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CALCULATION OF REINFORCED CONCRETE BEAMS' SHEAR STRENGTHS AT AMBIENT AND FIRE CONDITIONS ACCORDING TO RUSSIAN DESIGN STANDARDS

Nguyen Truong Thang[a,*], Nguyen Trung Kien[b]

[a]*Faculty of Building and Industrial Construction, Hanoi University of Civil Engineering, 55 Giai Phong road, Hai Ba Trung district, Hanoi, Vietnam*
[b]*Vietnam Institute for Building Science and Technology (IBST), 81 Tran Cung street, Cau Giay district, Hanoi, Vietnam*

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Abstract

This paper introduces general principles and gives analytical clarifications of the stirrups design at ambient condition and shear strength analysis for reinforced concrete (RC) beams at fire condition according to Russian design standards SP 63.13330.2012 (SP 63) and SP 468.1325800.2019 (SP 468), respectively. The calculation method on inclined cross section (ISC) and the simplified approach on normal cross section (NSC) are clarified and developed so that the stirrups configuration can be directly designed at ambient condition and the deterioration of shear strength when the beam is exposed to ISO 834 fire can also be explicitly determined. Calculation results of a case study conducted in the paper show that although there are certain gaps between the designed results of stirrups using ICS and NCS methods, the fire resistances based on shear strength criteria obtained from the two methods are similar. With the systematic nature between the design standards TCVN 5574:2018, SP 63 and SP 468, it is rational to use SP 468 as a reference for RC structural fire design that is compatible with QCVN 06:2021/BXD and TCVN 5574:2018, before any advanced international code is decided to be applied in Vietnam.

Keywords: fire resistance; structure; member; reinforced concrete; shear; stirrups.

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1. Introduction

In reinforced concrete (RC) structural members, transverse reinforcement including stirrups and inclined reinforcing bars (conventionally abbreviated as rebars) are designed to avoid failure along inclined cracks due to shear forces. Since shear failure is brittle, studies on shear strength in RC structures are also of importance [1–4].

In the world, there are different approaches to calculate transverse reinforcement specified in various design codes and standards for concrete and RC structures. In the codes issued by the American Concrete Institute (ACI) [5], concrete, interlock effect of aggregates along the inclined cracks, stirrups, and dowel effect of longitudinal rebars are all accounted for in the calculation of shear resistance

*Corresponding author. *E-mail address:* thangnt2@nuce.edu.vn (Thang, N. T.)

in RC beams. The Eurocodes specify that the condition of principal compressive stress along inclined strips between inclined cracks at beam web is to be satisfied before any further calculation can be conducted. The strength condition on inclined sections shall be verified based on trust-and-tie model, which only takes into the consideration the strength of stirrups going through inclined sections having assumed inclined angles from 22° to 45° with the longitudinal axis of the beam, regardless the concrete contribution for shear resistance. Besides, anchorage of longitudinal rebars to supports shall also be checked with shear effect [6]. There are also other approaches of stirrups' calculation introduced in various versions of Russian design standards that have been transformed to the corresponding design standards in Vietnam during the past decades. The current Vietnamese design standard for concrete structures TCVN 5574:2018 [7] is based on the corresponding Russian standard SP 63.13330.2012 [8] and is introduced in a number of textbooks [1, 9]. Research works on shear strength at ambient condition to TCVN 5574:2018 have also been published recently [10, 11].

When subjected to fire exposure, all mechanical properties of concrete and steel reinforcement significantly deteriorate, resulting in the shear strength degradation of the heated RC beams, then endangering the safety of the whole structural system due to the brittle nature of the shear failure. Hence, the calculation of shear resistance at elevated temperatures is a dominant design requirement. However, there are only prescriptive rules specified in the Vietnam national code [12] and design standard [7] whereas there is still a need of more rational approaches for the structural fire design for RC structures. Various studies on shear strength of RC structures at elevated temperatures have been published in the world [13–21]. However, in the literature of RC structures in fire in Vietnam, there is limited related information since most of them are for RC members under the effects of compression and bending moments [22–29]. This fact motivates the authors to introduce general principles and clarify the calculation methods for shear strength analysis of rectangular RC beams at ambient condition and when subjected to ISO 834 standard fire based on SP 63.13330.2012 [8] (SP 63) and SP 468.1325800.2019 [30] (SP 468), respectively. A case study is conducted on a simply-supported RC beam to illustrate that the fire resistance of RC beams based on shear strength criteria can be explicit determined, from which a number of discussions are given in the latter part of the paper.

2. Principles for shear strength calculation at ambient condition according to SP 63

2.1. Principles for shear strength calculation

Current Vietnamese design standard for concrete structures TCVN 5574:2018 [7], which is basically transformed from SP 63.13330.2012 [8], specifies that the strength design of reinforced concrete members for shear forces is based on model of inclined sections. In the design based on this model, member shear strength should be provided of a strip between inclined cross sections (ICS). Fig. 1 illustrates the ICS for the analysis of shear strength of rectangular RC beams according to SP 63 [8].

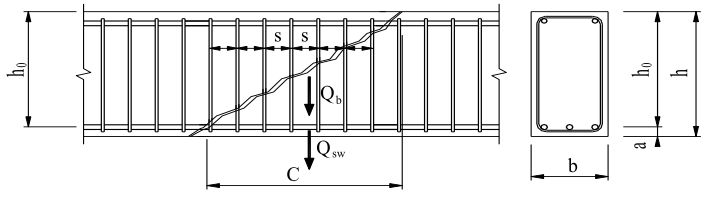


Figure 1. Inclined cross section for shear strength analysis

The condition for calculating transverse reinforcement is shown in Eq. (1):

$$Q_{b\,\min} = 0.5R_{bt}bh_0 \le Q_{\max} \le 0.3R_b bh_0 \tag{1}$$

where $Q_{\max}$ is the maximum shear force due to design loads; R_{bt} and R_b are the respective tensile and compressive strengths of concrete; b and h_0 are the geometric properties of the cross section of the rectangular beam shown in Fig. 1. The right hand side of Eq. (1) is to assure that concrete in the strips between inclined cracks at the beam web is not crushed due to principal compressive stresses.

According to SP 63 [8], verification of inclined section of bending members can be performed for ICSs placed along the length of a member at the most critical projection length of the inclined section C, considering in ultimate limit states that (Fig. 1):

$$Q \le Q_u = Q_b + Q_{sw} \tag{2}$$

where Q is the shear force in an ICS with the projection length C on the longitudinal axis of a member determined due to all external loads located at the same side of the referred ICS, while the most critical loading within the ICS should be considered; Q_u is shear strength; Q_b and Q_{sw} are the contribution of concrete in compression zone and stirrups for shear strength, respectively. It should be noted that the effects of compressive and tensile stresses on structures shall also be considered in Q_b.

SP 63 also adopts a simplified calculation method on normal cross sections (NCS) in which the shear strength may be verified neglecting inclined sections at determining shear force due to external load, from the condition:

$$Q_1 \le Q_{u,1} = Q_{b,1} + Q_{sw,1} \tag{3}$$

where Q_1 is the shear force in a NCS due to external design loads and can be directly taken from shear force diagram of the beam; $Q_{u,1}$ is the equivalent shear strength contributed by: (i) Concrete: $Q_{b,1} = 0.5R_{bt}bh_0$ which shall be multiplied by a coefficient of $2.5/(a/h_0)$ if Q_1 is considered at a position closed to support with a distance $a < 2.5h_0$; and (ii) Stirrups: $Q_{sw,1} = q_{sw}h_0$ which shall be multiplied by a coefficient of a/h_0 if $a < h_0$.

2.2. Stirrups design and shear strength analysis on inclined cross sections (ICS)

Consider a free body part due to an arbitrary inclined crack separated from a simply-supported RC beam under uniform loads as shown in Fig. 2.

The strength condition based on an inclined cross section in shear is:

$$Q_{\max} - q_1 \cdot C_o \le Q_u = Q_b + Q_{sw} \tag{4}$$

where: $q_1 = g + 0.5p$ is to account for the long-term value of the uniform live load p owing to favorable effect of the sign "-"; g is uniform dead load;

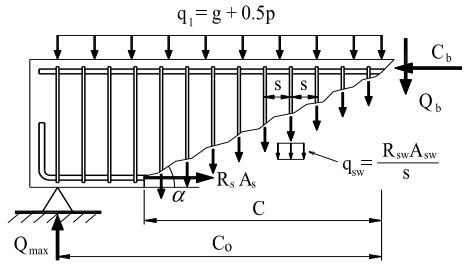


Figure 2. Principles for stirrups calculation on inclined cross sections C

Q_b is the shear strength of concrete:

$$Q_b = \frac{1.5R_{bt}bh_0^2}{C_o} \tag{5}$$

C_o: the projection length on the longitudinal axis of the beam from the upper end of the inclined crack to the support of the beam, $0.6h_0 < C_o < 3h_0$. This is to meet the code requirement that

$0.5R_{bt}bh_0 < Q_b < 2.5R_{bt}bh_0$;
Q_{sw} is the shear strength of all the stirrups on an inclined cracked section:

$$Q_{sw} = 0.75q_{sw}C \tag{6}$$

C: the projection length on the longitudinal axis of the beam from the start point to the end point of the inclined crack. It is observed from experiments that the following assumption can be made with acceptable errors: (i) If $C_o > 2h_0$ then $C = C_o$; and (ii) If $C_o > 2h_0$ then $C = 2h_0$;
q_{sw}: shear strength of a stirrup:

$$q_{sw} = \frac{R_{sw}A_{Sw}}{s} \geq q_{sw,\min} = 0.25R_{bt}b \tag{7}$$

where R_{sw} is the design strength for stirrups; A_{sw} is cross-sectional area of a stirrup, $A_{sw} = \frac{n_w\pi\phi_w^2}{4}$; n_w is the number of legs of stirrups; ϕ_w is the stirrup's diameter; s is the distance between two adjacent stirrups.

The strength condition in Eq. (4) on inclined sections for shear can be derived:

$$Q_{\max} \leq [Q] = \frac{1.5R_{bt}bh_0^2}{C_o} + 0.75q_{sw}C + q_1C_o \tag{8}$$

Considering the relationship between C and C_o, the above expression can be further derived for only variable C_o:

$$Q_{\max} \leq [Q]_1 = \frac{1.5R_{bt}bh_0^2}{C_o} + (0.75q_{sw} + q_1)C_o \text{ when } 0.6h_0 \leq C_o \leq 2h_0 \tag{9a}$$

$$Q_{\max} \leq [Q]_2 = \frac{1.5R_{bt}bh_0^2}{C_o} + q_1C_o + 1.5q_{sw}h_0 \text{ when } 2h_0 \leq C_o \leq 3h_0 \tag{9b}$$

There exit certain values of C_o, namely, C_1 and C_2, at which $[Q]_1$ and $[Q]_2$ respectively reaches minimum values. These values can be obtained by setting:

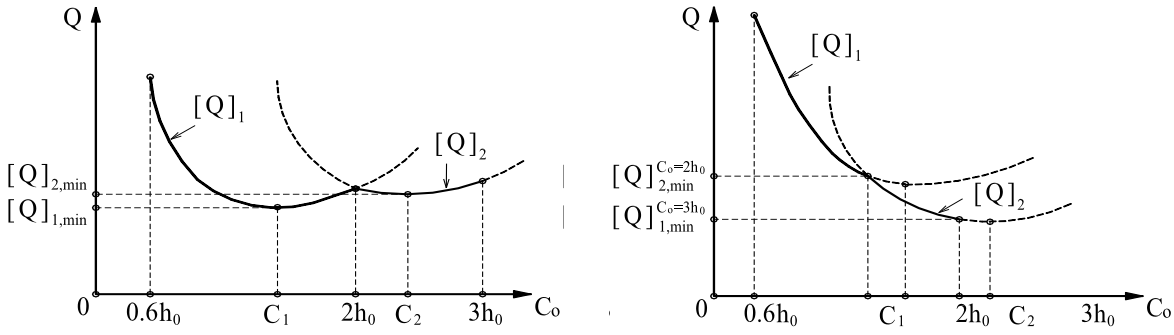
$$\frac{d[Q]_1}{dC_o} = 0 \Rightarrow C_1 = \sqrt{\frac{1.5R_{bt}bh_0^2}{(0.75q_{sw} + q_1)}}; \quad \frac{d[Q]_2}{dC_o} = 0 \Rightarrow C_2 = \sqrt{\frac{1.5R_{bt}bh_0^2}{q_1}} \tag{10}$$

Since C_1 is a function of the unknown variable q_{sw}, one can replace $q_{sw\,\min} = 0.25R_{bt}b$ into Eq. (10) to get the initial value of C_1. This is also the upper bound of C_1 since $q_{sw} \geq q_{sw\,\min} = 0.25R_{bt}b$ to avoid the brittle failure mode in shear.

If $0.6h_0 \leq C_1 \leq 2h_0$ and $2h_0 \leq C_2 \leq 3h_0$, then C_1 and C_2 can be put back into Eqs. (9a), (9b) to get the minimum values of $[Q]$ (Fig. 3):

$$Q_{\max} \leq [Q]_{1,\min} = \sqrt{6R_{bt}bh_0^2(0.75q_{sw} + q_1)} \quad \text{when } 0.6h_0 \leq C_1 \leq 2h_0 \tag{11a}$$

$$Q_{\max} \leq [Q]_{2,\min} = \sqrt{6R_{bt}bh_0^2q_1} + 1.5q_{sw}h_0 \quad \text{when } 2h_0 \leq C_2 \leq 3h_0 \tag{11b}$$



(a) C_1 and C_2 within condition range (b) C_1 and C_2 out of condition range

Figure 3. Diagrams of $[Q]_1$ and $[Q]_2$

Then:

$$q_{sw} \geq q_{sw1} = \frac{Q_{\max}^2}{4.5R_{bt}bh_0^2} - \frac{4}{3}q_1 \qquad \text{when } 0.6h_0 \leq C_1 \leq 2h_0 \tag{12a}$$

$$q_{sw} \geq q_{sw2} = \frac{Q_{\max} - \sqrt{6R_{bt}bh_0^2q_1}}{1.5h_0} \qquad \text{when } 2h_0 \leq C_2 \leq 3h_0 \tag{12b}$$

If$C_1 > 2h_0$ and $C_2 > 3h_0$, then functions $[Q]_1$ and $[Q]_2$ reach their minimum values at $C_1 = 2h_0$ and $C_2 = 3h_0$, respectively (Fig. 3). Hence:

$$Q_{\max} \leq [Q]_{1,\min}^{C_o=2h_0} = (0.75R_{bt}b + 1.5q_{sw} + 2q_1)\,h_0 \qquad \text{when } C_1 > 2h_0 \tag{13a}$$

$$Q_{\max} \leq [Q]_{2,\min}^{C_o=3h_0} = (0.50R_{bt}b + 1.5q_{sw} + 3q_1)\,h_0 \qquad \text{when } C_2 > 3h_0 \tag{13b}$$

Then:

$$q_{sw} \geq q_{sw1^*} = \frac{2}{3}\frac{Q_{\max}}{h_0} - \frac{1}{2}R_{bt}b - \frac{4}{3}q_1 \qquad \text{when } C_1 > 2h_0 \tag{14a}$$

$$q_{sw} \geq q_{sw2^*} = \frac{2}{3}\frac{Q_{\max}}{h_0} - \frac{1}{3}R_{bt}b - 2q_1 \qquad \text{when } C_2 > 3h_0 \tag{14b}$$

The calculation of q_{sw} can be finally derived as follows:

$$\text{If } 0.6h_0 \leq C_1 \leq 2h_0 \text{ and } 2h_0 \leq C_2 \leq 3h_0 \text{ then } q_{sw} \geq \max\left(q_{sw1}, q_{sw2}, q_{sw,\min}\right) \tag{15a}$$

$$\text{If } 0.6h_0 \leq C_1 \leq 2h_0 \text{ and } C_2 > 3h_0 \text{ then } q_{sw} \geq \max\left(q_{sw1}, q_{sw2^*}, q_{sw,\min}\right) \tag{15b}$$

$$\text{If } C_1 > 2h_0 \text{ and } 2h_0 \leq C_2 \leq 3h_0 \text{ then } q_{sw} \geq \max\left(q_{sw1^*}, q_{sw2}, q_{sw,\min}\right) \tag{15c}$$

$$\text{If } C_1 > 2h_0 \text{ and } C_2 > 3h_0 \text{ then } q_{sw} \geq \max\left(q_{sw1^*}, q_{sw2^*}, q_{sw,\min}\right) \tag{15d}$$

The stirrups configuration is then chosen by its diameter ϕ_w and number of legs n_w to calculate the distance satisfying that: $s_{tt} = \dfrac{R_{sw}A_{sw}}{q_{sw}} \leq s_{\max} = \dfrac{R_{bt}bh_0^2}{Q}$ and not exceed the detailing distance specified for concrete classes not higher than B60 shown in Eqs. (16).

$$s_{ct} = \min\left(0.5h_0, 300 \text{ mm}\right) \text{ when } Q > Q_{b\,\min} \tag{16a}$$

$$s_{ct} = \min\left(0.75h_0, 500 \text{ mm}\right) \text{ when } Q \leq Q_{b\,\min} \tag{16b}$$

It should be noted from the ICS calculation method that when the structure is subjected to concentrated loads, the term q_1 is set to zero in above derivations and particular inclined sections from the concentrated load shall be considered in the relation with the distance from the load to the support.

Having obtained the designed configuration of stirrups in terms of q_{sw}, the shear strength on ICS at ambient condition of RC beams can be analyzed based on Eqs. (2)–(7).

2.3. Stirrups design and shear strength analysis on normal cross sections (NCS)

From Eq. (3), a simplified procedure based on the concept of NCS is proposed for stirrups calculation as shown in Fig. 4.

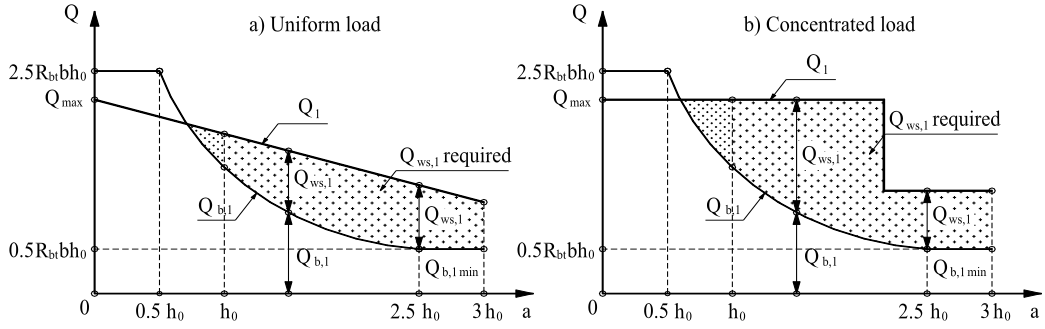


Figure 4. Simplified stirrups calculation on normal cross sections a

- Consider a NCS at a distance a to the support satisfying: $0 \le a \le 3h_0$;
- Draw the diagram of shear force Q_1 in NCS;
- Draw the diagram of $Q_{b,1}$ in NCS following Eq. (12):

$$+\ Q_{b,1} = 2.5R_{bt}bh_0 \text{ when } 0 \le a \le 0.5h_0 \tag{17a}$$

$$+\ Q_{b,1} = 0.5R_{bt}bh_0 \times \frac{2.5}{a/h_0} = \frac{1.25R_{bt}bh_0^2}{a} \text{ when } 0.5h_0 \le a \le 2.5h_0 \tag{17b}$$

$$+\ Q_{b,1} = 0.5R_{bt}bh_0 \text{ when } 2.5h_0 \le a \le 3h_0 \tag{17c}$$

- At a certain NCS at a distance a from support, determine $Q_{sw,1} = Q_1 - Q_{b,1}$

+ If $Q_{sw,1} < 0$ then stirrups can be arranged following detailing requirement;
+ If $Q_{sw,1} \ge 0$ then stirrups shall be calculated:

$$q_{sw} = \frac{Q_1 - Q_{b,1}}{h_0} : \frac{a}{h_0} = \frac{Q_1 - Q_{b,1}}{a} \quad \text{when } 0 \le a \le h_0 \tag{18a}$$

$$q_{sw} = \frac{Q_1 - Q_{b,1}}{h_0} \quad \text{when } h_0 \le a \le 3h_0 \tag{18b}$$

Having obtained the designed configuration of stirrups including A_{sw}, R_{sw} and s, calculate the corresponding value of $q_{sw} = \frac{R_{sw}A_{sw}}{s} \ge 0.25R_{bt}b$, the shear strength on NCS at ambient condition of RC beams can be analyzed based on Eq. (3) as follows:

$$Q_1 \le Q_{u,1} = 2.5R_{bt}bh_0 + q_{sw}a \quad \text{when } 0 \le a \le 0.5h_0 \tag{19a}$$

$$Q_1 \le Q_{u,1} = 1.25R_{bt}bh_0^2/\mathrm{a} + q_{sw}a \quad \text{when } 0.5h_0 \le a \le h_0 \tag{19b}$$

$$Q_1 \le Q_{u,1} = 1.25R_{bt}bh_0^2/\mathrm{a} + q_{sw}h_0 \quad \text{when } h_0 \le a \le 2h_0 \tag{19c}$$

$$Q_1 \le Q_{u,1} = 0.5R_{bt}bh_0 + q_{sw}h_0 \quad \text{when } 2h_0 \le a \le 3h_0 \tag{19d}$$

3. Principles for shear strength analysis under standard fire according to SP 468

3.1. Standard fire exposure

The standard ISO 834 fire exposure [31] is adopted in SP 468 for all fire resistance analyses. In European countries, this fire curve has also been conventionally used to determine temperature distribution in the cross-section of structural components. In this standard fire exposure, the temperature-time relationship is expressed as follows:

$$T_g = 20 + 345 \log_{10}(8t + 1) \tag{20}$$

where T_g (°C) is the temperatures and t is the in-minute time counted from the fire starts.

3.2. Materials' mechanical properties under standard fire exposure

It is specified in SP 486 that when subjected to standard fire curve in Eq. (20), the temperature-dependent mechanical properties of concrete and reinforcing steel are defined by multiplying the specified strengths at ambient condition by working condition factors to calculate at accidental limit states as follows:

$$R_{btn,T} = \gamma_{bt,T} \cdot R_{btn}; R_{bn,T} = \gamma_{bn,T} \cdot R_{bt} \text{ and } E_{b,T} = \beta_{b,T} \cdot E_b \tag{21}$$

$$R_{sn,T} = \gamma_{s,T} \cdot R_{sn}; R_{snw,T} = \gamma_{s,T} \cdot R_{snw} \text{ and } E_{s,T} = \beta_{s,T} \cdot E_s \tag{22}$$

where R_{bn}, R_{btn} are the respective specified compressive and tensile strengths of concrete; R_{sn}, R_{snw} are the specified reinforcing steel tensile strength of longitudinal rebars and stirrups, respectively; E_b and E_s are the respective elastic moduli of concrete and rebars at ambient condition. All the terms having subscript "T" are the corresponding temperature-dependent specified strengths and elastic moduli. The working condition factors are also reduction factors as shown in Tables 1 and 2.

Table 1. Working condition factors for compressive concrete and reinforcing steel

Materials	Reduction factor	Temperature (°C)							
		20	200	300	400	500	600	700	800
Siliceous aggregate concrete	$\gamma_{b,T}$	1.00	0.98	0.95	0.85	0.80	0.60	0.20	0.10
	$\beta_{b,T}$	1.00	0.70	0.50	0.40	0.30	0.20	0.10	0.05
Calcareous aggregate concrete	$\gamma_{b,T}$	1.00	1.00	0.95	0.90	0.85	0.65	0.30	0.15
	$\beta_{b,T}$	1.00	0.75	0.55	0.45	0.35	0.25	0.15	0.10
Hot-rolled reinforcing steel	$\gamma_{s,T}$	1.00	1.00	1.00	0.85	0.60	0.37	0.22	0.10
	$\beta_{s,T}$	1.00	0.92	0.90	0.85	0.80	0.77	0.72	0.65

Table 2. Working condition factor for concrete tensile strength according to SP 468

Calcareous aggregate concrete	Temperature (°C)								
	20	50	100	150	200	300	400	500	600
$\gamma_{bt,T}$	1.00	0.80	0.75	0.70	0.65	0.50	0.35	0.20	0.05

It should be noted that the temperatures in Table 2 are the average values of the whole cross-section when applied for shear strength calculation in fire, that can be determined from Appendix B of SP 468 [30].

3.3. Temperature profile on member cross-section under standard fire exposure

In fire tests and real situation, beams are usually heated from three beneath sides. With standard fire exposure, it is assumed that at a certain time t, the temperatures at all the points on the heated surfaces of the beams follow Eq. (20). Then, temperatures of different points within the beam's cross-section can be determined by thermal analysis based on heat balance analysis. Since it takes time for the heat transfer process, the temperatures at inner points are lower than that of the outer point as well as the standard-fire temperature at the surfaces. In RC members, thermal analysis can be conducted based on materials' thermal properties and heat transfer methods such as radiation, convection, and conduction. For simplification, it is assumed that the temperature in a rebar is equal to that of its surrounding concrete, meaning that the effect of rebars in concrete is ignorable.

Appendix B of SP 468 [30] provides temperature profiles for some typical RC beam cross-sections. An example of temperature files at 30, 60, 90, 120, 180 and 240 min of ISO 834 fire exposure for a half of $b \times h = 300 \times 600$ (mm) rectangular RC beam using siliceous agregates are shown in Fig. 5 for illustration.

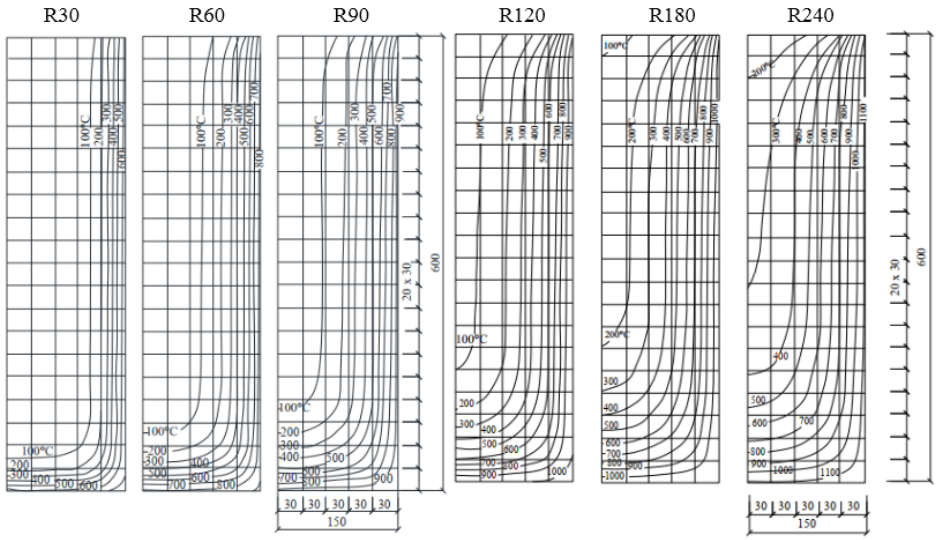


Figure 5. Temperature profile on a half of a rectangular beam according to SP 468

3.4. Fire resistance analysis based on shear strength criteria

It is specified in Clause 8.5 of SP 468 that the fire resistance based on shear strength criteria R in minute (i.e., R30, R60, R90, R120, R180 and R240) can be determined in accidental limit states as follows:

$$Q_{nT} \leq Q_{uT} \tag{23}$$

where Q_{nT} is the maximum specified shear force of the beam including permanent and temporary loads considered in accidental situation and Q_{uT} is the temperature-dependent shear resistance at the

corresponding time of 30, 60, 90, 120, 180 and 240 min in the ISO 834 standard fire exposure, which can be calculated based on the deteriorated value of materials' specified strengths shown in Eqs. (21)–(22) as well as the specification of the effective dimensions of the beam's cross-section as shown in Fig. 6.

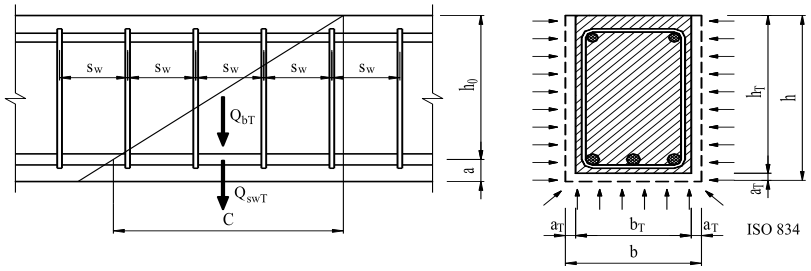
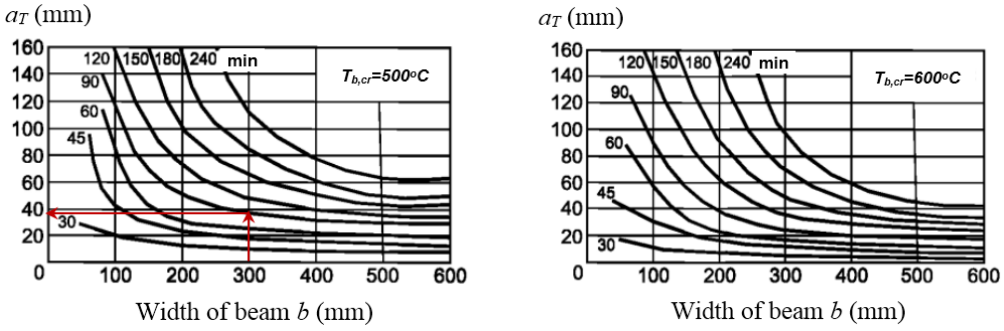


Figure 6. Calculation principle for shear strength in fire according to SP 468 [30]

In Fig. 6, the effective depth and effective height of the beam's cross-section are calculated as bellows:

$$b_T = b - 2a_T \quad \text{and} \quad h_T = h - a_T \tag{24}$$

where a_T is the thickness of the outer concrete layer having temperature higher than a certain critical temperature $T_{b,cr}$ (Fig. 7).



(a) Siliceous-aggregate concrete (b) Calcareous-aggregate concrete

Figure 7. Determination of a_T to SP 468 [30]

Figs. 7(a) and 7(b) depict that the critical values $T_{b,cr}$ for the determination of a_T are 500 and 600 °C for siliceous and calcareous aggregate concrete, respectively. For example, a 300 mm-width beam cast from siliceous aggregate gains the value of $a_T = 39$ mm at R90 (Fig. 7(a)).

4. Case study

4.1. Data for calculation

Consider a simply-supported RC beam spanning over 4.5 m with the rectangular cross-section of $b \times h = 300 \times 600$ (mm). There are two symmetrical point loads P at a distance of 1.5 m from the supports. This value consists of the specified dead load $G = 100$ kN and specified live load $Q = 70$ kN. The safety factors for ultimate limit states (ULS) at ambient condition are $n_1 = 1.1$ and

n_2 = 1.2. Then, the corresponding design value for ULS is $P = n_1G + n_2Q$ = 194 kN. This is also the maximum shear force in the beam at ambient condition. Siliceous concrete grade used for the beam is B20, with design strengths R_b = 11.5 MPa, R_{bt} = 0.9 MPa and the corresponding specified strengths R_{bn} = 15 MPa, R_{btn} = 1.35 MPa. The reinforcing steel for stirrups is CB240-T, with design shear strength R_{sw} = 170 MPa and the corresponding specified shear strength R_{swn} = 194 MPa. The CB300-V longitudinal rebars are 3Φ25 + 3Φ22, with the axis distance of a = 70 mm and the corresponding effective depth h_0 = 530 mm. The concrete cover to the centroid of the stirrups (having diameter of d) is 28.5 mm. At the one-third ends and the middle zones of the beam, the stirrups distances are a_1 and a_{i2} (in mm), respectively (Fig. 8).

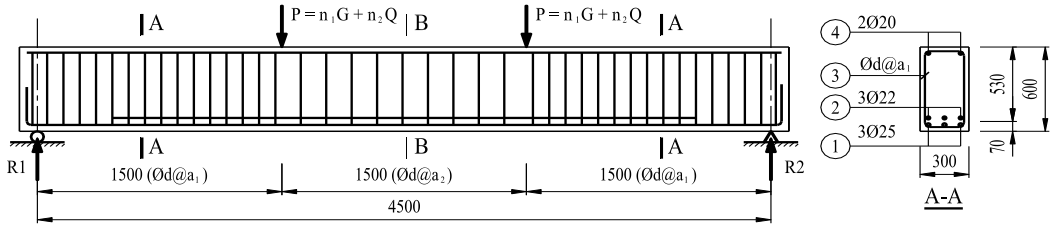


Figure 8. Simply-supported beam for case study

4.2. *Fire resistance analysis on inclined cross sections (ICS)*

At ambient condition, based on the SP 63's specifications introduced in Sections 2.1 and 2.2, with certain stirrups configuration including the diameter and the distance, one can establish the diagram of the resistance $Q(C_o)$ function as shown in Fig. 9. Based on the analysis on ICS introduced in Sections 2.1 and 2.2, the stirrups configurations are determined as d = 8 mm; a_1 = 100 mm and a_2 = 150 mm. It is clearly illustrated that with the stirrups configuration of $\phi 8a100$ and q_{sw} = 170.816 N/mm, the shear strength $Q(C_o)$ curve is totally above and closed to the line representing the maximum shear force of Q = 194 kN. The other configurations such as $\phi 8a125$ and $\phi 8a150$ do not satisfy the shear strength requirement at ambient condition since the minimum points of these curves are all below the line Q = 194 kN.

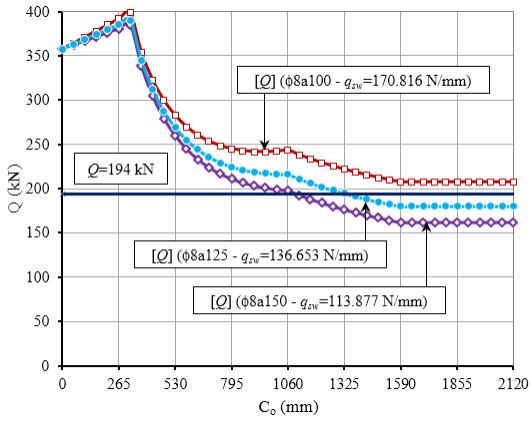


Figure 9. Shear strength analysis on ICS at ambient condition

When the beam is subjected to ISO 834 fire, all the deteriorated parameters of mechanical properties can be determined based on SP 468 specifications introduced in Section 3 as shown in Table 3.

In Table 3, the average temperature in concrete is calculated from temperatures at gridlines of 30×30 (mm) that can be linear-interpolated from the temperature profiles given in Appendix B of SP 468 (Fig. 5).

It is noted that the load safety factors in fire are n_1 = 1.0 and n_2 = 0.5 as specified in accidental limit states. Then, the corresponding specified value is $P = n_1G + n_2Q$ = 135 kN. This is also the maximum shear force in the beam $Q_{n,T}$ in fire situation.

Table 3. Temperature-dependent parameters for calculation

Calculation parameters	R0	R30	R60	R90	R120	R180	R240
a_T (mm)	0	10	24	39	50	85	112
b_T (mm)	300	280	252	224	200	130	76
h_T (mm)	600	590	576	561	450	415	338
$T_{stirrups}$ (°C)	20	215	520	667	7670	899	-
$\gamma_{s,T}$	1.0	1.0	0.554	0.446	0.14	0	-
$R_{snw,T}$ (MPa)	194	194	107.5	86.5	27.2	0	-
T concrete average (°C)	20	143	233	285	334	417	498
$\gamma_{bt,T}$	1.0	0.707	0.616	0.523	0.449	0.325	0.203
$R_{btn,T}$ (MPa)	1.35	0.954	0.832	0.706	0.606	0.439	0.274

Then, replacing all the temperature-dependent parameters of mechanical properties given in Table 3 to the corresponding terms at ambient condition in Eqs. (4)–(16), the results are shown in Fig. 10.

It can be observed from Fig. 10 that as the time goes by in the ISO 834 fire, the continuous degradation of the materials' specified strengths and the effective cross-section dimensions lead to the corresponding deterioration of the shear strength, that is represented by the gradual lower-down movement of the curves with an increase of R. Since all parts of the curve R60 is above the line $Q_{n,T}$ = 135 kN, whereas the latter parts of the curve R90 is below the line, it can be considered that the fire resistance of the beam based on shear strength criteria cannot reach to R90.

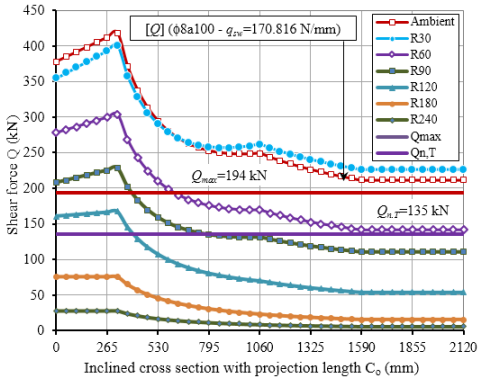


Figure 10. Shear strength analysis on ICS in fire

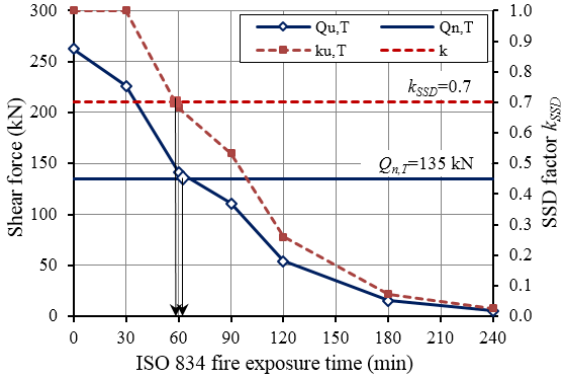


Figure 11. Fire resistance analysis based on shear strength on ICS

In order to determine the approximate time that the beam fails due to shear strength criteria, the minimum values of shear strength curves $Q_{u,T}$ are drawn in Fig. 11. It can be seen that the regressing curve intersects with the line representing the acting shear force $Q_{n,T}$ = 135 kN at 62 min, which is the analysis outcome. The analysis can also be conducted based on the concept of shear strength deterioration (SSD) factor, which is the ratio between the shear strength calculated at accidental limit states in fire and the shear strength at ultimate limit states as shown in the secondary vertical axis. With the conventional limit factor of k_{SSD} = 0.7, the similar result of about 59 min can be obtained (Fig. 11).

4.3. *Fire resistance analysis on normal cross sections (NCS)*

Based on the simplified analysis on NCS introduced in Sections 2.1 and 2.3, the stirrups configurations are determined as $d = 10$ mm; $a_1 = 110$ mm and $a_2 = 150$ mm, which form the basis to establish the curves representing the shear resistance as shown in Fig. 12.

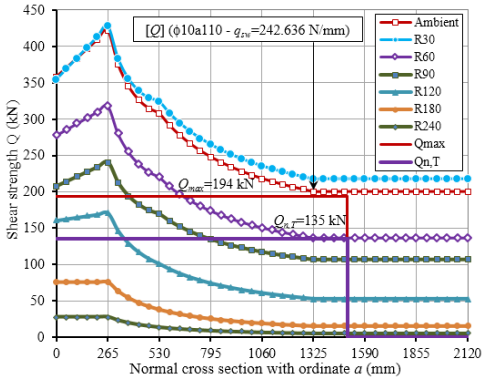


Figure 12. Shear strength analysis on NCS in fire

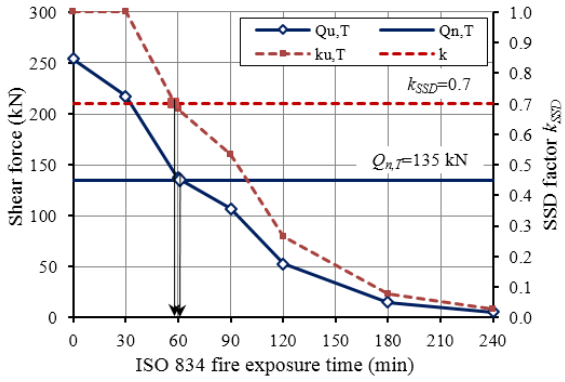


Figure 13. Fire resistance analysis based on shear strength on NCS

It can be observed from the figure that with the stirrups configuration of $\phi 10a110$ and $q_{sw} = 242.636$ N/mm, the shear strength curve is totally above and closed to the line representing the maximum internal shear force of $Q = 194$ kN. It can also be noted that this result of the simplified analysis on NCS is higher than that of ICS, which is $\phi 8a100$. Besides, the horizontal axis of the chart is the coordinate a, which is the absolute distance from the normal cross-section to the nearest support, instead of a relative projection length C_o considered in the ICS analysis.

When the beam is exposed to ISO 834 fire, all the deteriorated parameters of mechanical properties determined based on SP 468 specifications introduced in Section 3 and shown in Table 3 are used for the fire resistance analysis.

Similar to Fig. 10, it can be observed from Fig. 12 that the fire resistance of the beam following shear strength criteria cannot reach to R90 when ICS analysis is involved.

In order to determine the approximate time that the beam fails due to shear strength criteria, the minimum values of shear strength curves $Q_{u,T}$ are drawn in Fig. 13.

In Fig. 13, the regressing curves of shear force and SSD factor intersect with the lines representing the acting shear force $Q_{n,T} = 135$ kN and the conventional reduction factor of $k_{n,T} = 0.7$ at 61 and 59 min, respectively. These are also the failure times of the beam obtained from the analysis based on shear strength criteria.

4.4. *Discussions*

From the results obtained, a number of discussions are introduced as follows:

- Currently in Vietnam, QCVN 06:2021 [12] only specifies prescriptive rules for fire resistance of RC structures. Since they are based on standard fire tests conducted in other countries in the past and on other nominal information, these rules are implicit and may not closed to real fire situations;

- The calculations introduced and clarified in this paper, which are based on Russian design standards for RC structures, are rational since all the mechanical and geometrical properties of RC beams are explicitly taken into consideration. Hence, the disadvantages of the prescriptive rules can be overcome;

- From Eqs. (2) and (3), it seems that ICS and NCS methods are based on the same theory. However, the pronounced difference here is that the variable C_o in ICS representing the projection length of an arbitrary inclined cross section whereas the term a in NCS is an absolute distance from a normal section to the nearest support so that the equivalent shear strength Q_1 can be compared to the shear force diagram of the beam. This is also an effort to simplify the shear strength calculation but it is shown in this paper that NCS may lead to more conservative results at ambient condition;

- The principle to determine RC beams' fire resistance based on shear strength criteria is to calculate the time when the specified maximum shear force reaches the minimum value of the regressing shear strength at accidental limit states. The analytical derivations and case study presented in this study explicitly show that the RC beams' shear strength at elevated temperatures significantly depends on the contributions of concrete and stirrups, among which the latter is more vulnerable to fire since they are only at a certain distance to the beam surface by a concrete cover so that the stirrups quickly sustain reduction in strength. Hence, more understanding of the structural fire analysis is provided here to help practicing engineers to proactively calculate RC beams to meet a certain requirement of fire resistance, which is also the performance-based approach that has been commonly used internationally but not yet in Vietnam.

5. Conclusions

This paper introduces the analysis approaches for stirrups conducted on inclined cross sections (ICS) and normal cross sections (NCS) of rectangular reinforced concrete (RC) beams according to the recent Russian design standard for RC structures SP 63, which is also the basis of TCVN 5574:2018, followed by the introduction of the main principles for structural fire analysis specified in SP 468. Within the scope of the case study conducted, it is shown that:

- At ambient condition, the simplified approach on NCS is more practicing but may provide more conservative design of stirrups at the positions of high shear forces of the investigated RC beam, compared to the conventional method on ICS;

- The fire resistance times of the investigated RC beam based on shear strength criteria, that are obtained from the analyses on NCS and ICS are quite similar;

- With the systematic natures between SP 468 and SP 63 as well as between SP 63 and TCVN 5574:2018, it is rational and practical to use SP 468 as a reference for the fire structural analysis in Vietnam condition to suit to current specifications of QCVN 06:2021/BXD and TCVN 5574:2018;

- Further experimental, analytical and parametric studies shall be conducted in local conditions to investigate the effects of material strengths, reinforcement ratios, axial force, shear span, load combination, fire exposures, etc., so that the simplified calculation method based on SP 468 can be applicable for RC structural fire design in Vietnam.

In future, other advanced international design provisions such as the Eurocodes and the codes issued by the American Concrete Institute (ACI) shall also be studied together with the Russian standards in a comprehensive manor to get a reasonable decision for the long-term development of design standards for reinforced concrete structures at ambient and fire conditions in Vietnam.

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