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MECHANICAL PROPERTIES OF BITUMINOUS MIXTURES UNDER SINUSOIDAL CYCLIC LOADINGS: EXPERIMENT AND MODELLING

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Abstract

Mechanical behaviour of bituminous mixtures is characterized by the great thermal sensitivity and the large viscous effects. This paper focuses on the linear viscoelastic (LVE) behaviour of bituminous mixtures that is considered for pavement design. The studied material is a GB3 mix (GB in French is "Grave Bitume") which is often used for base course construction in France. Complex modulus tests are performed to determine the LVE properties of bituminous mix. Sinusoidal cyclic loadings in tension and compression for small strain amplitudes (up to 10^{-4} m/m) are applied on cylindrical samples at different temperatures (from −23.4°C to 39.1°C) and different frequencies (from 0.03 to 10 Hz). The complex modulus E^* and complex Poisson's ratio ν^* are obtained for these large ranges of temperature and frequency. From all these data, it is shown that within the linear viscoelastic domain and in the 3D case, the Time Temperature Superposition Principle (TTSP) is applicable and verified. A model with a continuum spectrum called 2S2P1D (2S2P1D means two Springs, two Parabolic elements, one Dashpot), developed at the Ecole Nationale des Travaux Publics de l'Etat (ENTPE), is used to simulate the 3D LVE behaviour of tested bituminous mixture.

Keywords: linear viscoelasticity; bituminous mixture; modelling; complex modulus; complex Poisson's ratio.

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1. Introduction

Mechanical behaviour of bituminous mixtures used in road construction is very complex. It is characterized by the great thermal sensitivity and the large viscous effects. Di Benedetto [1] proposed Fig. 1 to qualify bituminous mixtures behaviour considering the applied cyclic strain amplitude and number of cycles. This paper focuses only on the linear viscoelasticity (LVE) behaviour, which can be considered for number of loadings up to a few hundred cycles and strain amplitude lower than 10^{-4} m/m [2–4]. The LVE behaviour of bituminous materials can be characterized by the complex modulus E^* and complex Poisson's ratio ν^*. These two parameters can be determined by applying sinusoidal cyclic loadings on the tested specimens in small strain domain. Within LVE domain, many authors confirmed that the Time Temperature Superposition Principle (TTSP) can be applied as a good approximation for bituminous materials [5–11]. In addition, the behaviour of bituminous materials can be modelled using the linear viscoelastic theory. The aim of the study is the experimental

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measurement and modelling of the complex modulus E^* and complex Poisson's ratio ν^* of bituminous mixture. In Vietnam, the research on this topic is still limited since it requires high accuracy of test device.

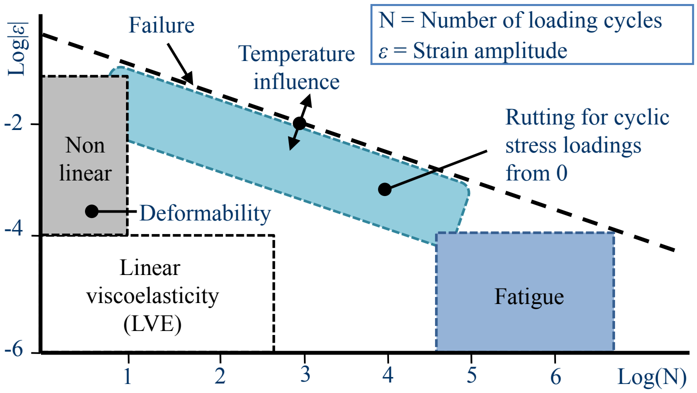

Figure 1. Typical domains of behaviour for bituminous mixtures [1]

In order to characterize the mechanical properties of bituminous mixtures under sinusoidal cyclic loadings, tension-compression modulus tests were carried out on cylindrical samples at the laboratory of the Ecole Nationale des Travaux Publics de l'Etat (ENTPE), France. In the first part of the paper, the tested material and complex modulus test are introduced. With this test developed at ENTPE, the complex modulus E^* and complex Poisson's ratio ν^* are measured for bituminous mixtures. In the second part, experimental results and modelling are presented. A LVE model with a continuum spectrum called 2S2P1D (two Springs, two Parabolic elements, one Dashpot), developed at the ENTPE, is used to simulate the 1Dim and 3Dim linear viscoelastic behaviour of tested asphalt mixture.

2. Material and specimens

The tested material, called GB3 (GB in French is "Grave Bitume"), are provided by EIFFAGE Travaux Publics Company. GB3 is often used for base course construction in France.

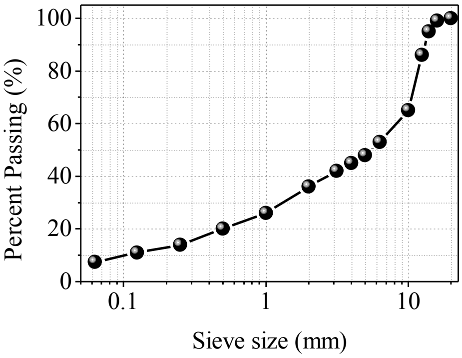

Figure 2. Aggregates grading curve of tested bituminous mixture

Bituminous mixture is dosed at 4.5% (aggregate weight) with a 35-50 (penetration grade by the standard penetration test) pure bitumen. The aggregates grading curve is presented in Fig. 2. The aggregates is 0/14 mm and typical for base course construction in France. The aggregates are diorite from La Noubleau quarry. The cylindrical specimens are 140 mm high and 74 mm in diameter. Six specimens were cored and sawn from a slab (14 cm × 40 cm × 60 cm) made with a French LPC ("Laboratoire des Ponts et Chaussées") wheel compactor, according to the European standard [12]. Three directions (called 1, 2, 3) are indicated in Fig. 3. In this study, one specimen was tested. The air void of tested specimen presented in this paper is 2.1%.

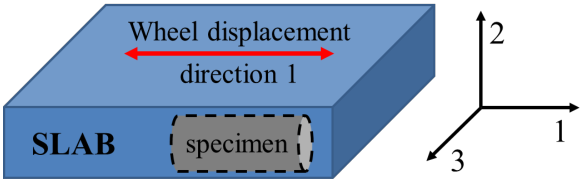

Figure 3. The material directions

3. Complex modulus test

The complex modulus test was performed using an hydraulic press (Fig. 4) where the sinusoidal loading in tension and compression is applied on the specimen. The loading is applied in material direction 1 (see Fig. 3). The measurement of radial strain is realised in material directions 2 and 3. Three extensometers and four non-contact sensors were set up to measure the axial strain and radial strains of the specimen (Fig. 5). Both extensometers and non-contact sensors are fixed in the center part of the specimen (Fig. 5). The length of extensometers is 75 mm. The extensometers are positioned at each 1/3 perimeter around the specimen. The non-contact sensors measure the radial deformation of the specimen. Two sensors are fixed opposite each other on the same diameter direction (direction 2 or 3) that allows obtaining the radial strain of the specimen in these directions (Fig. 5).

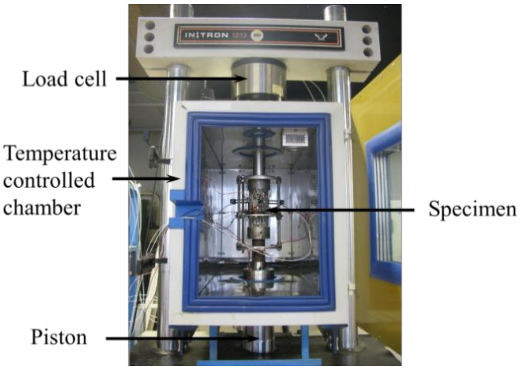

Figure 4. The hydraulic press and the test set up

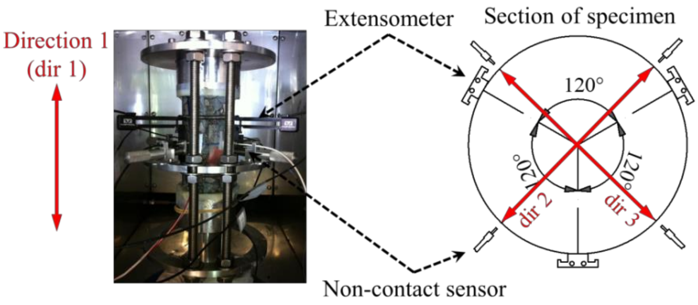

Figure 5. Measurements of axial strain, radial strains on the specimen

The complex modulus test presented in this study is a part of an international research project RILEM TC 237 – SIB. Temperatures (T) and frequencies (f) used in the test are listed below:

- Temperatures: −23.4°C, −14.4°C, −5.4°C, 3.7°C, 12.7°C, 17.2°C, 21.6°C, 30.1°C and 39.1°C.
- Frequencies: 0.03 Hz, 0.1 Hz, 0.3 Hz, 1 Hz, 3 Hz and 10 Hz.

At each couple of temperature and frequency, the sinusoidal axial strain in direction 1, called ε_1, was applied on the specimen. The number of cycles applied on the specimen is quite small (< 10 at 0.03 Hz and < 100 at 10 Hz), then the effect of self-heating on the mechanical properties of bituminous mixture is negligible [13–16]. Sinusoidal axial stress in direction 1 is called σ_1. The radial strains in direction 2 and 3 are named ε_2 and ε_3 respectively. The axial stress, axial and radial strains are expressed by Eqs. (1) to (3). The complex modulus (E^*) and complex Poisson's ratios in direction 2 and 3 (ν_2^* and ν_3^*) are then calculated using the Eqs. (4) to (5)

$$\varepsilon_1(t) = \varepsilon_{01}\sin(2\pi f t) \tag{1}$$

$$\sigma_1(t) = \sigma_{01}\sin(2\pi f t + \phi_E) \tag{2}$$

$$\varepsilon_i(t) = -\varepsilon_{0i}\sin(2\pi f t + \phi_{\nu i}) \quad (i = 2, 3) \tag{3}$$

$$E^* = \frac{\sigma_{01}}{\varepsilon_{01}} e^{j\phi_E} = \left|E^*\right| e^{j\phi_E} \tag{4}$$

$$\nu_i^* = \frac{\varepsilon_{0i}}{\varepsilon_{01}} e^{j\phi_{\nu j}} = \left|\nu_i^*\right| e^{j\phi_{\nu i}} \quad (i = 2, 3) \tag{5}$$

where f is the frequency; ε_{01} is axial strain amplitude in direction 1; ε_{02}, ε_{03} is the radial strain amplitudes in directions 2 and 3; σ_{01} is the axial stress amplitude in direction 1; ϕ_E is the phase angle between ε_1 and σ_1; $\phi_{\nu 2}$, $\phi_{\nu 3}$ are phase angles between ε_1 and the opposite of ε_2 and ε_3; $\left|E^*\right|$ is the norm of complex modulus and complex Poisson's ratios; $\left|\nu_2^*\right|$, $\left|\nu_3^*\right|$ are the norm of complex Poisson's ratios; j is complex number ($j^2 = -1$).

A measurement example of axial stress, axial strain and radial strains signal at the temperature of 12.7°C and at the frequency of 0.1 Hz is presented in Fig. 6.

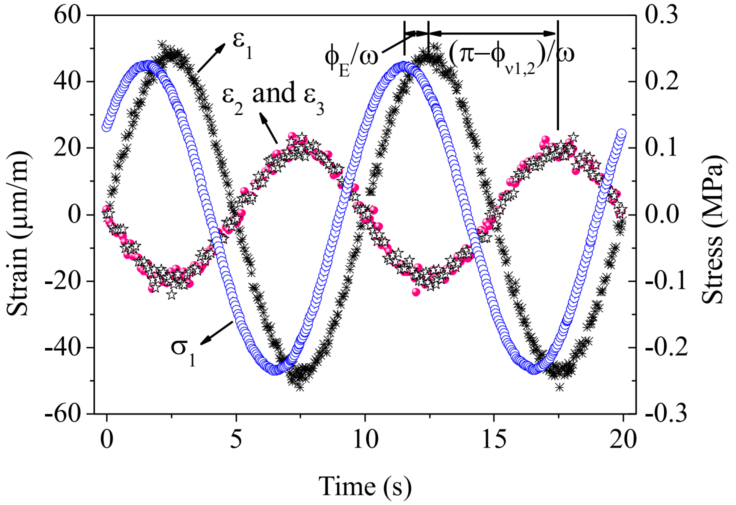

Figure 6. Example of measurements (2 cycles of stress and strains at 0.1 Hz and 12.7°C)

4. Experimental results

When the materials respect the Time Temperature Superposition Principle (TTSP), the master curves can be plotted for both norm and phase angle of the complex modulus of the material. The shifting procedure at a fixed reference temperature T_{ref} was used to build these master curves (see Fig. 7). The norm and phase angle master curves are presented as a function of the equivalent frequency (or sometimes called reduced frequency) f_e. The reduced frequency fe is expressed and calculated by Eq. (6):

$$f_e = a_T\left(T, T_{ref}\right) \times f \tag{6}$$

where a_T is the shift factor at temperature T, T_{ref} is the reference temperature ($a_T\left(T, T_{ref}\right) = 1$), f is the real frequency of solicitation. Fig. 7 presents the E^* master curves (norm and phase angle) for the tested asphalt mixture at the reference temperature $T_{ref} = 12.7°\text{C}$. It should be underlined that the master curves of complex Poisson's ratio ν^* (norm and phase angle) were obtained considering the same shift factor as complex modulus (Fig. 8). This result shows that the TTSP is verified and

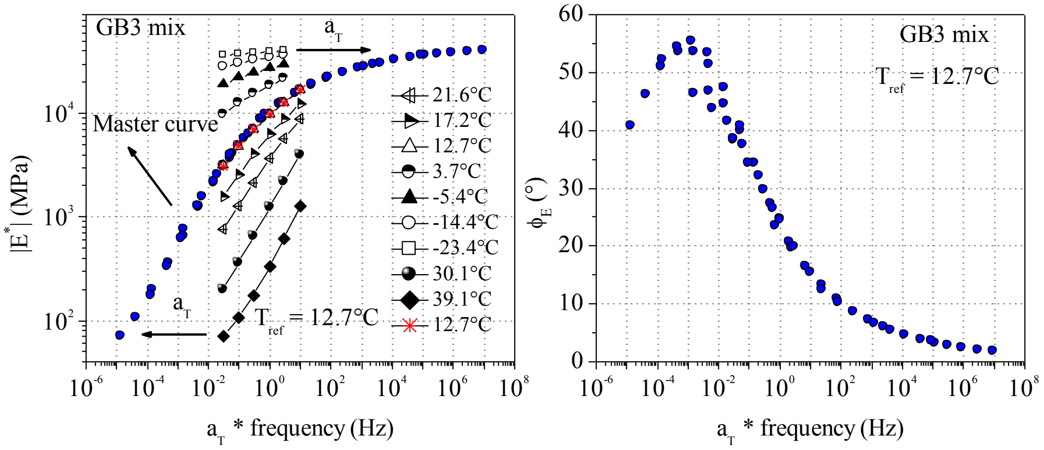

Figure 7. Master curves of $|E^*|$ (left) and phase angle ϕ_E (right) at the reference temperature 12.7°C

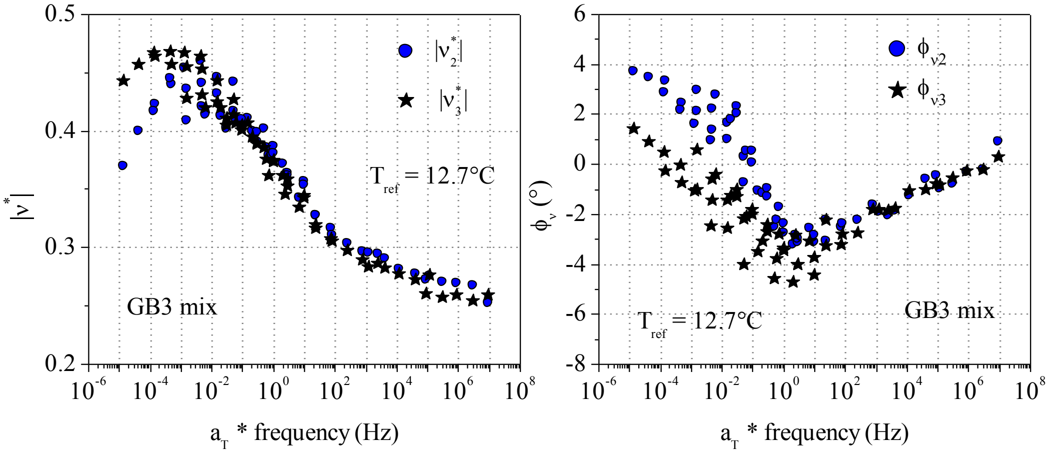

Figure 8. Master curves of $\left|\nu^*\right|$ (left) and phase angle ϕ_ν (right) at the reference temperature 12.7°C

applicable in the 3D case. It can be observed that the values of $\left|\nu^*\right|$ varies from about 0.25 at low temperatures and at high frequencies to 0.45 at high temperatures and low frequencies. It means that the values of $\left|\nu^*\right|$ is not constant across the test temperature and frequency range. However, in the literature, the complex Poisson's ratio is generally considered as a constant. In addition, the phase angle of the complex Poisson's ratio is difficult to be measured. The obtained results on the phase angle of the complex Poisson's ratio show that this value is not 0 as generally considered in literature. However, the phase angle of the complex Poisson's ratio is quite small and negative except for some measurements at high temperatures.

The shift factor a_T used in the procedure of master curves construction is presented in Fig. 9 as a function of the temperature at the reference temperature of 12.7°C. The Williams, Landel and Ferry law [5] (WLF) expressed in Eq. (7) was used to simulate the variation of a_T with the temperature.

$$\log(a_T) = \frac{-C_1(T - T_{ref})}{C_2 + T - T_{ref}} \tag{7}$$

where two constants of the WLF law (C_1 and C_2) should be determined. It should be noted that the identical shift factor a_T was used for both E^* and ν^* master curves. Some other studies [8, 17, 18] have confirmed this obtained result.

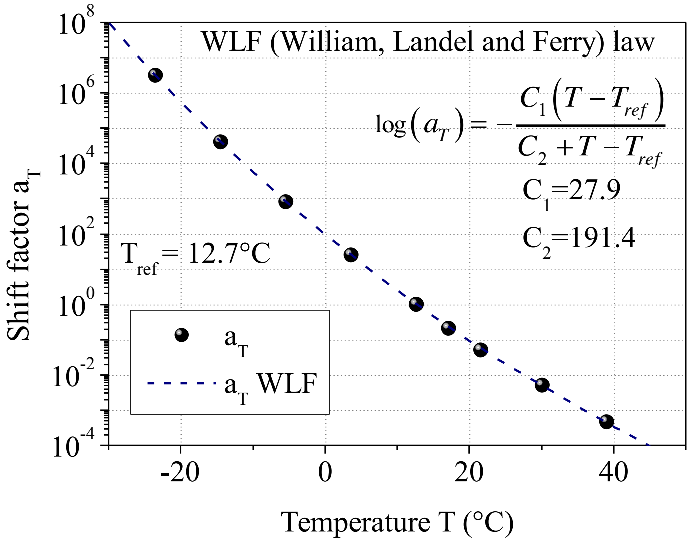

Figure 9. Shift factor a_T of GB3 mix and WLF law fitting

5. Modelling

Extensive works on the modelling of complex modulus of bituminous materials [7, 8, 19, 20] allowed to formulate a general 1D and 3D linear viscoelastic model, called 2S2P1D. 2S2P1D stands for 2 Springs, 2 Parabolic elements and 1 Dashpot) which are components of the model (Fig. 10). 2S2P1D is linear viscoelastic model with a continuum spectrum. In 1D case, the 2S2P1D model consists of a generalization of the Huet-Sayegh model [21]. Many researches have showed that the 2S2P1D model can simulate the 1D and 3D LVE behaviour of bituminous materials (such as binders,

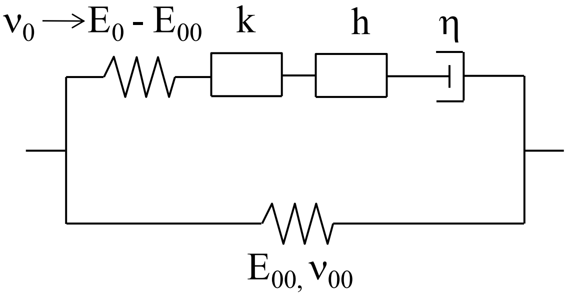

Figure 10. Representation of 2S2P1D model

mastics and mixtures) over a large range of temperatures and frequencies [7, 8, 22]. The expression of the complex modulus and complex Poisson's ratio of 2S2P1D model is given in Eqs. (8) and (9).

$$E^*(\omega) = E_{00} + \frac{E_0 - E_{00}}{1 + \delta (j\omega\tau)^{-k} + (j\omega\tau)^{-h} + (j\omega\beta\tau)^{-1}} \tag{8}$$

$$\nu^*(\omega) = \nu_{00} + (\nu_0 - \nu_{00}) \frac{E^*(\omega) - E_{00}}{E_0 - E_{00}} \tag{9}$$

where $\omega = 2\pi f$ (f is the frequency) is the pulsation; k and h are two exponents respecting $1 > h > k > 0$; δ and β are two constants; E_{00} is the static value of modulus obtained when the frequency tends toward 0; E_0 is the glassy value of modulus obtained when the frequency tends toward ∞; $\eta = (E_0 - E_{00})\beta\tau$ is Newtonian viscosity; τ is time parameter. This time parameter varies with the temperature and can be calculated as follow (if the TTSP is respected); $\tau(T) = a_T(T) \times \tau_0$ where $\tau_0 = \tau\left(T_{ref}\right)$ is the time parameter at T_{ref}; is the static value of Poisson's ratio when the frequency tends toward 0; is the glassy value of Poisson's ratio when frequency tends toward ∞. In 1D case, the simulation of the LVE of the tested bituminous materials using 2S2P1D model requires seven constants for 2 Springs (E_{00} and E_0), 2 Parabolic elements (δ, k and h), 1 Dashpot (β) and time parameter (τ). In 3D case, two Poisson's ratio parameters (and) are added. As a function of the shift factor, the time parameter τ can be simulated by a WLF law (Eq. (7)). Thus, two constants of the WLF law (C_1 and C_2) should be included in the simulation.

Figs. 11 and 12 present the simulation results using the 2S2P1D model. It can be observed that for both E^* (Fig. 11) and (Fig. 12), the simulation indicates a good result. It means that the high performance of 2S2P1D model is confirmed. This model can be also used to predict the LVE behaviour of bituminous mixtures from binder properties [23]. The simulation using 2S2P1D model allows obtaining the complex modulus values at any considered temperature-frequency that is required for asphalt pavement structure design [24]. The constants of 2S2P1D model used for the simulation are presented in the Table 1.

Fig. 11 presents the results for the complex modulus E^* and Fig. 12 is for the complex Poisson's ratio . Compared to the complex modulus E^*, the complex Poisson's ratio is generally more difficult to be measured because of the higher accuracy needed for the experimental device. Thus, values are obtained with higher dispersion. At very high temperature/low frequency, the stiffness of the material is quite small (see Fig. 7) and the measurement becomes more and more difficult. The value shows a decrease at high temperature/low frequency (see Fig. 12) and in some cases, this value increases that can be explained by high requirements of accuracy and the anisotropic behavior of the material especially at high temperatures/low frequency. For the 2S2P1D model, it can be seen in the Fig. 10

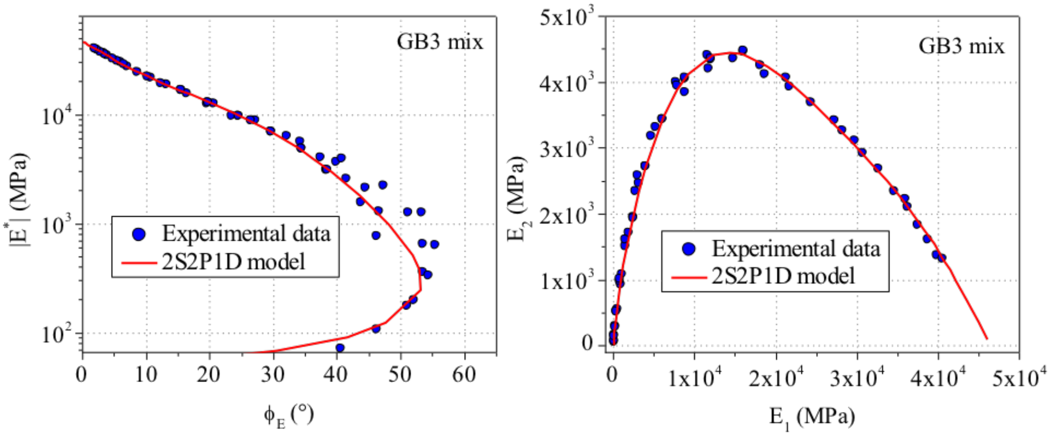

Figure 11. Simulation results for the complex modulus: Black diagram (left) and Cole-Cole diagram (right)

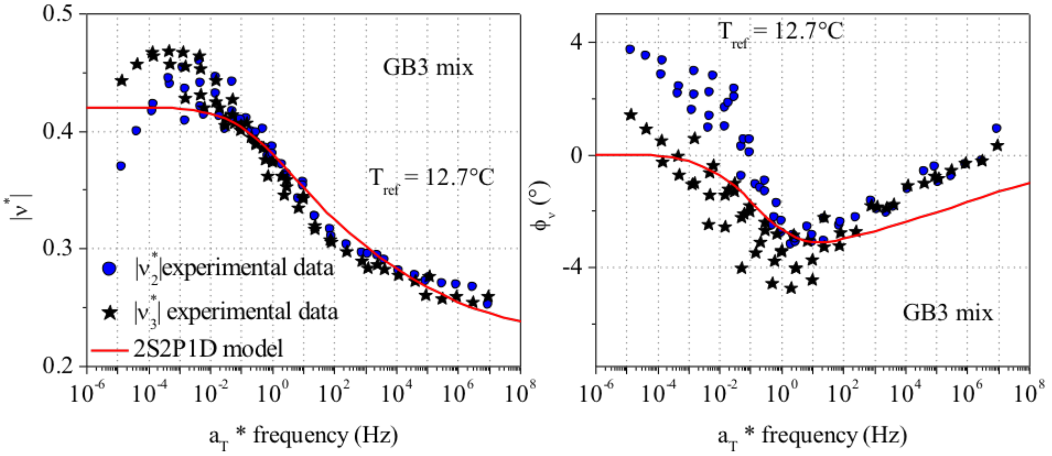

Figure 12. Simulation results for the complex Poisson's ratio: the norm ν^* master curve (left) and the phase angle master curve (right)

(and Eqs. (8), (9)) that at very low frequency ($\omega = 0$), the stiffness E^* of two parabolic elements and the dashpot become 0. Only the spring E_{00}, works such as an elastic material. Thus, the complex Poisson's ratio tends toward a constant value ν_{00} and its phase angle is a constant of 0.

Table 1. The constants of 2S2P1D model

E_{00} (MPa)	E_0 (MPa)	k	h	δ	τ (s)	β	ν_{00}	ν_0
50	46400	0.16	0.52	1.66	0.049	110	0.42	0.22

6. Conclusions

This paper focuses on the LVE behaviour of bituminous mixtures. From the presented results, the following conclusions can be given.

- The values of complex Poisson's ratio ν^* were measured in the small strain amplitude domain. The values of $|\nu^*|$ is not constant and is temperature and frequency dependent. The phase angle of the complex Poisson's ratio is quite small and negative except for some measurements at high temperatures.

- The complex modulus and complex Poisson's ratio master curves of tested bituminous mixture were plotted. Shift factors used for both master curves of E^* and ν^* are identical. The Time Temperature Superposition Principle can be applied for tested material in both 1D and 3D case.

- Shift factor a_T is simulated using the WLF law.

- The 2S2P1D model, developed at the ENTPE, shows a good performance and a good simulation of both 1D and 3D LVE behaviour of the tested mixture.

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