Journal of Science and Technology in Civil Engineering, HUCE (NUCE), 2022, 16 (3): 18–28

NAVIER SOLUTION FOR STATIC AND FREE VIBRATION ANALYSIS OF SANDWICH PLATE WITH AUXETIC HONEYCOMB CORE RESTING ON PASTERNAK ELASTIC FOUNDATION

Nguyen Tat Thang[a], Nguyen Van Long[a,*], Tran Minh Tu[a], Nguyen Hoang Nam[b], Mai Chau Anh[a]

[a] *Faculty of Civil and Industrial Construction, Hanoi University of Civil Engineering, 55 Giai Phong road, Hai Ba Trung district, Hanoi, Vietnam*
[b] *Key Research Group of Mechanics of Advanced Materials and Structures (MAMS), Hanoi University of Civil Engineering, 55 Giai Phong road, Hai Ba Trung district, Hanoi, Vietnam*

Article history:
Received 04/4/2022, Revised 16/5/2022, Accepted 20/5/2022

Abstract

This study investigates the bending behaviour and free vibration response of a rectangular sandwich plate with auxetic honeycomb core and functionally graded material (FGM) face sheets. Governing equations are established from Hamilton's principle in the framework of the first-order deformation theory (FSDT). The Navier solution has been utilized to determine the deflection and the natural frequency of the simply supported sandwich plate. The reliability and accuracy of the proposed model have been validated via verification studies. The numerical investigations are conducted to investigate the influence of geometric parameters of auxetic material, volume fraction index of FGM, the thickness ratio of layers, and elastic foundation stiffness on deflection and fundamental frequency of the FG-Auxetic sandwich plate.

Keywords: bending analysis; free vibration; sandwich plate; auxetic material; functionally graded material (FGM).

https://doi.org/10.31814/stce.huce(nuce)2022-16(3)-02

1. Introduction

Sandwich shells/plates are multi-layered structures composed of two thin outer skins of strong and stiff material separated by a thick core made from a low-density material [1]. Due to possessing many outstanding properties, such as high strength and stiffness-to-weight ratios, lightweight, and excellent absorption capacity in comparison with equivalent solid plates, sandwich plates are successfully utilized in aerospace, defense, civil, transportation, marine, and many mechanical engineering applications. By choosing the appropriate material and layer thickness ratio, the designer can get the desired sandwich structures, serving different purposes. The thin face sheets play important roles as not only the main bearing function but also a protective surface layer from the impact of the environment. The core materials of sandwich structures are commonly based on porous, lightweight foam such as metal foam, polystyrene foam or honeycomb form which possess superior energy absorption

*Corresponding author. *E-mail address:* longnv@huce.edu.vn (Long, N. V.)

capability [2]. These types of sandwich plates are commonly employed in constructing soundproofing, heat insulation walls and floors. Therefore, many studies on the kinetic and static behavior of sandwich structures with skin layers and a core made from different materials are conducted [3–10].

Auxetic materials are a class of advanced materials that exhibit numerous enhanced characteristics, both physical and mechanical [11]. Due to their superior material properties compared to conventional solid materials such as durability, lightweight, high stiffness and strength-to-weight ratio, good energy-absorption capacity, etc., they are widely used in various industrial branches. In particular, honeycombs are typical auxetic materials/structures that have many configurations such as triangular, square, circular, hexagonal, re-entrant, chiral, . . . Sandwich plates with honeycomb core have potential application in aerospace, civil, and defense, thus studies on their mechanical behavior are attracted significant interest from scientists.

Whal et al. [12] examined shear stress distribution in sandwich plates with an aluminium honeycomb core by using an analytical solution, finite element (FE) method and experimental validation. Streck et al. [13] studied the dynamic response of a sandwich panel with the auxetic core. Ruzzene et al. [14] analyzed vibration of and wave propagation in a sandwich plate with the cellular core by using the FE model. Based on the equivalent model in combination with 3D FEM, Wang et al. [15] presented free vibration response of sandwich plates with a hierarchical sandwich core in the form of a square honeycomb. In the framework of FSDT and FEM, Tran et al. [16] investigated the dynamic analysis of auxetic honeycomb core sandwich composite plates placing on an elastic foundation. Nam et al. [17] presented free vibration response, buckling analysis and dynamic instability of auxetic honeycomb plates using graphene platelets as reinforcement to skin layers. Mazaev and Shitikova [18] investigated the stressed state of sandwich plates with tetrarchical honeycomb core under static bending. The nonlinear vibrational characteristic of an auxetic laminated plate under blast loading is reported by Dat et al. [19] using an analytical approach. Quoc et al. [20] studied free vibration and dynamic response of sandwich plates with isotropic face sheets and auxetic core.

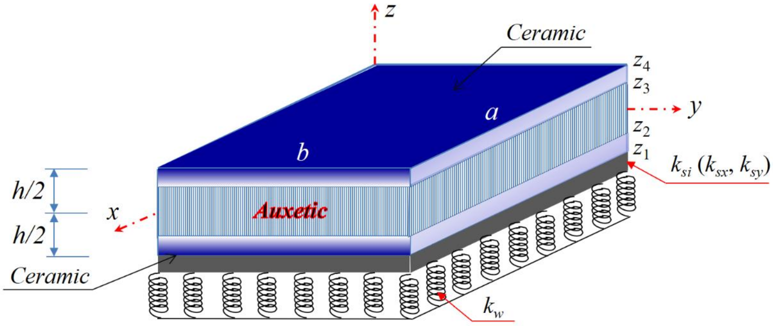
From the above comprehensive literature review, this paper proposes a model of the sandwich plate with FG face sheets and auxetic honeycomb core (re-entrant) applying for static and free vibration analysis for the first time. Following the FSDT, the Navier solution is utilized to predict deflection and natural frequencies of simply supported rectangular sandwich plates. The influence of face-sheet volume fraction index, geometric parameters of auxetic core, elastic foundation coefficients, layer's thickness ratios on deflection, and fundamental natural frequency is investigated in detail.

2. Theoretical formulations

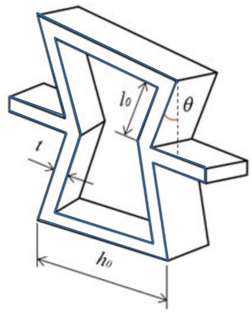
2.1. The FG-Auxetic sandwich plate

In this study, a sandwich plate with an auxetic core and two FGM face sheets is considered. The plate total thickness is h, the width and length are a and b, respectively. The plate is rested on the Pasternak elastic foundation (Fig. 1) with the Winkler stiffness of K_w and the shear stiffness of K_{si} $(i = x, y)$.

The thickness of face sheets and core layers are $(z_2 - z_1) = (z_4 - z_3) = h_F$, and $(z_3 - z_2) = h_C$, respectively. The FGM face sheets are made of two constituents: ceramic and metal, the effective material characteristics are adopted to vary along with thickness direction according to a simple power law. The core layer is made of a negative Poisson ratio material called auxetic honeycomb. The mechanical properties of the sandwich plate are defined as follows:



(a) The FG-auxetic sandwich plate



(b) Auxetic honeycomb unit cell

Figure 1. Configuration of an FG-auxetic sandwich plate resting on elastic foundtion

a. The top FGM face sheet (T): $z_3 \leq z \leq z_4$

$$E^T(z) = E_m + (E_c - E_m)\left(\frac{z - z_3}{z_4 - z_3}\right)^p; \quad \rho^T(z) = \rho_m + (\rho_c - \rho_m)\left(\frac{z - z_3}{z_4 - z_3}\right)^p \tag{1}$$

b. The bottom FGM face sheet (B): $z_1 \leq z \leq z_2$

$$E^B(z) = E_m + (E_c - E_m)\left(\frac{z_2 - z}{z_2 - z_1}\right)^p; \quad \rho^B(z) = \rho_m + (\rho_c - \rho_m)\left(\frac{z_2 - z}{z_2 - z_1}\right)^p \tag{2}$$

in which $p \geq 0$ is the volume fraction index of FGM, E_m, ρ_m are Young's modulus and mass density of the ceramic constituent, respectively; and E_c, ρ_c are modulus of elasticity and mass density of the metal constituent, respectively. For simplicity, the Poisson ratio of FGM is assumed to be constant.

c. Auxetic core (C): $z_2 < z < z_3$

An auxetic honeycomb-structured core type "*C*" with a hexagonal unit cell is shown in Fig. 1(b). Material properties are defined as follows [16]:

$$\begin{aligned}
E_1^C &= E\eta_3^3 \frac{\cos\theta}{[\eta_1 + \sin\theta]\sin^2\theta}; \quad E_2^C = E\eta_3^3 \frac{[\eta_1 + \sin\theta]}{\cos^3\theta} \\
\nu_{12}^C &= \frac{\cos^2\theta}{[\eta_1 + \sin\theta]\sin\theta}; \quad G_{12}^C = E\eta_3^3 \frac{[\eta_1 + \sin\theta]}{\eta_1^2[2\eta_1 + 1]\cos\theta} \\
G_{13}^C &= G\eta_3 \frac{\cos\theta}{[\eta_1 + \sin\theta]}; \quad G_{23}^C = G\eta_3 \frac{1 + 2\sin^2\theta}{2\cos\theta[\eta_1 + \sin\theta]} \\
\rho^C &= \rho \frac{\eta_3(\eta_1 + 2)}{2\cos\theta[\eta_1 + \sin\theta]}; \quad \eta_1 = \frac{h_0}{l_0}, \eta_3 = \frac{t}{l_0}
\end{aligned} \tag{3}$$

where E, G and ρ are modulus of elasticity, shear modulus and mass density of original material, whereas l_0, h_0, θ, t represent the length of the inclined rib and vertical rib, the inclined angle and the thickness of the cell, respectively.

2.2. Displacement field and stress-strain relations

Following the FSDT, displacement components of an arbitrary point in Cartesian coordinates (x, y, z) of a plate can be expressed as [21]:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t); \\ v(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t); \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{4}$$

where t is the time variable; and u_0, v_0, w_0 represent the displacements at the mid-plane of the plate in the x, y, z directions; θ_x, θ_y are the transverse normal rotations about the y and x-axes, respectively.

The linear strain-displacement relations of the plate are given as:

$$\begin{aligned} &\left\{\begin{array}{c} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{array}\right\} = \left\{\begin{array}{c} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{array}\right\} + z\left\{\begin{array}{c} \theta_{x,x} \\ \theta_{y,y} \\ \theta_{x,y} + \theta_{y,x} \end{array}\right\} = \left\{\begin{array}{c} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{array}\right\} + z\left\{\begin{array}{c} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{array}\right\}; \\ &\left\{\begin{array}{c} \gamma_{xz} \\ \gamma_{yz} \end{array}\right\} = \left\{\begin{array}{c} w_{0,x} + \theta_x \\ w_{0,y} + \theta_y \end{array}\right\} = \left\{\begin{array}{c} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{array}\right\} \end{aligned} \tag{5}$$

Mark (,) alongside displacement components indicates partial derivatives of corresponding variables.

The stress components in the n^{th} layer are determined from the stress-strain relations:

$$\left\{\begin{array}{c} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{array}\right\}^{(k)} = \left[\begin{array}{ccc} Q_{11}^{(k)} & Q_{12}^{(k)} & 0 \\ Q_{21}^{(k)} & Q_{22}^{(k)} & 0 \\ 0 & 0 & Q_{66}^{(k)} \end{array}\right]\left\{\begin{array}{c} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{array}\right\}; \quad \left\{\begin{array}{c} \sigma_{xz} \\ \sigma_{yz} \end{array}\right\}^{(k)} = \left[\begin{array}{cc} Q_{55}^{(k)} & 0 \\ 0 & Q_{44}^{(k)} \end{array}\right]\left\{\begin{array}{c} \gamma_{xz} \\ \gamma_{yz} \end{array}\right\} \tag{6}$$

where:

$$Q_{11}^{(C)} = \frac{E_1^{(C)}}{1 - \nu_{12}^{(C)}\nu_{21}^{(C)}}; \quad Q_{22}^{(C)} = \frac{E_2^{(C)}}{1 - \nu_{12}^{(C)}\nu_{21}^{(C)}}; \quad Q_{12}^{(C)} = \frac{\nu_{12}E_2^{(C)}}{1 - \nu_{12}^{(C)}\nu_{21}^{(C)}};$$

$$Q_{66}^{(C)} = G_{12}^{(C)}; \quad Q_{55}^{(C)} = G_{13}^{(C)}; \quad Q_{44}^{(C)} = G_{23}^{(C)};$$

$$Q_{11}^{(i)} = Q_{22}^{(i)} = \frac{E(z)}{1 - \nu^2}; \quad Q_{12}^{(i)} = \frac{\nu E(z)}{1 - \nu^2}; \quad Q_{66}^{(i)} = Q_{55}^{(i)} = Q_{44}^{(i)} = \frac{E(z)}{2(1 + \nu)}; \quad i = T, B$$

2.3. Governing equations

By employing the FSDT, the motion equations of the plate can be given as follows [22]:

$$\begin{gathered} N_{x,x} + N_{xy,y} = I_0\ddot{u} + I_0\ddot{\theta}_x; \quad N_{xy,x} + N_{y,y} = I_0\ddot{v}_0 + I_0\ddot{\theta}_y; \quad Q_{x,x} + Q_{y,y} + q + f_e = I_0\ddot{w}_0; \\ M_{x,x} + M_{xy,y} - Q_x = I_1\ddot{u}_0 + I_2\ddot{\theta}_x; \quad M_{xy,x} + M_{y,y} - Q_y = I_1\ddot{v}_0 + I_2\ddot{\theta}_y \end{gathered} \tag{7}$$

in which, moments of inertia: $I_i = \int_{-h/2}^{h/2} \rho(z)z^i dz; (i = 0, 1, 2)$; f_e is the density of reaction force of foundation, which is determined by [23, 24]: $f_e = -k_w w_0 + k_{sx} w_{0,xx} + k_{sy} w_{0,yy}$.

Internal force resultants of the plate are determined by:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}; \quad \begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{55}^s & 0 \\ 0 & A_{44}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \tag{8}$$

The stiffness coefficients of the sandwich plate are defined by:

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}\left(1, z, z^2\right) dz \quad \text{with} \quad ij = 11, 12, 22, 66$$

$$A_{ij}^s = k_c \int_{-h/2}^{h/2} Q_{ij} dz \quad \text{with} \quad ij = 55, 44$$

where k_c is the shear correction factor (assumed to be $k_c = 5/6$).

Substitute the stress resultants of Eq. (8) into Eqs. (7), we obtain the equations of motions in terms of displacements:

$$\begin{aligned}
&A_{11}u_{0,xx} + A_{66}u_{0,yy} + (A_{12} + A_{66})\,v_{0,xy} + B_{11}\theta_{x,xx} + B_{66}\theta_{x,yy} + (B_{12} + B_{66})\,\theta_{y,xy} = I_0\ddot{u} + I_0\ddot{\theta}_x; \\
&(A_{12} + A_{66})\,u_{0,xy} + A_{11}v_{0,yy} + A_{66}v_{0,xx} + (B_{12} + B_{66})\,\theta_{x,xy} + B_{11}\theta_{y,yy} + B_{66}\theta_{y,xx} = I_0\ddot{v}_0 + I_0\ddot{\theta}_y; \\
&A_{44}^s w_{0,yy} + A_{44}^s w_{0,xx} + A_{44}^s\theta_{x,x} + A_{44}^s\theta_{y,y} - k_w w_0 + k_{sx}w_{0,xx} + k_{sy}w_{0,yy} + q = I_0\ddot{w}_0; \\
&B_{11}u_{0,xx} + B_{66}u_{0,yy} + (B_{12} + B_{66})\,v_{0,xy} + D_{11}\theta_{x,xx} + D_{66}\theta_{x,yy} + (D_{12} + D_{66})\,\theta_{y,xy} \\
&\qquad - A_{44}^s\left(\theta_x + w_{0,x}\right) = I_1\ddot{u}_0 + I_2\ddot{\theta}_x; \\
&(B_{66} + B_{12})\,u_{0,xy} + B_{66}v_{0,xx} + B_{11}v_{0,yy} + (D_{66} + D_{12})\,\theta_{x,xy} + D_{66}\theta_{y,xx} + D_{11}\theta_{y,yy} \\
&\qquad - A_{44}^s\left(\theta_y + w_{0,y}\right) = I_1\ddot{v}_0 + I_2\ddot{\theta}_y;
\end{aligned} \tag{9}$$

The above five equations are the governing equations with five unknowns $u_0, v_0, w_0, \theta_x, \theta_y$ that can be used to investigate the static and free vibration response of the FG-Auxetic sandwich plate resting on Pasternak's elastic foundation.

3. Navier solution

Considering a simply supported rectangular sandwich plate with the length a and width b. Boundary conditions of the plate according to FSDT are obtained as:

$$\begin{aligned}
&x = 0, a : v_0 = 0, w_0 = 0, \theta_y = 0, N_x = 0, M_x = 0; \\
&y = 0, b : u_0 = 0, w_0 = 0, \theta_x = 0, N_y = 0, M_y = 0
\end{aligned} \tag{10}$$

The displacement unknowns satisfying the boundary conditions (10) are selected as:

$$\begin{aligned}
u_0 &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} u_{0mn} e^{i\omega t}\cos\alpha x\sin\beta y; \quad v_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} v_{0mn} e^{i\omega t}\sin\alpha x\cos\beta y;\\
w_0 &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{0mn} e^{i\omega t}\sin\alpha x\sin\beta y;\\
\theta_x &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \theta_{xmn} e^{i\omega t}\cos\alpha x\sin\beta y; \quad \theta_y = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \theta_{ymn} e^{i\omega t}\sin\alpha x\cos\beta y
\end{aligned} \tag{11}$$

where $u_{0mn}, v_{0mn}, w_{0mn}, \theta_{xmn}, \theta_{ymn}$ are unknown coefficients, and need to be determined; $\alpha = \dfrac{m\pi}{a}, \beta = \dfrac{n\pi}{b}; m, n = 1, 3, 5, ...$

The transverse distributed load $q(x, y)$ can be also expanded in the double trigonometric series as:

$$\begin{aligned}
q(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn}\sin\alpha x\sin\beta y;\\
q_{mn} &= \frac{1}{4ab}\int_0^a\int_0^b q(x,y)\sin\alpha x\sin\beta y dx dy
\end{aligned} \tag{12}$$

Substituting Eqs. (11) and Eq. (12) into the equations of motion Eqs. (9), we get linear algebraic equations ($\forall m, n$) as follows:

$$\left(\begin{bmatrix} s_{11} & s_{12} & 0 & s_{14} & s_{15}\\ s_{12} & s_{22} & 0 & s_{24} & s_{25}\\ 0 & 0 & s_{33} & s_{34} & s_{35}\\ s_{14} & s_{24} & s_{34} & s_{44} & s_{45}\\ s_{15} & s_{25} & s_{35} & s_{45} & s_{55} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & 0 & m_{14} & 0\\ 0 & m_{22} & 0 & 0 & m_{25}\\ 0 & 0 & m_{33} & 0 & 0\\ m_{14} & 0 & 0 & m_{44} & 0\\ 0 & m_{25} & 0 & 0 & m_{55} \end{bmatrix} \right) \begin{Bmatrix} u_{0mn}\\ v_{0mn}\\ w_{0mn}\\ \theta_{xmn}\\ \theta_{ymn} \end{Bmatrix} = \begin{Bmatrix} 0\\ 0\\ q_{mn}\\ 0\\ 0 \end{Bmatrix} \tag{13}$$

in which:

$$\begin{aligned}
& s_{11} = A_{11}\alpha^2 + A_{66}\beta^2; \quad s_{12} = (A_{12} + A_{66})\alpha\beta; \quad s_{14} = B_{11}\alpha^2 + B_{66}\beta^2;\\
& s_{15} = (B_{12} + B_{66})\alpha\beta; \quad s_{22} = A_{66}\alpha^2 + A_{11}\beta^2; \quad s_{24} = (B_{12} + B_{66})\alpha\beta;\\
& s_{25} = B_{66}\alpha^2 + B_{11}\beta^2; \quad s_{33} = A_{44}^s\alpha^2 + A_{66}\beta^2 + k_w + k_{sx}\alpha^2 + k_{sy}\beta^2; \quad s_{34} = A_{44}^s\alpha;\\
& s_{35} = A_{66}\beta; \quad s_{44} = D_{11}\alpha^2 + D_{66}\beta^2 + A_{44}^s; \quad s_{45} = s_{54} = (D_{12} + D_{66})\alpha\beta;\\
& s_{55} = D_{66}\alpha^2 + D_{11}\beta^2 + A_{66}; \quad m_{11} = m_{22} = m_{33} = I_0; \quad m_{14} = m_{25} = I_1; \quad m_{44} = m_{55} = I_2
\end{aligned}$$

For the static analysis, set $\omega = 0$, we obtain:

$$\begin{bmatrix} s_{11} & s_{12} & 0 & s_{14} & s_{15}\\ s_{21} & s_{22} & 0 & s_{24} & s_{25}\\ 0 & 0 & s_{33} & s_{34} & s_{35}\\ s_{41} & s_{42} & s_{43} & s_{44} & s_{45}\\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55} \end{bmatrix} \begin{Bmatrix} u_{0mn}\\ v_{0mn}\\ w_{0mn}\\ \theta_{xmn}\\ \theta_{ymn} \end{Bmatrix} = \begin{Bmatrix} 0\\ 0\\ q_{mn}\\ 0\\ 0 \end{Bmatrix}; \quad \forall m, n \tag{14}$$

By solving these equations, we can determine the unknown coefficients $\{Q_{mn}\} = \{u_{0mn}, v_{0mn}, w_{0mn}, \theta_{xmn}, \theta_{ymn}\}^T$, from which the displacements, strains, stresses can be obtained for static analysis.

For the free vibration analysis, set $q_{mn} = 0$, the eigenvalue problem is obtained as:

$$\left[\begin{array}{ccccc} s_{11} & s_{12} & 0 & s_{14} & s_{15} \\ s_{21} & s_{22} & 0 & s_{24} & s_{25} \\ 0 & 0 & s_{33} & s_{34} & s_{35} \\ s_{41} & s_{42} & s_{43} & s_{44} & s_{45} \\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55} \end{array}\right] - \omega^2 \left[\begin{array}{ccccc} m_{11} & 0 & 0 & m_{14} & 0 \\ 0 & m_{22} & 0 & 0 & m_{25} \\ 0 & 0 & m_{33} & 0 & 0 \\ m_{41} & 0 & 0 & m_{44} & 0 \\ 0 & m_{52} & 0 & 0 & m_{55} \end{array}\right] = 0 \tag{15}$$

The eigenvalue problem (15) is solved by using the standard eigenvalue algorithm and the fundamental angular frequency can be determined.

4. Numerical results and discussion

4.1. Verification examples

a. Verifying the deflection of the FGM sandwich plate

Table 1 shows the non-dimensional deflection of a square FGM sandwich plate with two FGM face sheets (face-sheet/core/face-sheet thickness ratio: 1-0-1) with $a = b = 10h$; $E_c = 151$ GPa; E_m = 70 GPa; $\nu = 0.3$. The plate is subjected to a bi-sinusoidal load $q = q_0 \sin\dfrac{\pi x}{a} \sin\dfrac{\pi y}{b}$. The present numerical results are verified with the analytical solutions by Zenkour [25] using the Navier solution approach. The non-dimensional deflection $\bar{w}$ is defined as follows:

$$\bar{w} = \frac{10hE_0}{q_0 a^2} w_0\left(\frac{a}{2}, \frac{b}{2}\right); \quad E_0 = 1 \text{ GPa} \tag{16}$$

Table 1. Non-dimensional deflection $\bar{w}$ of a square FGM sandwich (1-0-1) subjected to a bi-sinusoidal load

Source	p				
	0	1	2	5	10
Zenkour [25]	0.19605	0.32349	0.37319	0.40905	0.41750
Present	0.19607	0.32484	0.37514	0.41120	0.41919

b. Verifying fundamental frequencies of the FGM sandwich plate

Table 2 indicates the non-dimensional fundamental frequencies $\bar{\omega}$ of a square FGM sandwich plate composed of two FGM face sheets (1-0-1). The input data for the FGM plate: $a = b = 10h$; E_c = 380 GPa, ρ_c = 3800 kg/m^3; E_m = 70 GPa, ρ_m = 2707 kg/m^3; ν = 0.3. The present numerical results are validated with 3D solution by Li [26] using the Ritz method. The non-dimentional frequency is defined as follows:

$$\bar{\omega} = \omega_f \frac{a^2}{h}\sqrt{\frac{\rho_0}{E_0}}; \quad \rho_0 = 1 \text{ kg/m}^3; \quad E_0 = 1 \text{ GPa} \tag{17}$$

Table 2. Non-dimensional fundamental frequencies of a square FGM sandwich plate (1-0-1)

Source	p			
	0.5	1	5	10
3D - Li [26]	1.44614	1.24470	0.94476	0.92727
Present	1.44168	1.24032	0.94256	0.92508

c. Verifying the natural frequency of a sandwich plate composed of isotropic face sheets and Auxetic core

A simply supported (SSSS) rectangular FGM sandwich plate, which consists of two isotropic face sheets and an auxetic core, is considered. The thickness of the upper face-sheet and the lower face-sheet is assumed to be constant and is equal to h_F, while the core's thickness is h_C. It is assumed that the face sheets and the core layer are made from the same material, and the foundation is homogenous and isotropic ($k_{sx} = k_{sy} = k_s$). The following material and geometric properties of the sandwich plate are adopted: $h = h_C + 2h_F$, $h_C = 1.5h_F$, $E = 69$ GPa, $G = 26$ GPa, $\nu = 0.33$, and $\rho = 2700$ kg/m^3. The unit cell of hexagonal honeycomb with inclined angle θ and various ratios of h_0/l_0 is considered. The fundamental natural frequencies of the sandwich plate are computed and then compared with the results of Thanh Trung and co-workers [16] using the finite element method (FEM) following the FSDT (see Table 3).

Table 3. Fundamental natural frequency $f = \dfrac{\omega}{2\pi}$ (Hz) of a sandwich plate consists of isotropic face-sheets and auxetic core ($h = 0.1$ m, $a = b = 20h$, $t/l_0 = 0.01385, \theta = -55°$)

(k_w, k_s)	Source	$h_0/l_0 = 0.5$	$h_0/l_0 = 1$	$h_0/l_0 = 2$	$h_0/l_0 = 4$
(0, 0)	Tran et al. [16]	158.6420	142.8576	150.7676	151.7532
	Present	155.9038	140.3700	148.1536	149.1236
(0.1 GPa/m, 0)	Tran et al. [16]	206.6414	185.9790	196.3308	197.6211
	Present	204.5937	184.1132	194.3731	195.6519
(0.1 GPa/m, 0.05 GPa.m)	Tran et al. [16]	293.6156	264.1513	278.9102	280.7500
	Present	291.8352	262.5256	277.2061	279.0363

From three verification examples, it can be observed that the present results are in very good agreement with those gained by other authors. Thus proposed model, as well as the self-written Matlab code, are reliable.

4.2. Influence of plate parameters and foundation properties on the deflection and fundamental frequency of the FG-Auxetic sandwich plate

A rectangular FG-Auxetic sandwich plate composed of two FGM (Al/Al_2O_3) face sheets and an auxetic honeycomb core made of the same metal component of the FGM face sheets. The plate is rested on an homogenous and isotropic elastic foundation with input data: $h = 0.1$ m, $a/h = 20$, $b/a = 1$, $\eta_3 = 0.0138571$. For the static analysis, the uniformly distributed load $q = -10^4$ Pa is applied.

The influences of elastic foundation stiffness on the deflection and fundamental frequencies of the FG-Auxetic sandwich plate (1-2-1) are depicted in Figs. 2 and 3, respectively. As expected, an increase in elastic foundation stiffness (k_w, k_s) can enhance the plate stiffness. As a result, the deflection of the sandwich plate reduces, while the natural frequency goes up.

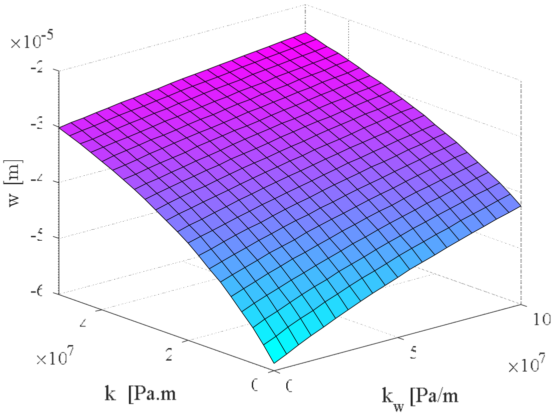


Figure 2. Influence of elastic foundation stiffness on deflection $w(a/2, b/2)$ of the FG-Auxetic sandwich plate

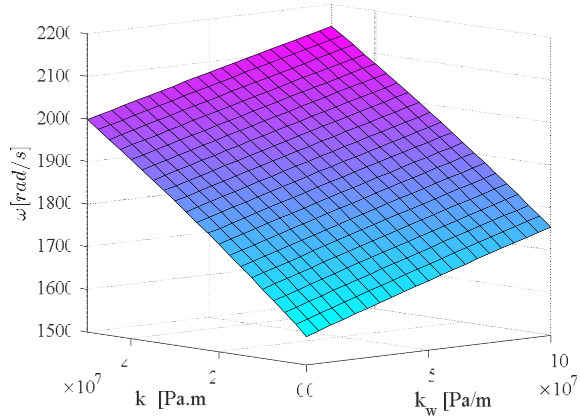


Figure 3. Influence of elastic foundation stiffness on fundamental frequency ω of the FG-Auxetic sandwich plate

Figs. 4 and 5 illustrate the influence of volume fraction index and face sheet-core-face sheet thickness ratio on deflection, and fundamental natural frequency ω of the FG-Auxetic sandwich plate resting on elastic foundation, respectively. It is clear that an increase in volume fraction index p results in increasing the metal content of FGM, leading to a decrease in the stiffness of the plate. As a result, deflection decreases, and fundamental natural frequency increases. Furthermore, the Pasternak stiffness k_s has more effect on increasing fundamental natural frequency and reducing deflection than the Winkler stiffness k_w.

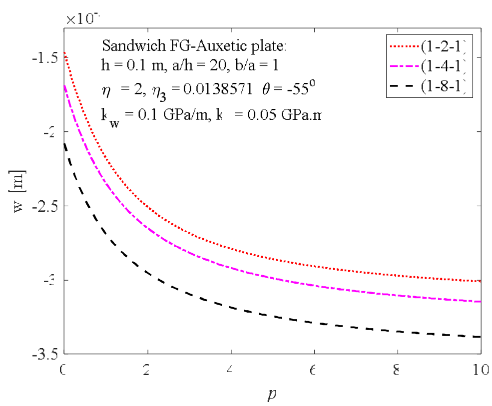


Figure 4. Influence of volume fraction index and face sheet-core-face sheet thickness ratio on the deflection of the sandwich plate

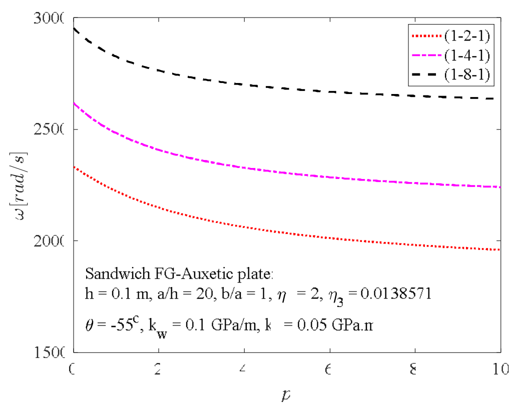


Figure 5. Influence of volume fraction index and face sheet-core-face sheet thickness ratio on the fundamental frequency of the sandwich plate

In addition, the significant effect of face sheet-core-face sheet thickness ratio on deflection, and fundamental natural frequency also can be observed in Figs. 4 and 5. The thicker core layer, the lower deflection, and the higher natural frequency.

The influences of inclined angle θ of auxetic honeycomb unit cell on the deflection and the natural frequency of the FG-Auxetic sandwich plate are illustrated in Figs. 6 and 7, respectively. It can be seen that the inclined angle θ has a significant effect on the natural frequency ω. In particular, a considerable increase in the natural frequency is observed followed by varying the inclined angle from 80° to −40°. This effect is extraordinary $\eta_1 = 1$. Nevertheless, the change in the inclined angle takes a very slight influence on the deflection of the FG-Auxetic sandwich structure.

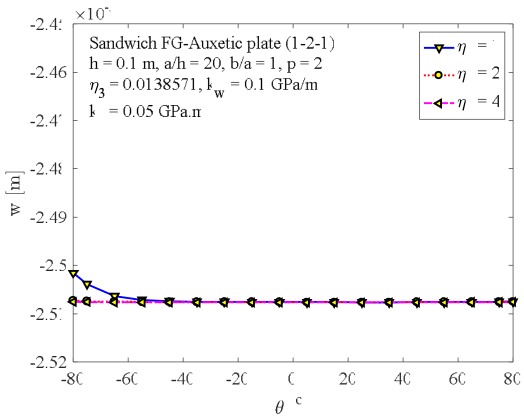


Figure 6. Effect of inclined angle θ on the deflection of the FG-Auxetic sandwich plate

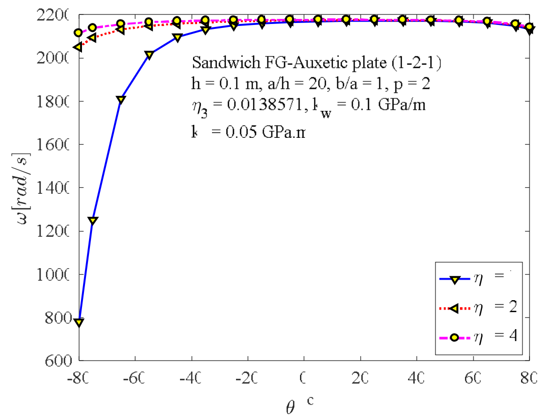


Figure 7. Influence of inclined angle θ on the natural frequency of the FG-Auxetic sandwich plate

5. Conclusions

Following the FSDT, in conjunction with Navier's technique, the bending and free vibration behavior of simply supported rectangular sandwich plates have been studied. The FGM face-sheet sandwich plate with an auxetic honeycomb core resting on Pasternak elastic foundation is considered. The reliability of Navier's solution and self-written Matlab code has been verified for the deflection of the FGM sandwich plate, and the natural frequency of the auxetic sandwich plate. The significant effect of volume fraction index of FGM, the inclined angle of auxetic honeycomb unit cell, face sheet-core-face sheet thickness ratio, elastic foundation stiffness on the deflection and the natural frequency of the FG-Auxetic sandwich plate is explored through numerical examples.

Acknowledgements

This research is funded by the Hanoi University of Civil Engineering (HUCE) under Grant number: 47-2021/KHXD-TĐ.

References

[1] Ha, K. H. (1990). Finite element analysis of sandwich plates: An overview. *Computers & Structures*, 37 (4):397–403.
[2] Heimbs, S. (2012). Foldcore Sandwich Structures and Their Impact Behaviour: An Overview. In *Dynamic Failure of Composite and Sandwich Structures*, Springer Netherlands, 491–544.
[3] Yuan, W. X., Dawe, D. J. (2002). Free vibration of sandwich plates with laminated faces. *International Journal for Numerical Methods in Engineering*, 54(2):195–217.
[4] Swanson, S. R. (2000). Response of Orthotropic Sandwich Plates to Concentrated Loading. *Journal of Sandwich Structures & Materials*, 2(3):270–287.

[5] Bennoun, M., Houari, M. S. A., Tounsi, A. (2015). A novel five-variable refined plate theory for vibration analysis of functionally graded sandwich plates. *Mechanics of Advanced Materials and Structures*, 23(4): 423–431.

[6] Benferhat, R., Daouadji, T. H., Abderezak, R. (2021). Effect of porosity on fundamental frequencies of FGM sandwich plates. *1*, 3(1):25.

[7] Natarajan, S., Manickam, G. (2012). Bending and vibration of functionally graded material sandwich plates using an accurate theory. *Finite Elements in Analysis and Design*, 57:32–42.

[8] Zenkour, A. M., Sobhy, M. (2010). Thermal buckling of various types of FGM sandwich plates. *Composite Structures*, 93(1):93–102.

[9] Joseph, S. V., Mohanty, S. C. (2017). Free Vibration of a Rotating Sandwich Plate with Viscoelastic Core and Functionally Graded Material Constraining Layer. *International Journal of Structural Stability and Dynamics*, 17(10):1750114.

[10] Hadji, L., Avcar, M. (2021). Free vibration analysis of FG porous sandwich plates under various boundary conditions. *Journal of Applied and Computational Mechanics*, 7(2):505–519.

[11] Novak, N., Vesenjak, M., Ren, Z. (2016). Auxetic Cellular Materials - a Review. *Strojniški vestnik – Journal of Mechanical Engineering*, 62(9):485–493.

[12] Wahl, L., Maas, S., Waldmann, D., Zürbes, A., Frères, P. (2012). Shear stresses in honeycomb sandwich plates: Analytical solution, finite element method and experimental verification. *Journal of Sandwich Structures & Materials*, 14(4):449–468.

[13] Strek, T., Jopek, H., Nienartowicz, M. (2015). Dynamic response of sandwich panels with auxetic cores. *physica status solidi (b)*, 252(7):1540–1550.

[14] Ruzzene, M., Mazzarella, L., Tsopelas, P., Scarpa, F. (2002). Wave Propagation in Sandwich Plates with Periodic Auxetic Core. *Journal of Intelligent Material Systems and Structures*, 13(9):587–597.

[15] jing Wang, Y., jia Zhang, Z., min Xue, X., Zhang, L. (2019). Free vibration analysis of composite sandwich panels with hierarchical honeycomb sandwich core. *Thin-Walled Structures*, 145:106425.

[16] Tran, T. T., Pham, Q. H., Nguyen-Thoi, T., Tran, T.-V. (2020). Dynamic Analysis of Sandwich Auxetic Honeycomb Plates Subjected to Moving Oscillator Load on Elastic Foundation. *Advances in Materials Science and Engineering*, 2020:1–16.

[17] Nguyen, N. V., Nguyen-Xuan, H., Nguyen, T. N., Kang, J., Lee, J. (2021). A comprehensive analysis of auxetic honeycomb sandwich plates with graphene nanoplatelets reinforcement. *Composite Structures*, 259:113213.

[18] Mazaev, A. V., Shitikova, M. V. (2021). Numerical analysis of the stressed state of composite plates with a core layer made of tetrachiral honeycombs under static bending. *Composites Part C: Open Access*, 6: 100217.

[19] Dat, N. D., Quan, T. Q., Duc, N. D. (2022). Vibration analysis of auxetic laminated plate with magneto-electro-elastic face sheets subjected to blast loading. *Composite Structures*, 280:114925.

[20] Quoc, T. H., Tu, T. M., Tham, V. V. (2021). Free vibration and dynamic response of sandwich composite plates with auxetic honeycomb core. *Journal of Science and Technology in Civil Engineering (STCE) - HUCE*, 15(4):1–14.

[21] Reddy, J. N. (2006). *Theory and Analysis of Elastic Plates and Shells*. CRC Press.

[22] Reddy, J. N. (2017). *Energy principles and variational methods in applied mechanics*. John Wiley & Sons.

[23] Zenkour, A. M. (2009). The refined sinusoidal theory for FGM plates on elastic foundations. *International Journal of Mechanical Sciences*, 51(11-12):869–880.

[24] Huang, Z. Y., Lü, C. F., Chen, W. Q. (2008). Benchmark solutions for functionally graded thick plates resting on Winkler–Pasternak elastic foundations. *Composite Structures*, 85(2):95–104.

[25] Zenkour, A. M. (2005). A comprehensive analysis of functionally graded sandwich plates: Part 1—Deflection and stresses. *International Journal of Solids and Structures*, 42(18-19):5224–5242.

[26] Li, Q., Iu, V. P., Kou, K. P. (2008). Three-dimensional vibration analysis of functionally graded material sandwich plates. *Journal of Sound and Vibration*, 311(1-2):498–515.