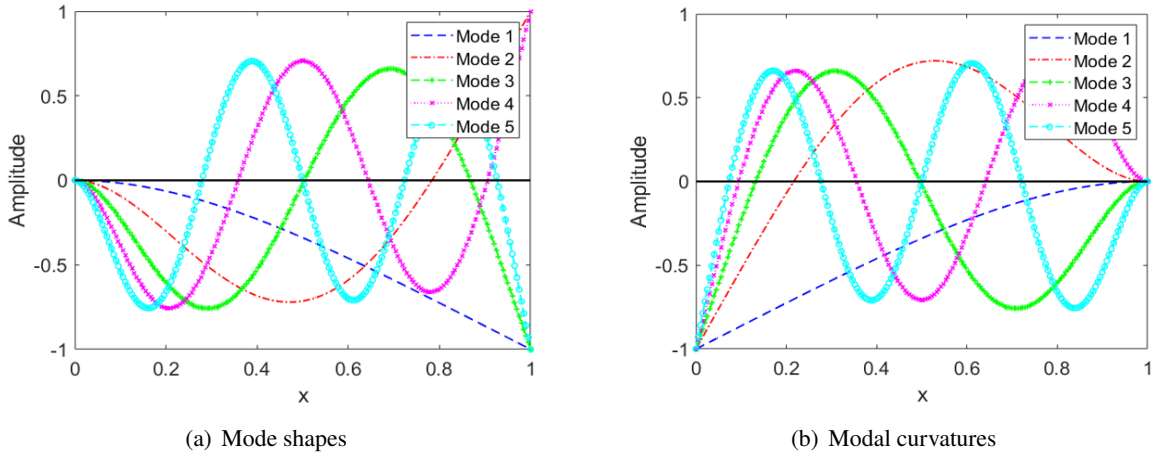


Figure 1. A cantilever beam

In the case of damage caused only by a change in bending stiffness and neglecting the effect of a change in mass, a simpler expression for the natural frequency variation can be represented as a function of squared modal curvature:

$$\frac{\Delta f_i}{f_i} \approx \zeta \phi_i''^2(x_0) \quad (2)$$

where ζ represents the damage coefficient and depends on the size of the damage.

The relative frequency shift is defined as:

$$\overline{\Delta f_i} = \frac{f_i - \bar{f}_i}{f_i} \quad (3)$$

Substituting Eq. (3) into Eq. (2), the relationship between damage coefficient and relative frequency shift is given as follows:

$$\zeta \approx \frac{\overline{\Delta f_i}}{\phi_i''^2(x_0)} \quad (4)$$

Thus, a curve of damage coefficients $\zeta_i(x)$ defined as follows:

$$\zeta_i(x) \approx \frac{\overline{\Delta f}_i}{\phi_i''^2(x)} \quad (5)$$

where x is the position on the beam and $\zeta_i(x)$ is the curve of damage coefficients for mode i .

Following Eq. (4), if $x = x_0$ then $\zeta_i(x) = \zeta_i(x_0) = \zeta \forall i$. Thus, the curves $\zeta_i(x)$ intersect at the point of coordinate (x_0, ζ) that gives the location x_0 of the damage.

Eq. (5) has been applied to damage detection in many studies [6, 14, 15]. The intersection of these damage coefficient curves was used to determine the location and severity of damage. This method is simple and convenient for single crack detection in beams. An illustration of the mutual intersection of damage coefficient curves is given in Fig. 2.

However, in some cases, the curves do not have a mutual intersection as presented in Fig. 3. To explain why the curves do not intersect, one can rewrite Eq. (5) as follows:

$$\overline{\Delta f}_i \approx \zeta_i(x) \phi_i''^2(x) \quad (6)$$

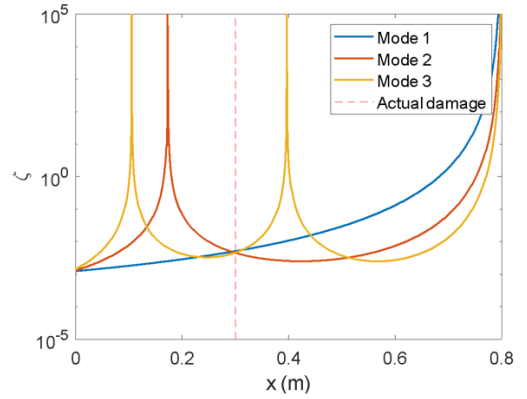


Figure 2. Example of the mutual intersection of damage coefficient curves

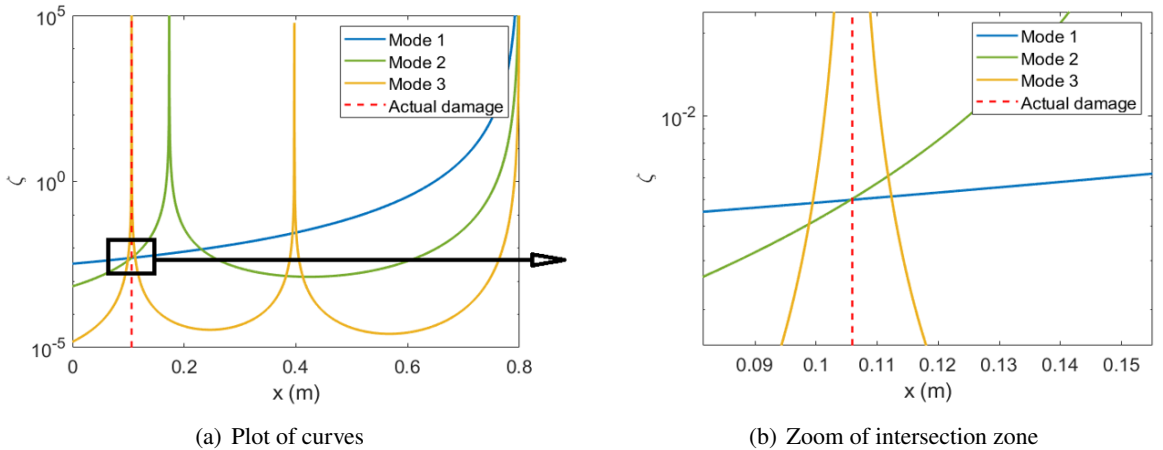


Figure 3. Damage at the location of zero modal curvature

It can be seen from Eq. (6) that when the damage location x_0 coincides with the location where the modal curvature of the i^{th} mode $\phi_i''^2(x_0)$ is zero, the natural frequency shift $\overline{\Delta f}_i$ should be zero. However, the natural frequency shift $\overline{\Delta f}_i$ can be non-zero due to numerical error or noise effect. Thus $\zeta_i(x_0)$ tends to be infinite as can be seen from Eq. (5). Therefore, in this particular case the mutual intersection does not exist, and the damage location cannot be detected using the intersection

of curves. Fig. 3 illustrates the case described above when damage occurs at the location where the curvature of the 3rd mode is zero.

The absence of a single clear point of intersection of the damage coefficient curves (Fig. 3) does not allow one to unambiguously determine the location of the damage. However, visually it is possible to determine a certain area of intersection of these curves. Thus, it is necessary to establish a procedure that will allow one to uniquely identify the point of intersection of these curves using numerical methods in order to facilitate the application of the method described above.

3. Enhanced procedure

To overcome these shortcomings, it is proposed to use a subset of curves to highlight their mutual intersection, which will ensure its presence. Moreover, damage identification will be more accurate and systematic if a numerical procedure is used to determine this intersection of curves.

3.1. Selection of curves

As presented above, when damage site is close to a location where the modal curvature is close to zero, its curve $\zeta_i(x)$ gives a peak at damage location and does not intersect the other curves at the damage position. To select a subset of curves that will uniquely define the intersection, it is proposed to perform the following steps:

- Draw the curves $\zeta_i(x)$ for all available modes on the same graph.
- Eliminate a curve if it has a vertex with an abscissa which is close to the abscissa of the common point of intersection of other curves.

3.2. Finding the intersection from the singular value decomposition

To accurately calculate the position of the mutual intersection of the curves, a new approach is proposed as follows:

- Divide the length of the beam into n positions. For each mode i at each location x_j one gets:

$$\zeta_{i,j} \approx \frac{\overline{\Delta f_i}}{\phi_i'^2(x_j)} \text{ with } i = 1 : m \text{ and } j = 1 : n \text{ with } m \ll n, \text{ where } m \text{ is the number of modes.}$$

- Define the following matrix $\mathbf{A}^{(j)}$ for each location x_j :

$$\mathbf{A}_{(m+1) \times m}^{(j)} = \begin{bmatrix} 1 & 1 & \dots & 1 & \dots & 1 \\ \zeta_{1,j} & \zeta_{2,j} & \dots & \zeta_{i,j} & \dots & \zeta_{m,j} \\ \zeta_{1,j} & \zeta_{1,j} & \dots & \zeta_{1,j} & \dots & \zeta_{1,j} \\ \zeta_{1,j} & \zeta_{2,j} & \dots & \zeta_{i,j} & \dots & \zeta_{m,j} \\ \zeta_{2,j} & \zeta_{2,j} & \dots & \zeta_{2,j} & \dots & \zeta_{2,j} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ \zeta_{1,j} & \zeta_{2,j} & \dots & \zeta_{i,j} & \dots & \zeta_{m,j} \\ \zeta_{i,j} & \zeta_{i,j} & \dots & \zeta_{i,j} & \dots & \zeta_{i,j} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ \zeta_{1,j} & \zeta_{2,j} & \dots & \zeta_{i,j} & \dots & \zeta_{m,j} \\ \zeta_{m,j} & \zeta_{m,j} & \dots & \zeta_{m,j} & \dots & \zeta_{m,j} \end{bmatrix} \quad (7)$$

- Take a singular value decomposition of matrix $\mathbf{A}^{(j)}$ to obtain m singular values for each location x_j :

$$s_1^{(j)} > s_2^{(j)} > \dots > s_m^{(j)} > 0 \quad (8)$$

when $x \rightarrow x_0$, all elements in matrix $\mathbf{A}^{(j)}$ tend to be 1, and thus:

$$s_1^{(j)} > 0 \text{ and } s_2^{(j)}, s_3^{(j)}, \dots, s_m^{(j)} \rightarrow 0 \quad (9)$$

- Plot the curve $p(x_j) = \frac{1}{s_2^{(j)}}$ over the length of the beam. Then, the peak at location $x = x_0$ is observed.

3.3. Enhanced procedure of single damage detection

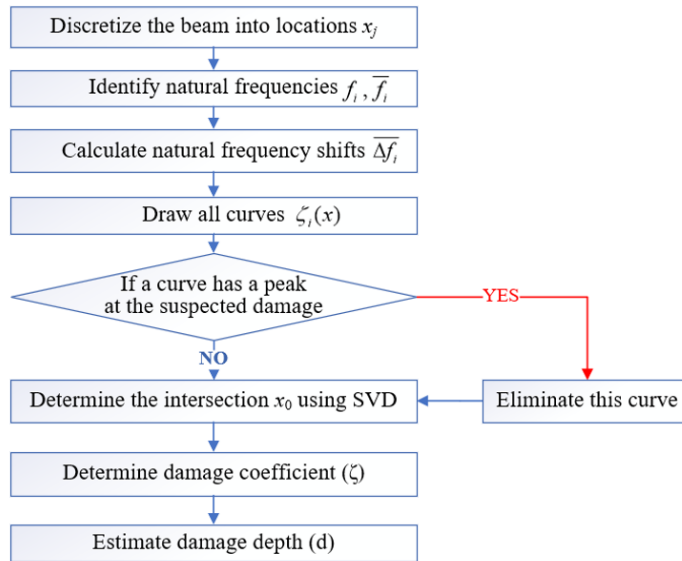


Figure 4. Enhanced procedure flowchart

Summarizing the above analysis, the following enhanced single damage identification procedure is proposed:

- Step 1. Discretize the beam into n positions x_j .
- Step 2. Identify natural frequencies f_i and \bar{f}_i of the intact and the damaged beam.
- Step 3. Calculate the relative frequency shifts $\Delta \bar{f}_i$
- Step 4. Select curves by drawing all curves for the modes on the same graph and exclude the curve if its vertex abscissa is close to the intersection point abscissa.
- Step 5. Determine the intersection x_0 of the remaining curves (the peak of $p(x)$ as in Section 3.2).
- Step 6. Determine the damage coefficient (ζ) using Eq. (4).
- Step 7. Estimate the damage depth (d). In order to quantify the damage depth, an inverse method is used to estimate the relationship between the coefficient (ζ) and the damage depth (d). Since the damage coefficient (ζ) only depends on the change in stiffness and width, it can be calculated as a function of these parameters. To establish the relationship between the damage coefficient (ζ) and the

damage depth (d), a fixed position x_0 and the width ΔL of the damage are used while the damage depth changes (for this work $\Delta L = 2$ mm is used for every case). The damage depth corresponding to any damage coefficient can then be interpolated from neighboring values using spline interpolation.

Fig. 4 shows a flowchart that summarizes the enhanced procedure for determining a single damage in a beam presented above.

4. Verification of proposed procedure by numerical investigation

In this section, the proposed procedure is validated by analyzing the model simulated in ANSYS Workbench which is presented in Fig. 5. The numerical tests were carried out for a steel beam of Young’s modulus $E = 200$ GPa, and mass density $\rho = 7850$ kg/m³. The beam of 800 mm long, 40 mm wide and 6 mm high was used for these simulations with different boundary conditions. All simulated damage sites are 2 mm wide.

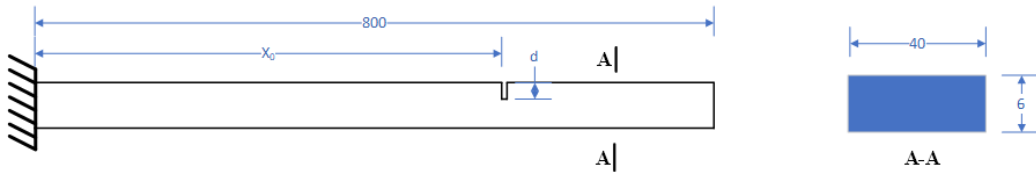
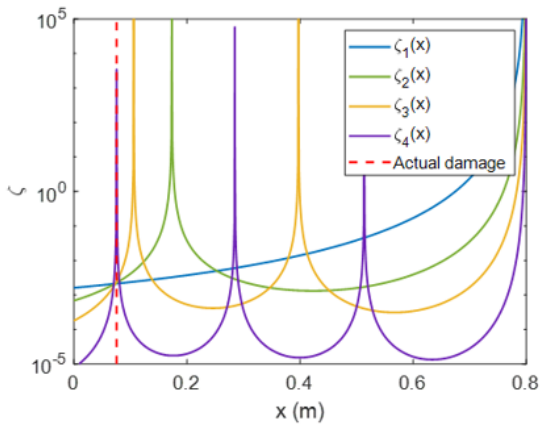


Figure 5. Cantilever beam

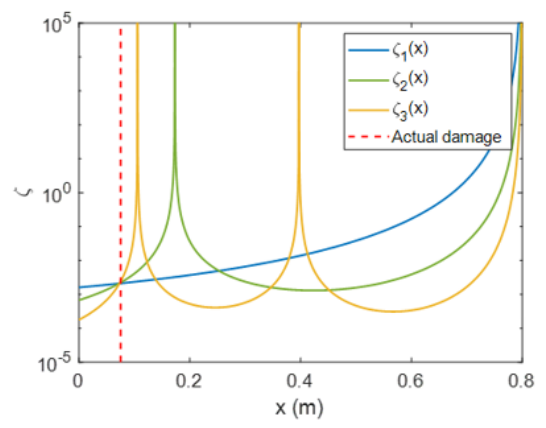
Six cases of damage are considered, which differ in their location and depth of the crack (see Table 1). Following the proposed procedure, the identified intersection point represents the damage location as shown in Figs. 6–11. In case 1, damage is modeled at the location where the modal curvature of the 4th mode is zero. In case 2, damage is modeled at the point where the modal curvature of the 3rd mode is zero. In case 3, damage is modeled at the location where the modal curvature of the 2nd mode is zero. In case 5, damage is modeled at the point where the modal curvature of the 3rd mode is zero. Thus, modes with a peak close to the intersection point are discarded from the damage identification process. However, for cases 4 and 6, the damage is modeled at the location where the modal curvature of all modes is nonzero, so all curves are used for the damage identification procedure.

Table 1. Damage scenarios of the numerical cantilever beam

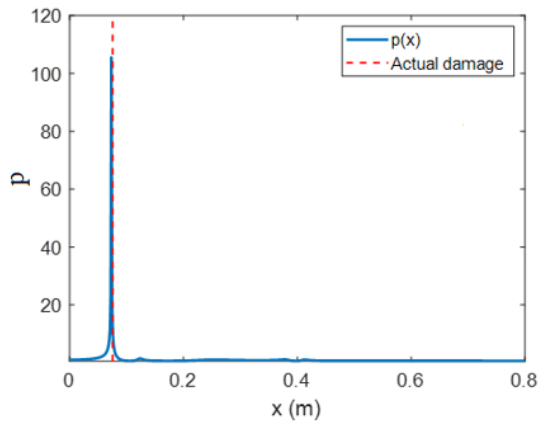
Case	Damage		Natural frequency (Hz)			
	x_0 (mm)	d (mm)	Mode 1	Mode 2	Mode 3	Mode 4
Intact	-	-	7.6850	48.149	134.79	264.08
1	75.6	10	7.6233	47.986	134.67	264.07
2	105.9	15	7.5567	47.981	134.78	263.44
3	173.3	20	7.5134	48.148	133.58	259.52
4	300.0	15	7.6375	47.838	133.83	263.86
5	397.1	15	7.6629	47.579	134.77	261.06
6	600	20	7.6820	47.753	131.69	258.98



(a) Plot of all curves

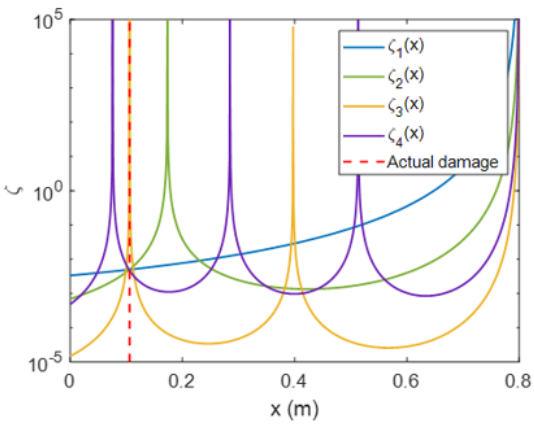


(b) Plot of a subset of curves

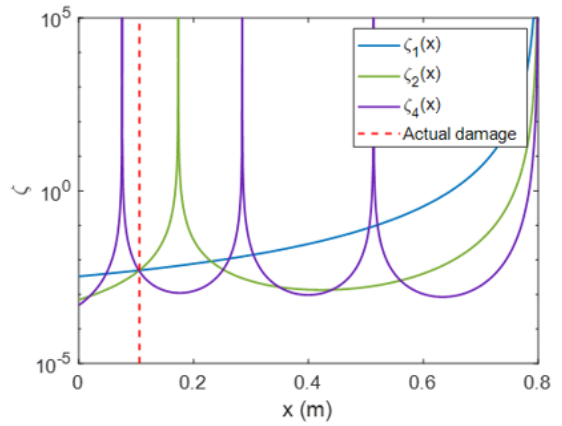


(c) Identified damage location

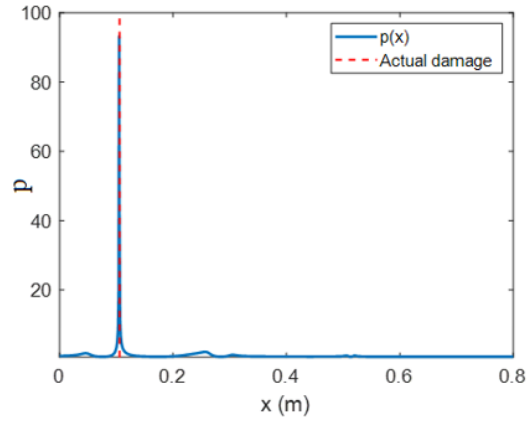
Figure 6. Damage detection for case 1



(a) Plot of all curves

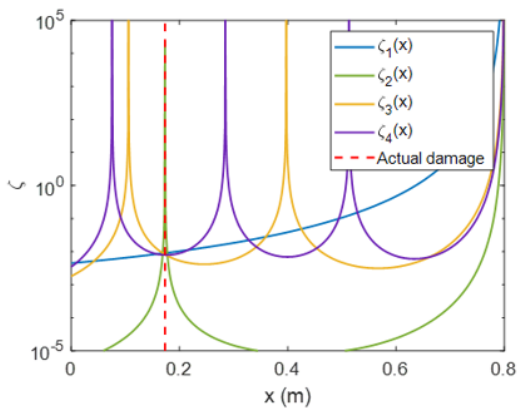


(b) Plot of a subset of curves

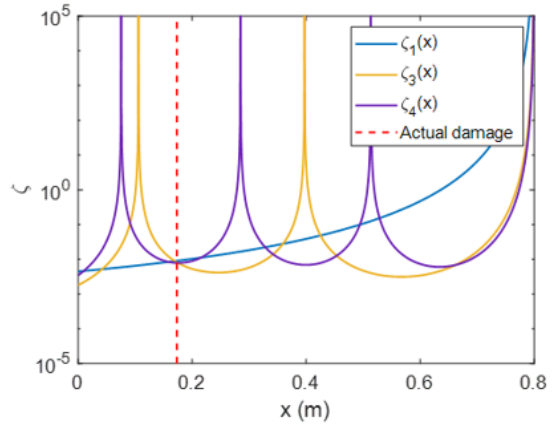


(c) Identified damage location

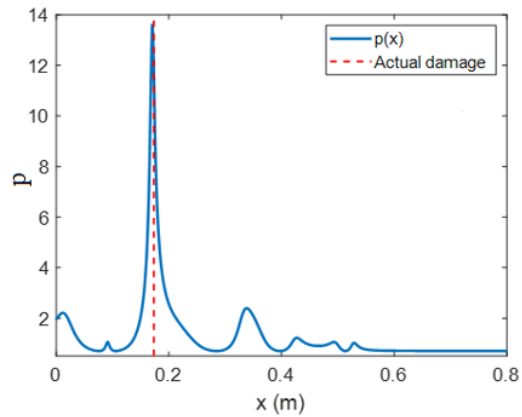
Figure 7. Damage detection for case 2



(a) Plot of all curves



(b) Plot of a subset of curves



(c) Identified damage location

Figure 8. Damage detection for case 3

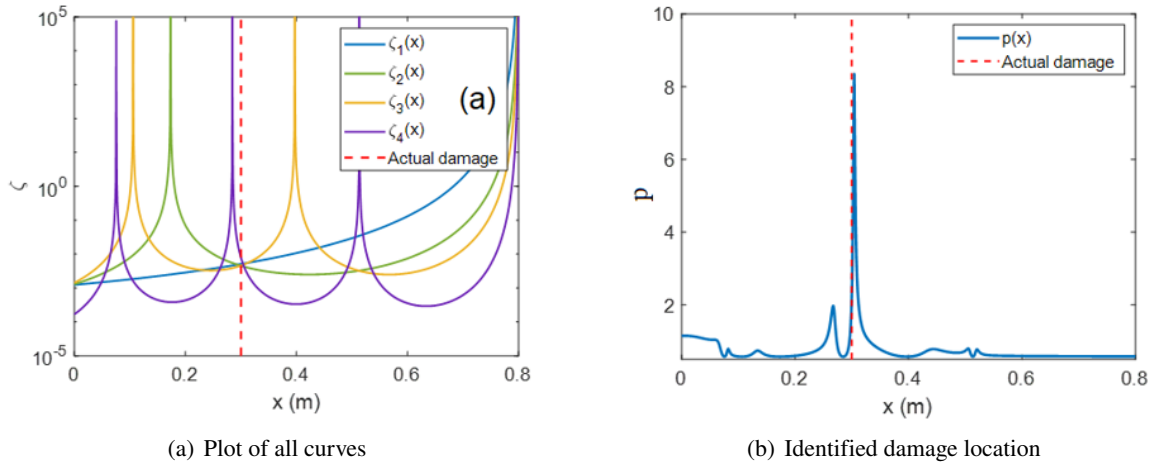


Figure 9. Damage detection for case 4

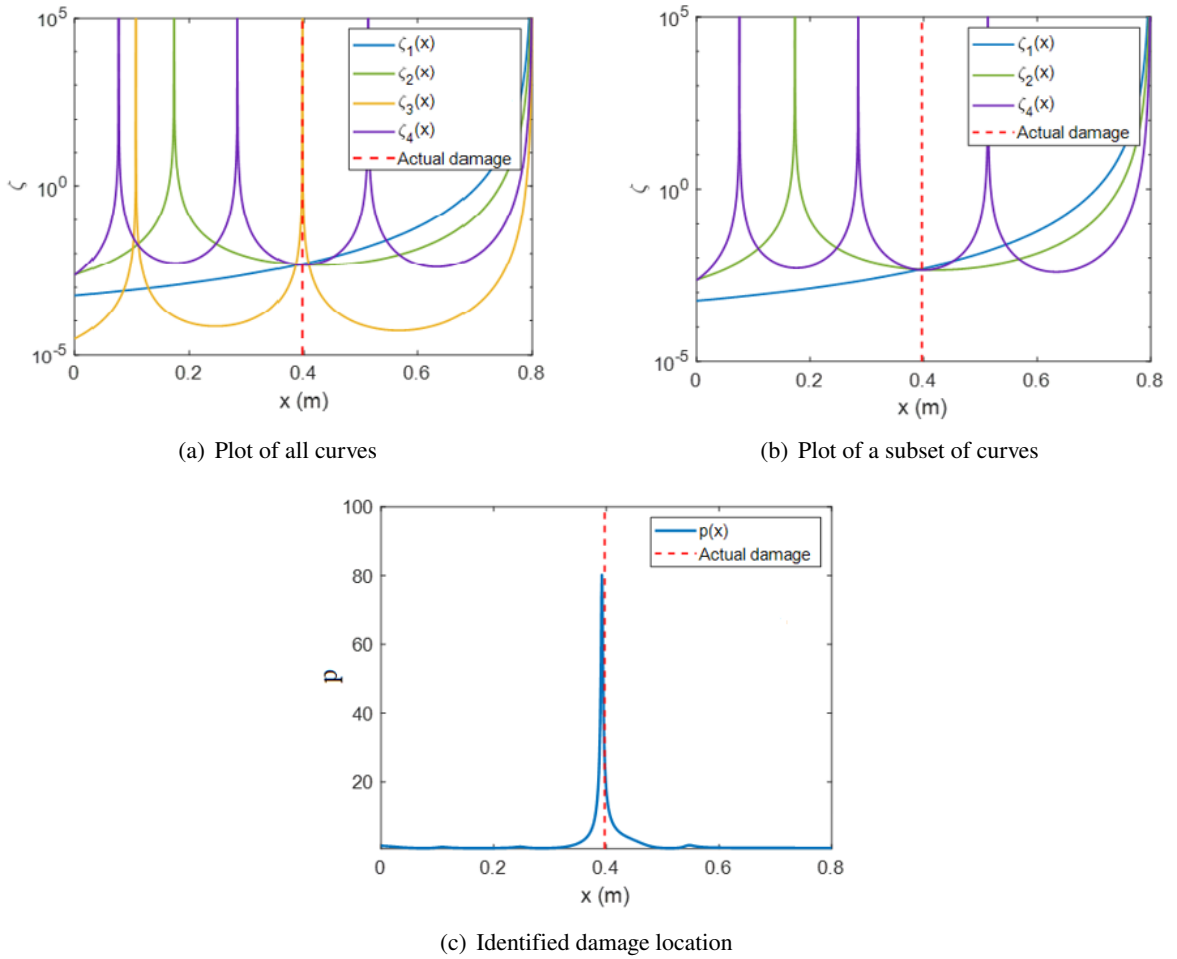


Figure 10. Damage detection for case 5

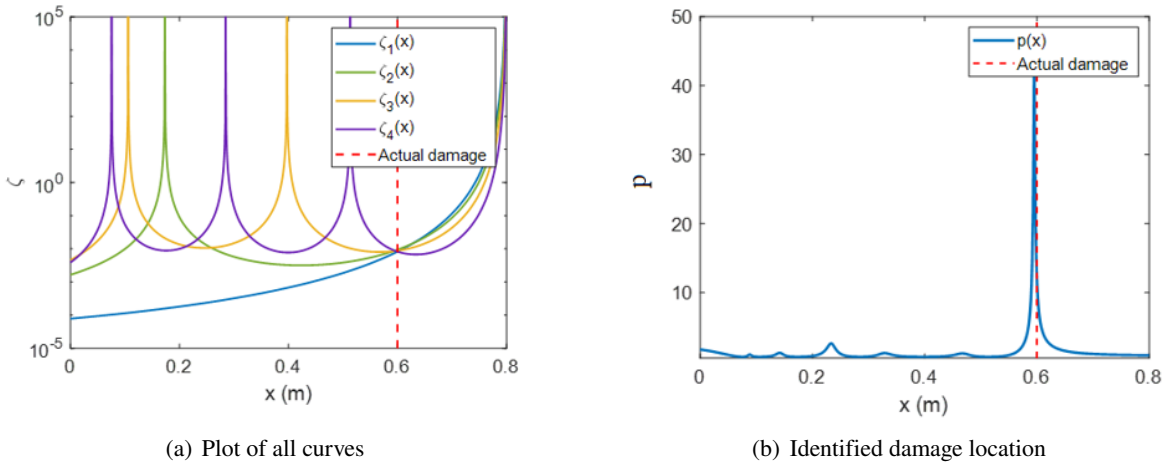


Figure 11. Damage detection for case 6

To estimate the damage depth, numerical simulations are performed to establish a relationship between the depths and their damage coefficients for the cantilever beam. The curve for this relationship is shown in Fig. 12.

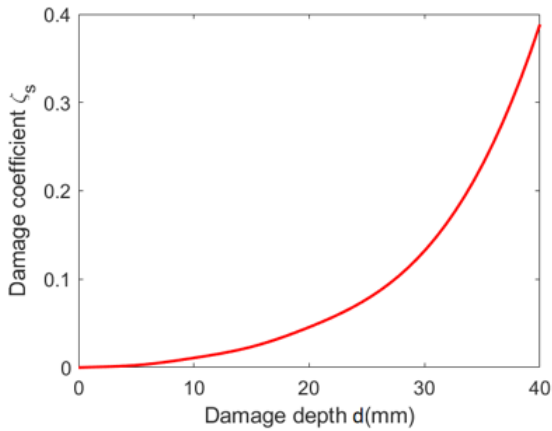


Figure 12. Damage coefficient vs. damage depth for the numerical test

Table 2. Results estimated from the numerical tests

No	x_0 (mm)	Error (%)	d (mm)	Error (%)
1	73.1	3.3	9.9	1.0
2	105.0	0.8	15.5	3.3
3	169.0	2.5	19.4	3.0
4	300.7	0.2	15.4	2.7
5	386.4	2.7	14.9	0.7
6	598.7	0.3	19.6	2.0

The results found are given in Table 2 indicating the location of the damage and the damage depth. It can be seen that they are close to real ones with an error in determining the location and the damage depth of less than 3.3%.

5. Verification of proposed procedure by experimental investigation

The beam used to perform the experimental studies presented in this article are steel cantilever beams. They have the following physical parameters: length $L = 1005$ mm, width $B = 42$ mm and height $H = 10$ mm. The material of the beam has Young's modulus $E = 200$ GPa, and density $\rho = 7850$ kg/m³.

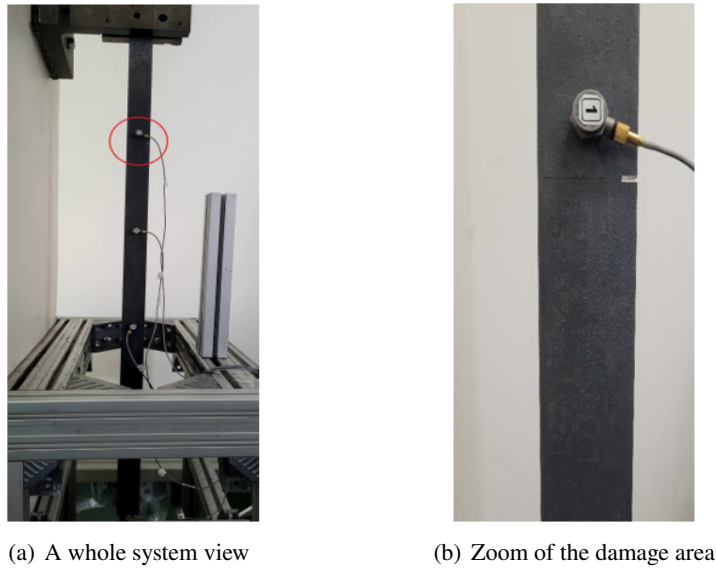


Figure 13. Experimental setup for the cantilever beam

For this test scenario, a cut 2 mm wide and 5 mm deep was created by machining. It was located at a distance of 220 mm from the support. The damage of the beam is shown in Fig. 13. The damage site is created close to the place where the modal curvature of the 2nd mode is equal to zero. Thus, this mode is excluded for damage identification procedure. To obtain the natural frequencies for the intact and damaged states of the beam, modal tests were carried out using an impact hammer. Time responses were recorded using five PCB piezoelectric accelerometers. Commercial B&K ConnectTM software was used to acquire the excitation and the beam responses. The modal parameters were determined using the Rational Fraction Polynomial (RFP) method implemented in the software. The measured frequencies of the beam are given in Table 3 while the identified location of the damage is shown in Fig. 14. The relationship between coefficient (ζ) and depth (d) is obtained by simulations (presented at Step 7 in Section 3.3) and shown in Fig. 15. The identified results are presented in Table 4. They are close to real values, which confirms the proposed method.

Table 3. Frequencies of the experimental test for the cantilever beam

No	Damage		Natural frequency (Hz)			
	x_0 (mm)	d (mm)	Mode 1	Mode 2	Mode 3	Mode 4
Intact	-	-	7.769	48.366	137.350	277.269
Damage	220	5	7.767	48.365	137.339	277.240

Table 4. Results estimated from the experimental test

No	x_0 (mm)	Error (%)	d (mm)
1	208	5.4	4.7

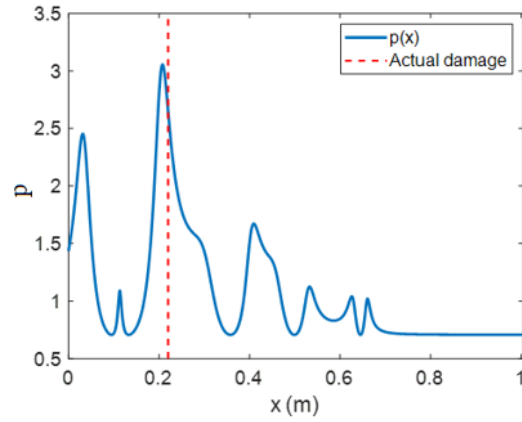
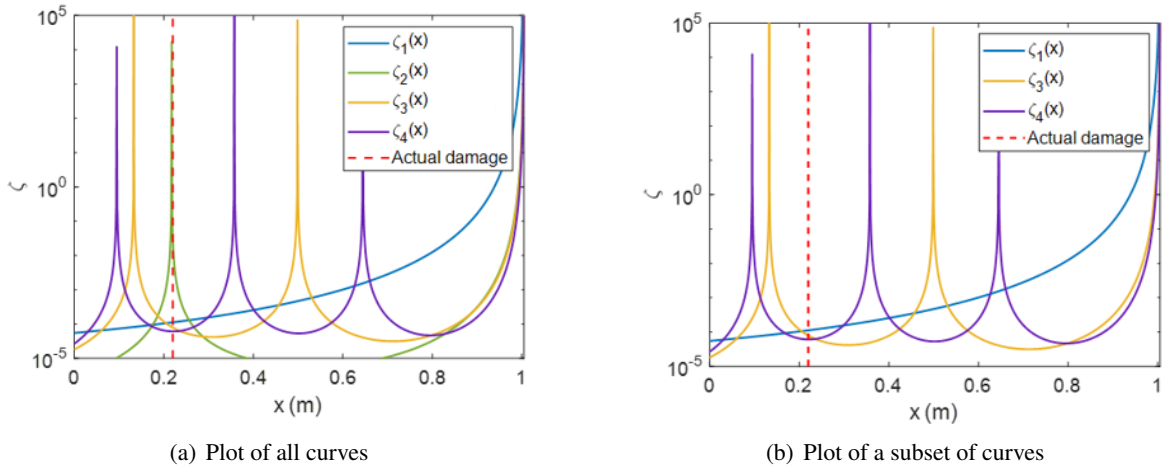


Figure 14. Damage detection for the experimental test

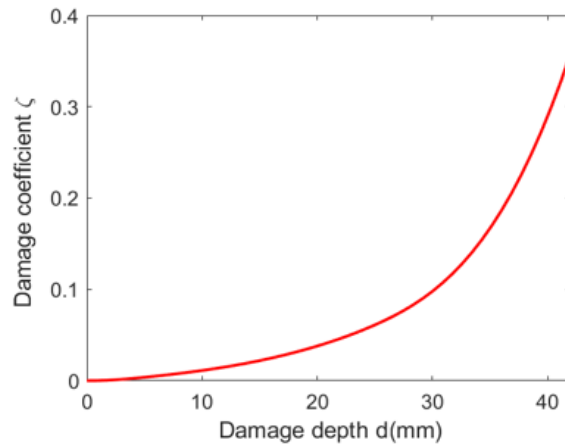


Figure 15. Damage coefficient vs. damage depth for the experimental test

6. Conclusions

This paper presents an improved procedure for detecting single damage in beam structures. First, curves of damage coefficients are constructed along the beam from natural frequency shifts and analytical modal curvatures. Then, the exact identification of the intersection of these curves gives the location of the damage. The numerical determination of the point of intersection of these curves is based on the singular value decomposition of matrices constructed from the ratios of the curves of damage coefficients. In consequence the degree of this damage is defined.

The proposed procedure was first applied to numerical examples. Good results of the identified damages (location and extent) were noted compared to exact ones on cantilever beams. Compared to the existing procedure, the proposed one makes the intersections more visual and simplifies their determination from the peaks of the inverse of the second singular value.

Finally, this procedure was tested on an experimental cantilever beam. The identified damage coincided well with the real one. This confirms the validity of the proposed method for detecting single damage in beam-like structures. The next step is to extend to multiple damages and first results can be seen in [20].

References

- [1] Chen, H.-P., Ni, Y.-Q. (2018). *Structural Health Monitoring of Large Civil Engineering Structures*. Wiley.
- [2] Doebling, S. W., Farrar, C. R., Prime, M. B., Shevitz, D. W. (1996). [Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review](#). Technical report.
- [3] Avci, O., Abdeljaber, O., Kiranyaz, S., Hussein, M., Gabbouj, M., Inman, D. J. (2021). [A review of vibration-based damage detection in civil structures: From traditional methods to Machine Learning and Deep Learning applications](#). *Mechanical Systems and Signal Processing*, 147:107077.
- [4] Lee, Y.-S., Chung, M.-J. (2000). [A study on crack detection using eigenfrequency test data](#). *Computers & Structures*, 77(3):327–342.
- [5] Patil, D. P., Maiti, S. K. (2005). [Experimental verification of a method of detection of multiple cracks in beams based on frequency measurements](#). *Journal of Sound and Vibration*, 281(1-2):439–451.
- [6] Gillich, G.-R., Praisach, Z.-I. (2014). [Modal identification and damage detection in beam-like structures using the power spectrum and time–frequency analysis](#). *Signal Processing*, 96:29–44.
- [7] Dahak, M., Touat, N., Benseddiq, N. (2017). [On the classification of normalized natural frequencies for damage detection in cantilever beam](#). *Journal of Sound and Vibration*, 402:70–84.
- [8] Sha, G., Radziński, M., Cao, M., Ostachowicz, W. (2019). [A novel method for single and multiple damage detection in beams using relative natural frequency changes](#). *Mechanical Systems and Signal Processing*, 132:335–352.
- [9] Surace, C., Bovsunovsky, A. (2020). [The use of frequency ratios to diagnose structural damage in varying environmental conditions](#). *Mechanical Systems and Signal Processing*, 136:106523.
- [10] Carden, E. P., Fanning, P. (2004). [Vibration Based Condition Monitoring: A Review](#). *Structural Health Monitoring*, 3(4):355–377.
- [11] Pandey, A. K., Biswas, M., Samman, M. M. (1991). [Damage detection from changes in curvature mode shapes](#). *Journal of Sound and Vibration*, 145(2):321–332.
- [12] Wahab, M. M. A., Roeck, G. D. (1999). [Damage detection in bridges using modal curvatures: Application to a real damage scenario](#). *Journal of Sound and Vibration*, 226(2):217–235.
- [13] Gorgin, R. (2020). [Damage identification technique based on mode shape analysis of beam structures](#). *Structures*, 27:2300–2308.
- [14] Dahak, M., Touat, N., Kharoubi, M. (2018). [Damage detection in beam through change in measured frequency and undamaged curvature mode shape](#). *Inverse Problems in Science and Engineering*, 27(1): 89–114.

- [15] Capecchi, D., Ciambella, J., Pau, A., Vestroni, F. (2016). [Damage identification in a parabolic arch by means of natural frequencies, modal shapes and curvatures.](#) *Meccanica*, 51(11):2847–2859.
- [16] Lee, J.-W. (2016). [Crack identification method for tapered cantilever pipe-type beam using natural frequencies.](#) *International Journal of Steel Structures*, 16(2):467–476.
- [17] Vakil-Baghmisheh, M.-T., Peimani, M., Sadeghi, M. H., Etefagh, M. M. (2008). [Crack detection in beam-like structures using genetic algorithms.](#) *Applied Soft Computing*, 8(2):1150–1160.
- [18] Moezi, S. A., Zakeri, E., Zare, A. (2018). [Structural single and multiple crack detection in cantilever beams using a hybrid Cuckoo-Nelder-Mead optimization method.](#) *Mechanical Systems and Signal Processing*, 99:805–831.
- [19] Khatir, S., Dekemele, K., Loccufier, M., Khatir, T., Wahab, M. A. (2018). [Crack identification method in beam-like structures using changes in experimentally measured frequencies and Particle Swarm Optimization.](#) *Comptes Rendus Mécanique*, 346(2):110–120.
- [20] Ta, D.-T. (2022). Structural health monitoring based on operational modal analysis. PhD thesis, Université Paris-Saclay.