Journal of Science and Technology in Civil Engineering, HUCE (NUCE), 2023, 17 (1): 16–25

PROBABILISTIC EVALUATION OF THE AXIS DISTANCE'S INFLUENCE ON THE FLEXURAL STRENGTH DETERIORATION OF REINFORCED CONCRETE BEAMS UNDER ISO 834 FIRE

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Article history:
Received 08/6/2022, Revised 31/8/2022, Accepted 06/9/2022

Abstract

The distance from the nearest concrete exposed surface to the centroidal axis of main longitudinal steel reinforcing bars, so-called axis distance, plays a critical role in ensuring the safety of reinforced concrete (RC) structures under fire, as it helps the rebars not being directly exposed to heating in a fire incident. However, a large axis distance value could reduce the effective height as well as the beam's flexural strength at ambient condition. In order to determine the appropriate values of axis distance, this article developes a data-driven method for predicting the flexural strength deterioration (FSD) of RC beams under ISO 834 standard fire based on the material and geometrical inputs. This method consists of two main stages: (i) Establishing a theoretical/experimental database by collecting experimental data from the literature; and (ii) Engineering a probabilistic model based on the Bayesian Neural Network. The results obtained show that the proposed approach is a practical tool that is capable of performing quick and reasonably accurate analysis such as degradation curves of FSD against exposure time. In addition, the uncertainty related to the prediction results is also evaluated, providing useful information for structural fire engineers to achieve conservative designs.

Keywords: reinforced concrete; structure, fire engineering; probabilistic; machine learning.

https://doi.org/10.31814/stce.nuce2023-17(1)-02

1. Introduction

Reinforced concrete (RC) is the most common structural material that is widely used in civil and industrial buildings, infrastructure systems and other subjects in construction field. Ensuring the safety of load-bearing RC structural members under fire conditions is of great importance, that helps save human life and valuable assets. For RC members, the fire safety is primarily considered by providing an appropriate covering concrete layer to protect the inner steel reinforcing bars (rebars), which can be characterized by the distance from the nearest concrete exposed surface to the rebars' centroidal axis, so-called axis distance. However, a relatively thick concrete covering layer will reduce the effective cross-section sizes, thus reducing the RC members' strength and stiffness. In the meantime, a thin concrete cover may not guarantee the particular fire resistance class that is required by

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design standards. To determine the adequate values of the axis distance, it is necessary to accurately estimate their influences on the structural performances of the considered RC members.

In the literature, there are a number of research works using different approaches, including experimental, theoretical, and numerical methods, to study the behavior of structural members subjected to standard fire conditions such as ISO 834 under different perspectives. Kodur et al. [1, 2] built a numerical model to estimate the flexural behavior of RC beams from elastic until failure using moment-curvature relationships. Thanks to the numerical model, parametric studies were carried out, showing the effects of different materials, fire scenarios, and geometric configurations without requiring expensive and time-consuming full-scale experiments. Thanaraj et al. [3] performed a series of experiments to explore the responses of RC beams with various concrete grades under fires. Recently, Gedam [4] proposed a theoretical method to estimate the RC beam load-bearing capacity based on the materials' thermally-induced properties, stress-strain relationship, and heat transfer model. In [5], the corresponding author analytically investigated the flexural strength deterioration (FSD) of RC beams, showing that the cross-section dimension and axis distance having a positive correlation with the FSD coefficient. Moreover, the authors also developed a calculation sheet to quickly calculate the FSD coefficient, which is useful for design practice [5]. Later, the authors [6] extended the theory for computing the FSD of RC T-beams, interestingly highlighting that the dimension of the beam's web and flanges have a negative correlation with FSD coefficients. Sun et al. [7] proposed a hybrid numerical method combining a finite element model for simulating members' thermal behavior with a one-dimensional spectral model for modeling RC beams response. The method significantly reduces the computational time while still provides competing results with more complex Finite Element models. Ozbolt et al. [8] proposed a thermo-mechanic three-dimensional (3-D) model to simulate the behavior of RC members under fire loads according to ISO 834. The model was successfully validated through comparison with experimental data from 4-point tests on RC beams. In addition to technical contribution for building a high-fidelity 3-D numerical model, the study provided noteworthy observations about the reduction in flexural stiffness of cracked beams due to thermal load after one hour of fire exposure and the occurrence of flexural shear failure after 1.5 hour. Zhang et al. [9] carried out both experimental and numerical studies to fully investigate the fire resistance of RC T-beam equipped with high strength reinforcement. The obtained results demonstrated that the flexural failure mode dominated with the presence of plastic hinges; hence, the full plastic analysis method is applicable for the design of RC T-beams with high-strength rebars.

On the other aspect, in the past decade, machine learning (ML) algorithms have been increasingly adopted by fire scientists and engineers to predict the behaviors of concrete-based structural members thanks to their practical, fast calculation and acceptable accuracy [10–12]. Naser [13] investigated the applicability of nine ML algorithms in analyzing fire-induced spalling of RC columns. Though obtained results were promising, the author emphasized the need for collaboration and sharing of data between research groups to address the data scarcity caused by the complexity of fire testing. Panev et al. [14] proposed a support vector machine-based algorithm to predict the fire resistance of composite shallow floor systems subjected to the ISO 834 standard fire. The proposed model could provide highly accurate results (up to 96%) about insulation ratings of the shallow system floor; on the other hand, the author also pointed out the limited extrapolation capacity of the method, i.e., if data points are too different compared to training data, prediction results may not be physically reasonable. Fu [15] developed a ML-framework specially designed for assessing the progressive collapse resistance of steel frame structures under fire, showing that the neural network-based model achieved better results than the counterparts given a sufficiently large dataset. Kodur et al. [16] explored a data-driven method to assess the fire hazard of bridges based on their geometric configurations (span, number of lanes), materials, and current operation states (damage, age). It was shown that the proposed method could be used as a low-budget tool to assess the fire vulnerability of bridges with similar patterns.

As pointed out by the aforementioned studies [10–16], a common major obstacle to the data-driven method is the scarcity of relevant data; this problem is more accentuated when studying structural members under fire compared to ambient condition. Even with data in hand, there exist unavoidable deviations between them because experiments and simulations were carried out by different authors in various conditions. Hence, the key contributions of this article are as follows:

- A probabilistic ML model on the basis of the Bayesian Neural Network rather than deterministic models introduced in the reviewed research works, is proposed for evaluating the effect of axis distance on the deterioration of RC beams' flexural strength. The advantage of such a probabilistic model is that it could not only predict quantities of interest, such as load, deformation, etc., but it is also able to estimate what amount of uncertainty is associated with prediction values; and

- With the help of the proposed ML model, the evolution curves of the FSD factors against exposure time can be computed for various values of axis distance, providing reference results for the subsequent studies.

The remainder of this article is organized as follows. Sections 2 and 3 respectively introduce the background and a database including both experimental and numerical data collected from the literature for RC beams' flexural strength deterioration under ISO 834 fire. After that, Section 4 describes the theoretical foundation and realization steps of the Bayesian Neural Network. Section 5 then demonstrates the calculation results obtained by BNN and resulting FSD curves for various axis distances. Finally, the conclusions and some ideas for future works are withdrawn in Section 6.

2. Flexural strength deterioration of RC beams exposed to ISO 834 fire

The European standard ISO-834 fire exposure [17] is one of the most common standards in the world, which describes in detail the temperature-time relation on the surface of the fire-exposed structural members, based on which the time-dependent temperature distribution and evolution within the cross-section of structural members can be further analyzed. According to the standard, the temperature at the beam's surface exposed to fire is determined by the following equation:

$$T = T_0 + 345 \log_{10}(8t + 1) \tag{1}$$

where t is time expressed in minutes, T is the temperature at time t in Celsius degree, and T_0 is the initial temperature, which is usually set to 20 °C.

In real situations, when a fire occurs, temperatures of any internal points within the beams' cross-sections will be lower than those of the points on the concrete surfaces which are directly exposed to fire, because heat requires a transfer process to reach these points. The transfer process is dependent on the materials' thermal properties. It is well known that the thermal conductivity of concrete is much lower than that of steel; therefore, it is reasonable to assume that the temperature in rebar is the same as that of the nearby concrete areas. As the temperature increases, the material strength will decrease accordingly, leading to the reduction in the beam flexural strength. This reduction is quantified through the FSD factor which is the ratio between the ultimate moment capacity at an elevated temperature T, i.e., $M_{u,T}$ with that at ambient temperature M_u as follows:

$$k_{FSD} = \frac{M_{u,T}}{M_u} \tag{2}$$

The value of $M_{u,T}$ could be experimentally determined indirectly through the ultimate load at failure or theoretically calculated using the 500 °C isotherm method as done in [5, 6].

3. Database on RC beams under ISO 834 fire

As a standard step when building any data-driven method, it is crucial to prepare a relevant database a priori, which would be used to train and validate the data-driven method later. The authors conducted a bibliography study from the literature and selected a number of published works which were directly related to the flexural behavior of beams at elevated temperatures. The criteria for selecting this database are the following three folds: (i) The considered concrete is ordinary Portland concrete (OPC), without any of other materials such as fly ash, geopolymer, fiber-reinforced polymer, etc.; (ii) Fire exposure is in conformity with the ISO 834 Standard; and (iii) The results should provide directly or indirectly information about the FSD of RC beams.

In summary, nine selected works are enumerated in Table 1, including the size of data, the cross-section of RC beams ($b \times h$), the tensile rebar ratio of the beam ($\mu_{A_{sbot}}$), the cube concrete strength (f_{cu}), duration (t) and axis distance (a).

It is noted that the tensile strength of the longitudinal reinforcement is not a variable of interest in the published research works since the reduction factors of hot-rolled reinforcing steel strength at elevated temperatures are identical between the steel classes [18]. However, in future this figure should be considered for the cold-worked reinforcing steel, which has different values of strength reduction at the same temperature compared to that of hot-rolled reinforcing steel.

Table 1. Summary of database on the flexural behavior of RC beams subjected to ISO 834 standard fire, collected from the literature

No	Study	Number of data	b (mm)	h (mm)	f_{cu} (MPa)	$\mu_{A_{sbot}}$ (%)	a (mm)	Time (min)
1	Kodur et al. [1]	9	300	500	37.5	0.63	50	0-240
2	Kodur and Dwaikat [2]	27	300	500	30	0.63	50	0-220
3	Thanaraj et al. [3]	35	200	200	20, 30, 40, 50	0.39	30	0-240
4	Gedam [4]	27	200	400	25	1.00	28, 48, 68	0-240
5	Nguyen et al. [5]	108	80, 160, 300, 500	150, 300, 600, 800	30	1.00	30	0-240
6	Nguyen et al. [6]	7	300	600	20	1.64	40	0-240
7	Sun et al. [7]	5	200	300	20	0.75	30	0-120
8	Ozbolt et al. [8]	5	200	300	20	0.75	30	0-120
9	Zhang et al. [9]	24	300	600	45	1.27	40, 60	0-220

For each work, one focused on the evolution of FSD against the temperature, relevant information is extracted and stored in a tabular format. These tabular data were concatenated together, forming a final database with 7 columns of features and 244 lines of data. Moreover, Fig. 1 displays histograms of features, showing clear visualization of the range of values as well as their distributions. It can be seen from Table 1 and Fig. 1 that the compressive strength of concrete varies in the range [20- 50 mm], the rebar ratio in [0.4-1.7%], the beam width in [80-500 mm], beam height in [160-500 mm], axis distance in [28-60 mm]. Next, the database is split into three non-overlapping datasets, namely training, validation and testing dataset with a ratio of 60:20:20.

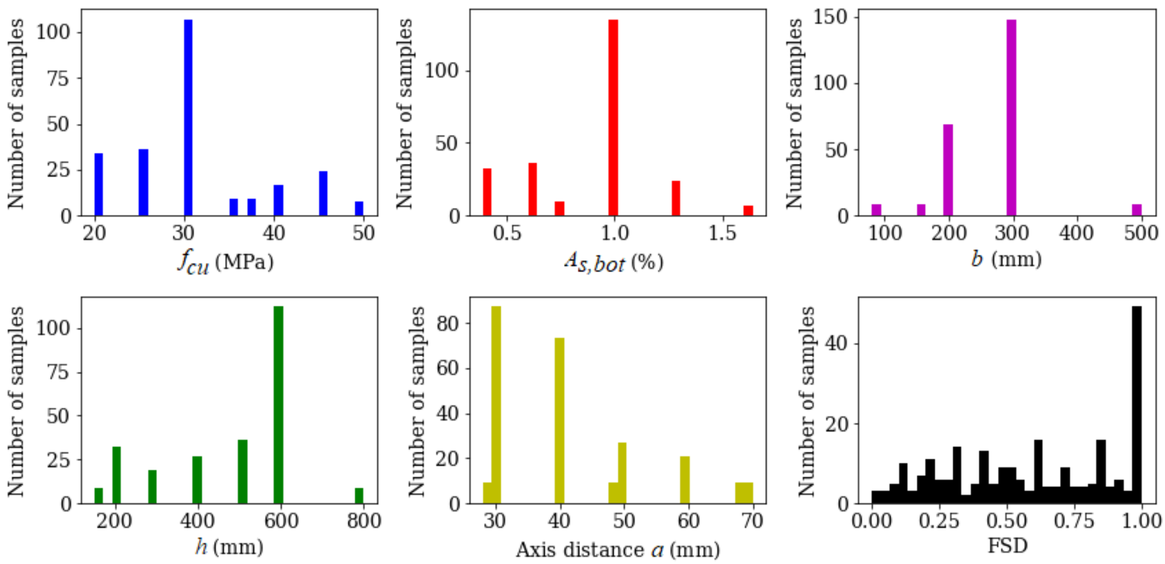

Figure 1. Statistical representation of database via histograms

4. Stochastic model using Bayesian Neural Network

Bayesian Neural Network (BNN) [19] is a probabilistic deep learning model that combines the high prediction performance of ANN with the ability to estimate uncertainty of the Bayes theory. In the authors' opinion, the model is especially suitable for working with not-so-abundant experimental data because two reasons: (i) In practice, similar series of experiments with identical input parameters still provide different results due to unavoidable uncertainty; and (ii) Fitting an ANN with many parameters to a limited database may cause the over-fitting problem, i.e., ANN is likely to yield low-accuracy results on new data despite being well trained. In other words, it is necessary to not only perform prediction of RC beam response but also to estimate how much confidence we have about the prediction results.

Specifically, the adopted architecture of the BNN in this study is 6/16/16/1, i.e., it consists of an input layer with 6 neurons, 2 hidden layers with 16 neurons and an output layer with 1 neuron corresponding to the FSD factor, as graphically illustrated in Fig. 2. For the input layer, there are six neurons corresponding to the concrete strength, beam width, beam height, rebar ratio, axis distance and exposure time. Because the data size is moderate, it is not reasonable to use either a too deep architecture of many hidden layers, or a wide layer with too many neurons, which could increase the number of parameters to determine significantly. On the other hand, as a rule of thumb, the number of neurons should be a power of 2 to be convenient with the binary memory structure of the computer; thus, one sets the number of neurons for the hidden layer as 16. Note that, for the BNN model, each neuron has two parameters to be determined as discussed previously, characterizing the probability distribution of its weight. At the beginning of learning, they are initialized as a normal distribution with zero mean and unity standard deviation.

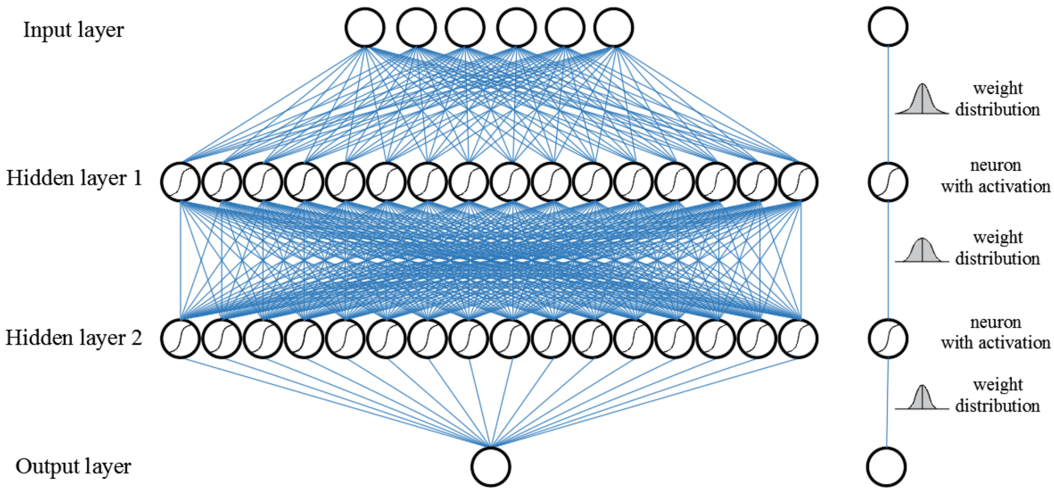

Figure 2. Graphical representation of Bayesian neural network whose weights are characterized by probability distributions

5. Calculation results on FSD factor using data-driven model

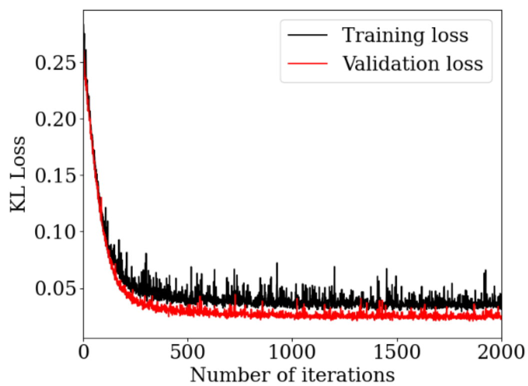

Figure 3. Evolution of KL loss function against number of iterations on training and validation datasets

The updating of the model's weights is realized with the help of the optimization algorithm Adam belonging to the first-order gradient descent optimization family which gradually adapts the model's weights by a small amount after each iteration to reduce the loss function. The amount of updates is controlled through a hyper-parameter, a.k.a, learning rate, which is set equal to 0.001. This value was determined via a preliminary test to ensure the learning process is convergent within a reasonable learning time. Note that a small learning rate will unnecessarily increase learning time, while a large value could lead to premature results. On the other hand, to obviate the scale difference issue between different features with different physical meanings, the pre-processing standardization is adopted. The implementation of the proposed data-driven framework is realized by the authors with the aid of the deep learning library Pytorch for building the overall framework, the deep probabilistic library Pyro [20] for building the BNN-based data-driven model, Pandas for data management, scikit-learn library [21] for data standardization and Matplotlib for data visualization.

Once the probabilistic model is built, it will be trained with the database prepared in the previous section associated with the aforementioned training setting. Fig. 3 depicts the learning curve of the BNN model, showing how loss functions evolve versus the number of training iterations, a.k.a., epochs, on both training and validation datasets. Apparently, the KL loss function quickly drops for epochs from 0 to 200, before gradually decreasing to values around 0.04 and 0.03 on the training and validation datasets, respectively. After that, a steady trend is observed, i.e., no clear improvement is obtained, until the number of epochs reaches 2000. It is also noticed that for epochs from 1600 to 1800, there is less fluctuation in loss function than for other intervals; hence, one selected the configuration at iteration 1600 as the final configuration of the BNN model.

Next, the final performance of the trained model is evaluated on the test dataset, whose results are demonstrated in Fig. 4. It can be seen in Fig. 4 that each data point (shown in the cross symbol), its X-coordinate denotes true FSD values from the database, while Y-coordinate is a value predicted by the model. Ideally, a perfect model will provide the same results as those from the database, as highlighted by the solid 45-degree inclined line. It can be seen that, predicted points lie closely to the ideal line. Statistically, the average of relative errors is about 6.5%. These results qualitatively and quantitatively confirm the viability of the BNN model.

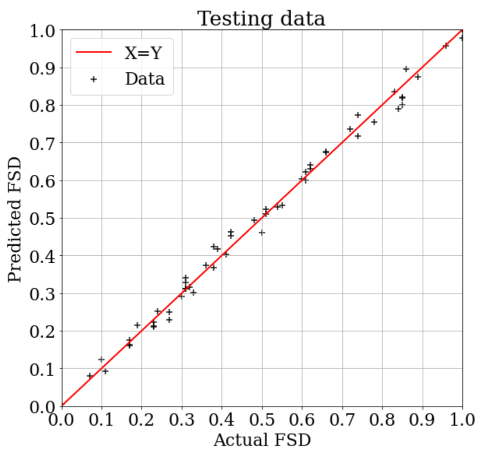

Figure 4. Comparison results between FSD values predicted by the data-driven models with true values from the testing dataset

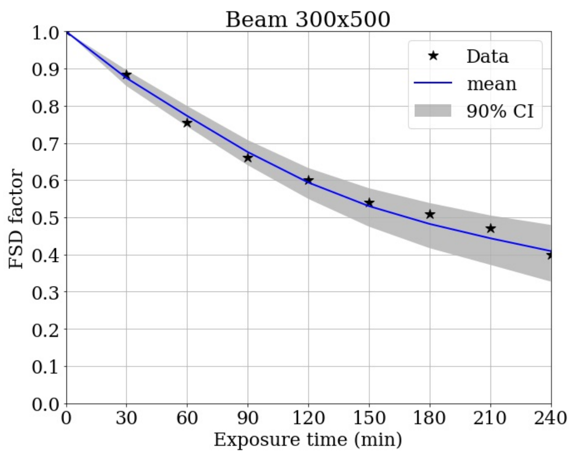

Figure 5. Representative example of FSD curve for a 300×500 beam with an axis distance of 30 mm and associated 90%-confidence interval, constructed by using the data-driven model

After that, the trained model is used to construct the FSD curve showing the evolution of the FSD factor against exposure time. Fig. 5 illustrates an example of computed FSD curves for a 300×500 (mm) beam with $\mu_{A_{sbot}} = 0.6\%$ and an axis distance of 50 mm. To obtain these results, the inference is repeated 100 times, then the average results (in solid blue line) and its 90% confidence interval (gray areas) are derived.

It is noted that for each inference, neurons' weights are randomly drawn from corresponding distributions; thus, they are not the same among inferences, leading to different prediction results. The 90% confidence interval (CI) area signifies that 90% of prediction results will fall within this area. Obviously, the CI area encompasses all data points (from the collected database). In other words, if there is any data point lying outside this CI area, it will be an anomaly uncorrelated with results from the literature, indicating potentially a miscalculation or an unexpected failure.

Next, the data-driven model is used to plot FSD curves for RC beams with $\mu_{A_{sbot}} = 1.0\%$ and with different values of axis distance as shown in Fig. 6 for beams of cross-sections 200×400, 350×500 and 400×600 (in mm). One can roughly divide a FSD curve into three parts. In the first part, the beam can still maintain its strength to some extent. After that, FSD decreases approximately linearly with a faster rate as curves shown in the leftmost subfigure. The third part corresponds to a short period before the failure occurs whose reduction rate is smaller than that of the second part. It is noted that with a low value of axis distance (a = 30 mm), the first part is nearly unnoticeable, i.e., the FSD drop quickly with exposure time; while for larger section and high values of a (a = 80 mm), it can maintain FSD values more than 90% for a period longer than 60 min as shown in the rightmost subfigure. Hence, it should apply adequate axis distance for RC beams to ensure required periods of fire resistance of structures.

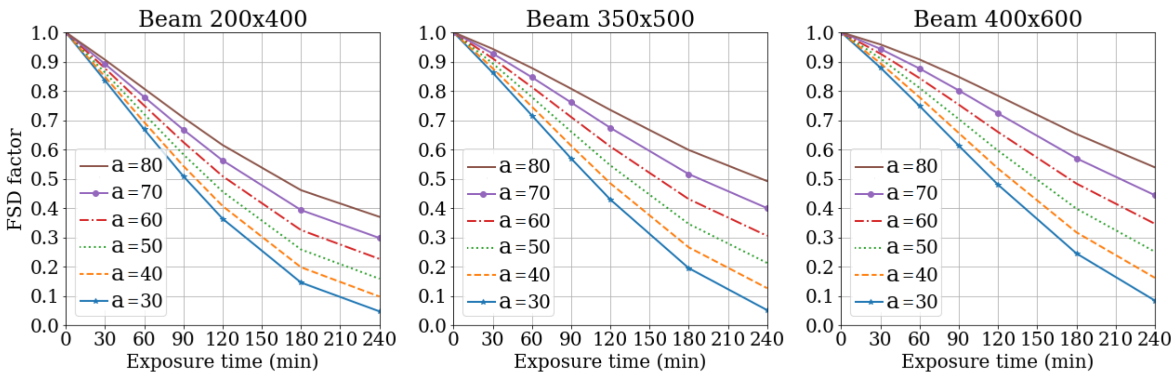

Figure 6. FSD curve for RC beams of various cross-section and different with axis distance values

In detail, the axis distance has a significant impact on the absolute value of the FSD factor, i.e., the larger the axis-distance, the larger the FSD factor. For example, for beam 200×400 (mm), FSD after 240 min for a = 30 mm and 80 mm are around 0.05, and 0.38, respectively. With larger cross-sections of beam (400×600 (mm)), after 240 min for a = 30 mm and 80 mm, FSD factor increases up to 0.1 and ~0.55. Besides, these subfigures placed side-by-side point out that the reduction rate of FSD factors of beams with large cross-sections is considerably lower than that of small beams.

It should be noted that although playing an active role on the FSD factor and the fire resistance of RC beams, the increment of axis distance requires some remarkable issues in the design as follows: (i) It reduces the effective height of the beam cross-section and leads to a reduction of the flexural strength of RC beams at ambient condition; thus the beams dimensions should be further increased for the ambient strength compensation and then influence to the designs of architecture, mechanical and plumping as well as the cost of the project; and (ii) With a relatively thick concrete cover, surface reinforcement mesh should be provided to avoid falling-off of the surface concrete layer when exposed to fire. For example, the Eurocode specifies that when the axis distance to the reinforcement is 70 mm or more, the surface reinforcement mesh should have a diameter not less than 4 mm and a spacing not greater than 100 mm [18].

6. Conclusions

The study presented in this article explores a data-driven method for assessing the flexural strength deterioration (FSD) of reinforced concrete (RC) beams subjected to ISO 834 fire exposure based on its geometric, material properties, and especially axis distance. The backbone of the proposed method is as follows: (i) An experimental and numerical database about the RC beams' flexural strength selectively gathered from accredited works; and (ii) A probabilistic machine learning model based on the Bayesian Neural Network. Throughout the article, an overview of RC beam's behavior at elevated temperatures, the theoretical foundation of the model, collected database, and key parameters of the proposed approach are described.

As one of the main results, the proposed model helps plot the FSD curves of RC beams for different values of axis distance and provide the associated confidence interval accounting for unavoidable uncertainties of input data and/or the prediction model. In addition, the utilization of the data-driven model is straightforward as it is built based on open source libraries and the user-friendly programming language Python without requiring any specialized software; thus, it could be used as a complementary tool to perform preliminary checks for any theoretical method or experimental set up for fire testing of RC beams.

For the next step of the study, one can incorporate into the database more material/geometric properties and enlarge the BNN model to a more generalized framework accounting for more types of RC beams such as T-beam, fly ash concrete, recycled concrete, etc. Another interesting direction is to extend the proposed method for other structural members such as RC columns, shear walls, steel-concrete composite members, etc. This can be done by using a similar procedure presented in this study, though for members featuring more complex behaviors such as combined shear and flexural behavior, it is required to consider simultaneously different metrics rather than only the FSD factor.

Acknoweledgement

The research presented in this paper was funded by Ministry of Construction (MOC Vietnam) under Grant No. RD 19-21. The financial support of MOC is gratefully acknowledged.

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