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EFFECTS OF LIGHTWEIGHT PARTICLE CONTENT ON THE MECHANICAL STRENGTH OF CYLINDRICAL AGGREGATES

Ho Van Nhat Phong[a], Vo Thanh Trung[b,*], Tran Dinh Minh[c], Nguyen Van My[c], Vu Thi Lo[d,e]

[a] *University of Science and Technology - The University of Danang, 54 Nguyen Luong Bang street, Lien Chieu district, Da Nang City, Vietnam*
[b] *Department of Research and International Affairs, Danang Architecture University, 566 Nui Thanh street, Da Nang city, Vietnam*
[c] *Faculty of Road and Bridge Engineering, University of Science and Technology - The University of Danang, 54 Nguyen Luong Bang street, Lien Chieu district, Da Nang city, Vietnam*
[d] *Division of Computational Mathematics and Engineering, Institute for Computational Science, Ton Duc Thang University, 19 Nguyen Huu Tho street, District 7, Ho Chi Minh city, Vietnam*
[e] *Faculty of Civil Engineering, Ton Duc Thang University, 19 Nguyen Huu Tho street, District 7, Ho Chi Minh city, Vietnam*

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Abstract

By using an extensive three-dimensional discrete element method, we numerically investigate the effects of lightweight particle content on the mechanical strength of the cylindrical aggregates, randomly composed of two different groups of primary particles: light (low stiffness) and heavy (high stiffness), and the stress contribution of the heavy-heavy, light-heavy, and light-light interactions on the mechanical behavior of such aggregates, subjected to the compression test by using a constant downward velocity. The interactions between primary particles are modeled by using the frictional-contact force law with a giving approximate analytical expression of capillary cohesion forces. We found that the lightweight particle content significantly affects the mechanical stress of aggregates and stress contribution obtained by different interaction types above, and the stress obtained by heavy-heavy interactions strongly dominates the mechanical strength of agglomerates. Remarkably, meanwhile, the stress obtained by the heavy-heavy contacts gradually decreases with increasing the lightweight particle content up to 20%, leading to the decrease of the mechanical strength, the stresses obtained by light-light and light-heavy contacts increase gradually. For higher lightweight particle contents, the results continuously show the same tendencies but with lower rates.

Keywords: discrete element method; capillary cohesion; compression test; lightweight particle content; stiffnesses.

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1. Introduction

Aggregates or cemented granular aggregates of light (soft) and heavy (rigid) primary particles are common cases of granular materials [1, 2]. Such aggregates are commonly found in nature due to

*Corresponding author. *E-mail address:* trungvt@dau.edu.vn (Trung, V. T.)

the accretion between soil grains with gravels/rocks under the action of the cohesion forces induced by the presence of the interstitial liquid between grains. In industry, lightweight aggregate concrete is a popular product, created by replacing a part of gravels with waste rubber, found as a technical and economical solution in order to reduce the increase of the waste rubber as a consequence of remarkably increasing of vehicles worldwide [3, 4]. Due to the presence of content of lightweight grains (as waste rubber), the mechanical strength of such aggregates is weakened [3].

In recent decades, a number of experimental works have been performed in order to understand the effects of lightweight (waste rubber) particles on the physical and mechanical properties of aggregates. Lv et al. [3] experimentally studied the influence of rubber lightweight particles on mechanical properties of aggregate concrete in different ages, showing that the compressive strength is reduced in the presence of light particles and significantly reduced when the percentage of rubber particles is lower than 50%. The compressive strength of rubber concrete is also significantly decreased by using different sizes of rubber particles, and the rate of this reduction increases with increasing the size of rubber grains [5]. As a result of the decrease of the mechanical strength of rubber concrete, the presence of rubber particles leads to the significant effects of the formation and development of cracks [6] In contrast to the decrease of the compressive strength of rubber concrete, the replacement of gravel by rubber particles (or waste rubber) could improve the ductility, the impact resistance as a consequence of increasing the absorption of impact energy, the tensile strength, and bending strength of rubber concrete [7, 8].

As mentioned above, studies focusing on the physical and mechanical properties of concrete by replacing a part of its aggregates with rubber particles were mainly carried by using the experimental investigations by uniaxial compression test. Thus, it is difficult to get a fundamental understanding of the interactions between particles/aggregates as well as the stress contribution obtained by these interactions in experimental works. In contrast, numerical investigations have the advantage of detecting different kinds of contacts based on the discrete nature of rigid and deformable particles (rubber grains). However, recent studies mainly investigated the compaction of only highly deformable particles or mixtures of rigid and deformable grains [9, 10].

In this paper, by replacing a part of heavy (high stiffness) particles with light (low stiffness) particles, we numerical investigate the effects of lightweight particle content on the mechanical strength of wet cylindrical agglomerates under uniaxial compression test. To simplify the model, the deformable characteristics of lightweight particles are neglected, implying that all particles are modeled as rigid grains. The interactions between particles are modeled by considering the frictional contact force law with the inclusion of the capillary cohesion forces. This modeling approach allows the application of the current numerical model to study the mechanical strength of the early-age rubber concrete. As we shall see, the lightweight particles significantly affect the mechanical behavior of lightweight aggregate concrete, and the different interactions between grains inside aggregates have different contributions to the strength of such aggregates, this can be explained due to the origins of the normal contact forces.

In the following, we briefly introduce the numerical method and model preparation in Section 2. Then, in Section 3, we numerically analyze the evolution of the mechanical strength of aggregates as well as the contribution of different kinds of contacts to the strength of such aggregates. Finally, we conclude in Section 4 with a short summary results and further research directions.

2. Number method

The discrete element method (DEM) has been used for simulations of granular materials in nature and industrial processes [11–13]. Due to the advantage of exactly reflecting the discrete nature of the material, the physical and mechanical properties of granular materials in different configurations are well determined. In DEM, all particles are modeled as rigid grains and interact with neighboring particles by considering the frictional contact force law [14, 15]. This numerical method is also possible to implement the interstitial liquid in the form of capillary bridges which induced the cohesion forces between grains. Thus, in advanced DEM, the equations of motion of all rigid particles are integrated according to the explicit step-wise scheme (as shown in Fig. 1) by taking into account the particle interactions under the action of the normal contact forces f_n, tangential forces f_t, and normal cohesion forces f_c, as shown in Fig. 2.

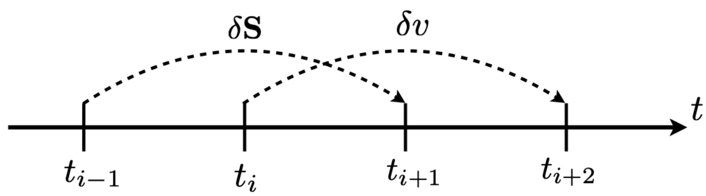
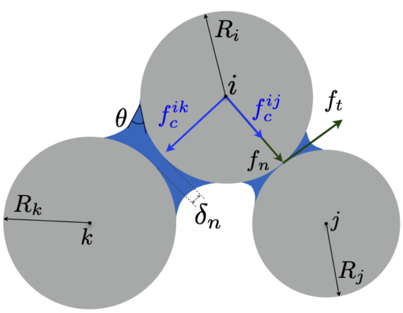
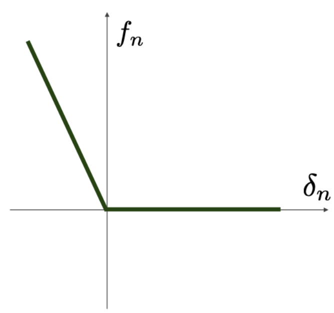


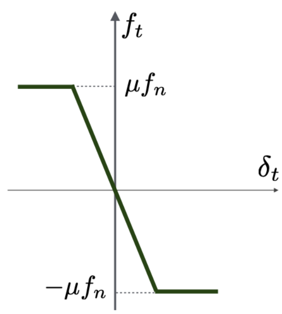
Figure 1. Time-stepping schematic drawing used in DEM



(a) Schematic drawings of the capillary bridges between particles



(b) The normal contact force



(c) The tangential contact force

Figure 2. The contact forces law used in DEM

Integration the equations of motion of particle i with displacement is shown by the following expression:

$$m_i \frac{d^2 \boldsymbol{s}_i}{dt^2} = \sum_i (f_n \boldsymbol{n} + f_t \boldsymbol{t}) + \sum_i f_c \boldsymbol{n} \tag{1}$$

and rotation is shown by the expression:

$$\boldsymbol{I}_i \frac{d\omega_i}{dt} = \sum_i f_t \boldsymbol{t} \times \boldsymbol{c} \tag{2}$$

where m_i and s_i are the mass and position vector of particle i. ω_i is the angular velocity of particle i. $\boldsymbol{I}_i$ is the inertial matrix of particle i. $\boldsymbol{n}$ and $\boldsymbol{t}$ are the normal and tangential unit vectors, respectively.

n has the direction that points from neighboring particles to particle i during contacting, whereas ***t*** has the direction opposite to the relative tangential displacement between these particles. ***c*** has the direction pointing from the center of particle i to the contact point with neighboring particles, this unit vector is to equivalent normal unit vector ***n*** when two particles in contact are spheres.

The normal contact force f_n is determined as a linear combination of the normal elastic force and normal damping force:

$$f_n = k_n\delta_n + \gamma_n\dot{\delta}_n \tag{3}$$

where, $k_n\delta_n$ is the normal elastic force, k_n is normal stiffness, δ_n is the separation distance between two particles, $\gamma_n\dot{\delta}_n$ is the normal damping force, γ_n denotes the damping coefficient, $\dot{\delta}_n$ is the relative normal velocity between two particles in contact.

The frictional contact force f_t is determined according to the Coulomb friction law, this force is the minimum of the sum of the tangential elastic force and tangential damping force and the friction threshold.

$$f_t = -\min\left\{\left|k_t\delta_t + \gamma_t\dot{\delta}_t\right|, |\mu_s f_n|\right\} \times \text{sign}\left(\dot{\delta}_t\right) \tag{4}$$

where, $k_t\delta_t$ denotes the tangential elastic force, $\gamma_t\dot{\delta}_t$ is the tangential damping force, $\mu_s f_n$ is the friction threshold according to Coulomb friction law, k_t and γ_t denote the tangential stiffness and tangential damping parameter, δ_t and $\dot{\delta}_t$ are the relative tangential displacement and the relative tangential velocity between two particles in contact, respectively. μ_s is the friction coefficient of particles.

The cohesion force f_c is an approximate solution of the Laplace–Young equation, as given following and plotted in Fig. 3 [15, 16].

$$f_c = \begin{cases} -\kappa R & \text{for } \delta_n \leq 0 \\ -\kappa R e^{-\frac{\delta_n}{\lambda}} & \text{for } 0 \leq \delta_n \leq d_{rupt} \\ 0 & \text{for } \delta_n \geq d_{rupt} \end{cases} \tag{5}$$

where, $\kappa = 2\pi\gamma_s\cos\theta$, is the pre-factor of the capillary cohesion force. $R = \sqrt{R_iR_j}$ is the mean radius of two particle i and j in contact. γ_s denotes the liquid-vapor surface tension. θ is the contact angle. d_{rupt} denotes the debonding distance that depends on the contact angle and volume V_b of the capillary bridge, as shown by the following expression:

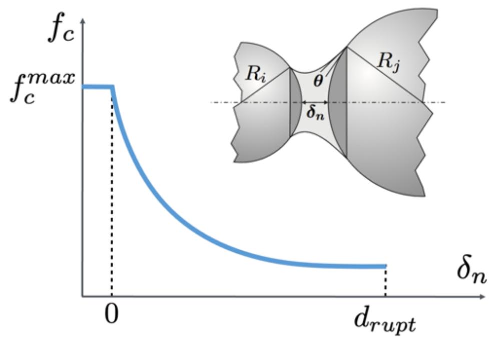
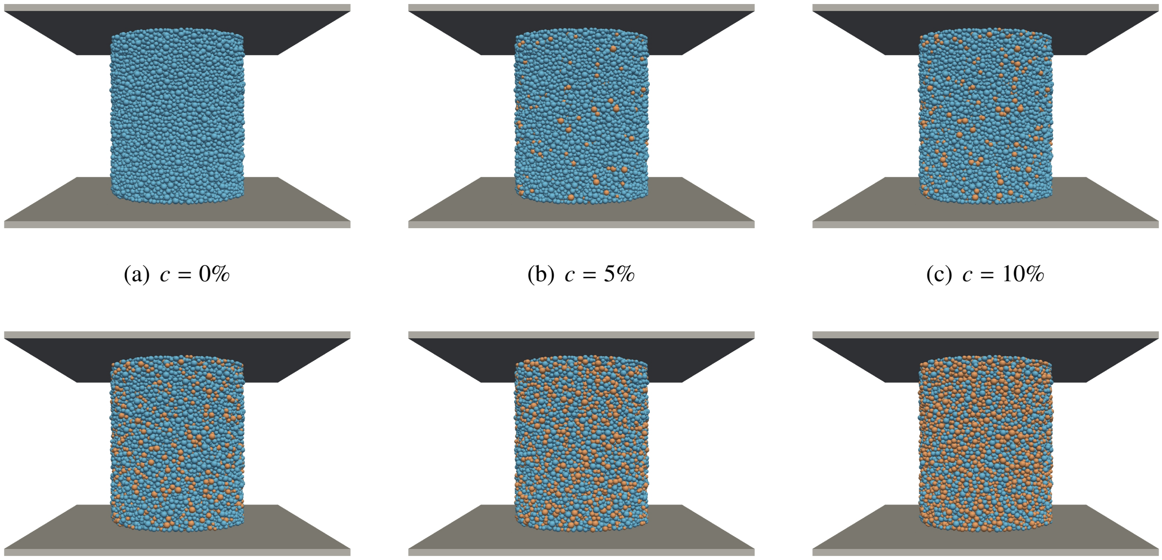


Figure 3. Schematic drawing showing the relationship between the liquid cohesion force and the debonding distance

$$d_{rupt} = \left(1 + \frac{1}{2}\theta\right)V_b^{\frac{1}{3}} \tag{6}$$

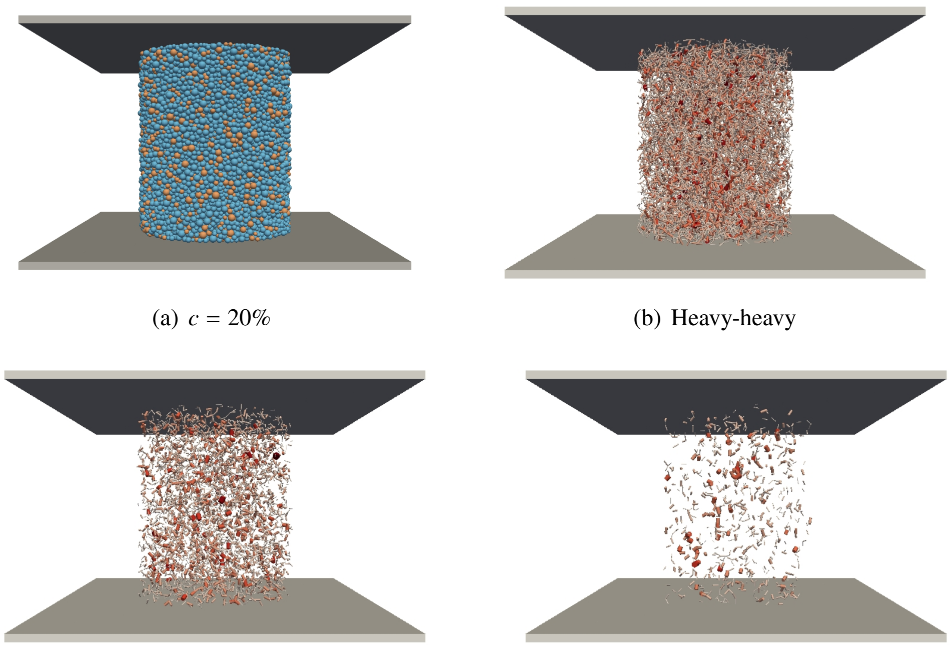
In order to create the sample of cylindrical aggregate, we first prepared a large cuboidal sample of nearly 70,000 primary particles under the isotropic compaction. The particle diameter is varied in a range $d_{\max} = 2\times d_{\min}$, with $d_{\min} = 6\times10^{-4}$ m. The particle size distribution is assumed to be uniformed by using the particle volume fraction, leading to a dense packing of granular materials in the sample. After reaching an equilibrium, a cylindrical probe was introduced along the vertical direction with its radius, reaching a cylindrical sample composed of 30,060 spherical particles. By systematically varying a broad range of values of the lightweight particle content, an appropriate number of heavy (high stiffness) particles is randomly replaced by the lightweight particles (low stiffness), as shown in



(a) c = 0% (b) c = 5% (c) c = 10%

(d) c = 20% (e) c = 33% (f) c = 50%

Figure 4. Snapshots representation the model of the uniaxial compression test with different values of the lightweight particle content c. The high stiffness particles marked in cyan, the low stiffness particles marked in yellow



(a) c = 20% (b) Heavy-heavy

(c) Light-heavy (d) Light-light

Figure 5. Snapshots representation the compression model with a given value of c, and the forces distribution in three different interaction types

Fig. 4. The cohesion forces between particles are then activated, the cylindrical aggregate then reaches the second equilibrium state before applying a compression velocity.

To investigate the effects of the lightweight particle content on the mechanical strength of aggregates, we performed a uniaxial compression test by applying a constant downward velocity on the top platen of the model, whereas the bottom platen is fixed, as shown in Fig. 4(a). In our simulations, we kept a constant value of the cohesion force f_c and a constant downward compression velocity $v = 0.01$ m/s, but the lightweight particle content c is varied in a broad range ($c = [0, 50]\%$). The particle gravity is absent. All other system parameters are given in Table 1.

Table 1. Schematic parameters of numerical simulations

Parameter	Symbol	Value		Unit
		heavy	light	
Normal stiffness	k_n	10^6	7.5×10^5	N/
Tangential stiffness	k_t	8×10^5	6×10^5	N/m
Normal damping	γ_n	0.5	0.35	Ns/m
Tangential damping	γ_t	0.5	0.35	Ns/m
Particle density	ρ_s	2600	1300	Kg/m^3
Coefficient of friction	μ_s	0.4	0.2	
Cohesion force	f_c	0.037		N
Time step	Δt	2×10^{-7}		sec.
Compression velocity	v	0.01		m/s

3. Results

Figs. 5(b), (c), and (d) display the forces distribution in three different contact types during the uniaxial compression test: heavy-heavy, heavy-light, and light-light interactions, respectively, with a given value of the lightweight particle content (20%), as shown in Fig. 5(a). As we can see, the heavy-heavy contacts are densest and strongest due to the domination of the high stiffness particles in the aggregate. The light-light interactions are much looser than the light-heavy contacts. All these contacts are quite homogeneous due to the homogeneous distribution of low and high stiffness particles inside the aggregate.

In order to study the evolution of the mechanical strength of the cylindrical aggregate during the compression test, the mean vertical stress is considered. This stress is measured by considering the stress tensor obtained from the vertical components of the force vectors that both normal and tangential forces between particles are involved and the branch vectors which joints two particle centers, leading to the following expression of the average vertical stress

$$\sigma_{zz} = \frac{1}{V_c} \sum_{k=1}^{N_c} f_z^k b_z^k \tag{7}$$

where V_c is the volume of the cylindrical aggregate, N_c denotes the number of contacts at the current stage. f_z^k and b_z^k are the z components of the force vectors and branch vectors at the contact k between two particles, respectively.

Fig. 6 shows the evolution of the average vertical stress σ_{zz} as a function of the cumulative vertical strain ε for different values of the lightweight particle content c. It is interesting to see that the average vertical stress increases rapidly and reaches a peak at a small strain of the cylindrical aggregate. The value of this peak stress decreases with increasing the content of the lightweight particles, this reflects the strong affect of the lightweight particle content on the mechanical behavior of aggregates. σ_{zz} then declines gradually when increasing the cumulative vertical strain due to losing cohesion contacts as a consequence of considering the irreversible breaking of such contacts. Interestingly, the reduction rate of the average vertical stress decreases with increasing the lightweight particle content, this can be explained due to the improvement of the ductility of the cylindrical aggregates. At the end of the compression test, σ_{zz} of the low lightweight particle content is higher than that of the high one due to the inherent properties of primary particles.

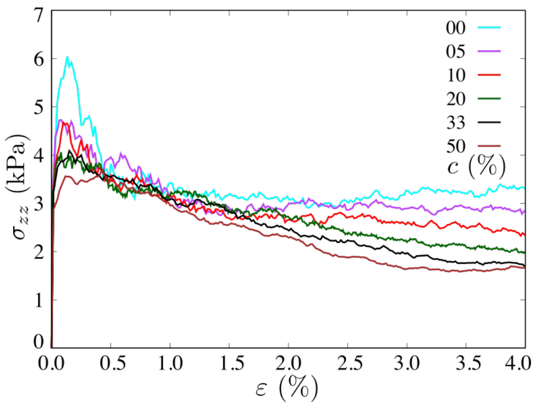


Figure 6. The evolution of the average vertical stress as a function of the cumulative vertical strain for different values of the lightweight particle content

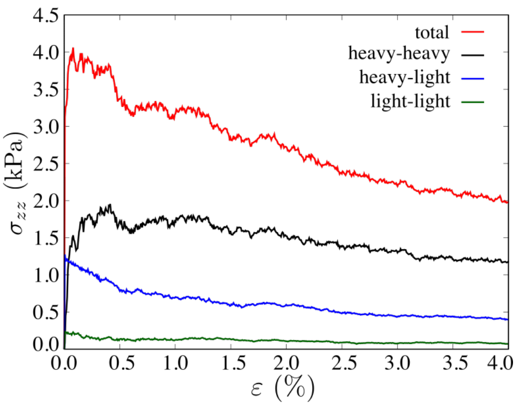
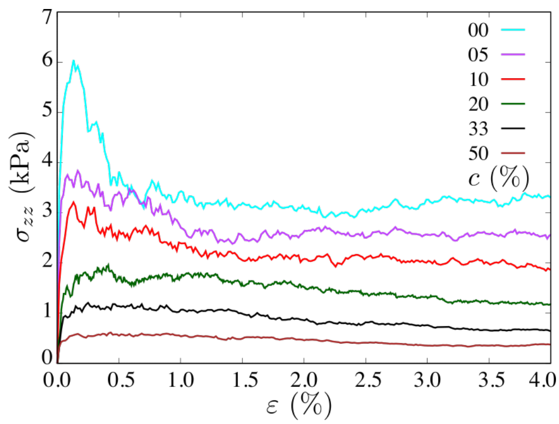


Figure 7. Evolution of the average vertical stress obtained in a whole system and by different contact types with a given value c = 20%

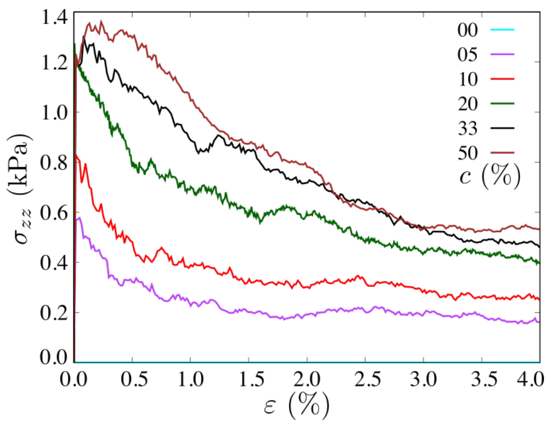
Although the results presented above describes well the effects of the lightweight particle content on the mechanical strength of the cylindrical aggregates, it is essential to check the stress contribution of three different contact types involving heavy-heavy, heavy-light, and light-light interactions between primary particles in order to see the specific contribution of the lightweight particles to the aggregates. We first show the contribution of these three different contact types above in a cylindrical aggregate to the mechanical strength with a given value of the lightweight particle content c = 20% (as shown in Fig. 7). It is remarkable to see that the total stress (measured for the whole system) is mainly due to the strong contribution of the heavy-heavy interactions as well as the heavy-light contacts, whereas the interactions of light-light particles have a relatively small contribution.

Figs. 8(a), (b), and (c) show the different contribution of the heavy-heavy contacts, heavy-light contacts, and light-light contacts to the mechanical strength of cylindrical aggregates, respectively, expressed as a function of cumulative vertical strain. The results show that average vertical stress obtained by heavy-heavy interactions in the case of without considering the participation of light particles reaches the largest value (see Fig. 8(a)). In the presence of light particles, this stress decreases dramatically and is almost constant with the evolution of vertical strain when the light particle content reaches 50%. In contrast, the average vertical stress obtained by heavy-light and light-light contacts steeply increases with the increasing of c, Figs. 8(b) and 8(c), respectively. These results clearly provide evidence for the re-contribution between different interaction types to the mechanical strength of granular materials.

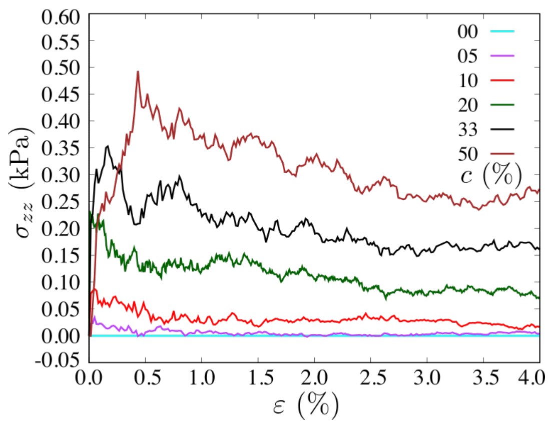
Fig. 9 shows the peak compressive stress σ_p obtained by different contact types and in a whole system as a function of the lightweight particle content c. With lightweight particle content c that



(a) Heavy-heavy



(b) Heavy-light



(c) Light-light

Figure 8. The vertical stress obtained by different interaction types for different values of the lightweight particle content

is varied from 0 to 20%, the peak stress obtained by the heavy-heavy interactions decreases significantly, whereas this stress obtained by the heavy-light interactions greatly increases and a slight increase recored by the light-light contacts, leading to the significant reduction of the peak stress in the whole aggregate. These can be explained due to the increase of the number of low stiffness particles inside aggregates, leading to increase the number of heavy-light and light-light contacts. For higher lightweight particle content, meanwhile, the contribution of the heavy-heavy interactions still keeps a dramatic decrease, the results represent the slight increase of the contribution of two other interactions, leading to the light decrease of the mechanical strength of aggregates. This reduction of the mechanical strength of aggregates represents a similar common trend with the results reported in previous experiment due to increasing the rubber particle content [3].

In order to ligh-light the origins of the different contributions of the heavy-heavy, heavy-light, and light-light interactions to the strength of agglomerates presented above, the average normal compressive forces $< f_n^+ >$ of these interaction groups are considered [1, 17]. Fig. 10 shows the average normal compressive forces of different couples of particles in contacts as a function of the lightweight particle content. It is interesting to see that $< f_n^+ >$ of heavy-heavy, heavy-light, and light-light interactions gradually decreases with increasing c in its whole range of values, whereas the average normal compressive force of cylindrical aggregates only decreases with c up to 20%, it then nearly reaches a plateau. These can be explained due to the increase of the low stiffness particle content, leading to the reduction of the normal compressive forces obtained by the inherent high stiffness of heavy particles.

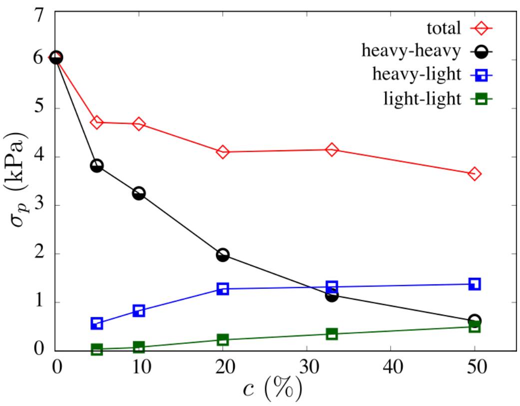


Figure 9. Peak compressive stress σ_p as a function of the lightweight particle content c for different interaction types

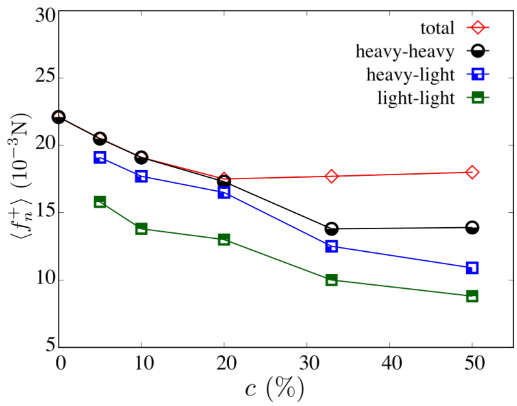


Figure 10. Average normal compressive force of different couples of particles in contacts as a function of the lightweight particle content

The present study has yielded several interesting perceptions in terms of the variation of the granular microstructure and how the heavy-heavy, heavy-light, and light-light particle interaction participates in maintaining the total vertical stress. It presents that the average normal compressive forces between particles of granular skeleton during compressive test plays an important role. Therefore, we propose the following classifications of granular materials depending on light particle content into three categories.

- 0 – 20% for lightweight particle content: the total vertical stress decreases immediately when lightweight particles are involved and continues to decrease while the lightweight particle content increased up to 20%. The content of heavy particles gradually decreases and the light particles are partially bound to heavy particles leading to creating heavy-light particle interaction that contributes to carrying the vertical stress. Yet light particles still is not number enough associated with the heavy particles, hence interactions between heavy-heavy particles mainly carry the vertical stress to participate in the compressive stress.

- 20 – 30% for lightweight particle content: the total vertical stress increases lightly. The lightweight particles are significantly linked to heavy particles and greatly replace heavy-heavy particle interaction. Interactions between light-light particles start contributing more and participates in carrying vertical tress. In this case, the main participant to vertical stress is assumed by heavy-heavy and heavy-light interactions.

- More than 30% for light particle content: the total vertical stress decreases gradually due to dispersing heavy particles in granular mixtures which restrict heavy-heavy particle interaction. Thus, heavy-light particle interaction continues to carry vertical stress while this stress is no longer dominated by heavy-heavy particle interaction.

It is should be noted that we do not perform the DEM simulations for samples that have lightweight particle content of more than 50% because according to Lv et al. [3], the total vertical stress of the samples decreases insignificantly as the percentage of rubber light particles exceeds 50%.

4. Conclusions

In this paper, we have studied the influence of lightweight particle content on mechanical properties of cylindrical aggregate materials with lightweight particle content up to 50% by means of the discrete element method. This study allows us to explain how lightweight particle content affects the compressive strength of aggregates and the following conclusions are drawn:

- The participation of light particles can reduce the total stress value in the aggregate and the interaction between heavy particles. This shows that the weight of the aggregate decreases with the light particle content.

- A significant decrease in the vertical stress of the heavy-heavy particles and a sharp increase in that of the heavy-light particles were observed in the aggregate samples. These changes occurred when the light particle content was lower than 20%.

- When the fine grain content is from 20% to 30%, the adhesion between particles becomes more stable and becomes a unified mass. In this case, the total stress increases slightly.

- When the fine particle content is greater than 30%, the interaction of heavy-heavy particles gradually disappears and is replaced by the interaction of heavy-light particles.

Within the scope of the paper research, it is found that the role of light particles has a significant influence on the mechanical strength of aggregates. Hence, the future for using light granular in lightweight concrete is also promising.

Acknowledgements

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