

EFFECT OF FIBER AMOUNT AND STIRRUP RATIO ON SHEAR RESISTANCE OF STEEL FIBER REINFORCED CONCRETE DEEP BEAMS

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Abstract

This paper deals with the effect of steel fiber amounts and the interaction between the fiber amount and stirrup ratio on the shear behavior and capacity of reinforced concrete (RC) deep beams with steel fibers. The experimental program was carried out on twelve deep beams with different fiber amounts (0, 30, 40, and 65 kg/m³) and stirrup ratios (0.1, 0.15, and 0.25%). The test results have shown that the use of steel fibers increased the shear resistance (up to 55%), reduced the shear crack width (up to 11 times) and deflection (up to 57%) of the tested deep beams. Also, it was found that using unsuitable steel fiber amount and stirrup ratio would reduce the efficiency of the fibers in a deep beam due to the interaction between the fibers and stirrups. Increasing the stirrup ratio in a deep beam with a high amount of steel fibers can reduce the efficacy of the fibers in enhancing the shear capacity of the beam. The most cost-effective steel fiber amount was found to be around 30 to 45 kg/m³.

Keywords: steel fibers; deep beam; shear capacity; fiber amount; stirrup ratio.

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1. Introduction

Several structural elements behave as deep beams; walls of bunkers, load-bearing walls in buildings, transfer beams in high-rise buildings, and pile caps are the typical examples [1, 2]. It is well-known that the behavior of long-span beams (Bernoulli beams) is governed mainly by flexure that results in the bending stress which is higher compared to the shear stress as observed in previous experimental studies [3–7]. However, for shorter span beams, particularly deep beams, due to their geometry, the shear behavior plays a decisive role. In these beams, the shear cracks may form in the web due to high diagonal tensile stress and precipitate failure before the ultimate flexural capacity of the beam is reached [8, 9].

Numerous investigations of reinforced concrete (RC) beams in shear or RC slabs in punching shear have proven that using an adequate dosage of steel fibers of appropriate shapes can significantly

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enhance the shear loading capacity of the structures thanks to a good crack control mechanism provided by the steel fibers [10–17]. Adding the steel fibers to concrete may also result in the change from a brittle shear failure mode to a ductile flexure mode [18–21]. Overall, the presence of steel fibers in the concrete matrix can improve the shear loading capacity, ductility, and capacity of energy absorption of the structures.

Steel fibers, which are uniformly distributed in the concrete, have some benefits compared to stirrups. Firstly, fibers' spacing is incomparably smaller than the spacing that can be practically achieved by using stirrups. This small spacing is greatly advantageous since the fibers can offer a better crack-bridging effect throughout the entire concrete matrix. The second benefit could be that deploying a sufficient amount of steel fibers can increase the tensile strength of the concrete matrix, which improves the shear-cracking resistance of the member [14]. Despite these evident benefits, up to now, relatively few works dealing with the use of steel fibers in deep beams have been published [8, 22–28]. All these works indicated the effectiveness of steel fibers in enhancing the shear capacity and ductility of deep beams. However, the questions related to the use of a reasonable fiber amount as well as the interaction between the fiber amount and stirrup ratio in deep beams remain unanswered so far and therefore, further research on this topic is desirable.

This paper presents an experimental study on the shear behavior of steel fiber reinforced concrete (SFRC) deep beams. The main investigated parameters are the amount of steel fibers and the stirrup ratio. The interaction between the fiber amount and stirrup ratio, shear cracking, and shear capacity of the beams are discussed in this study.

2. Experimental investigation

2.1. Tested specimens

A total of 12 beams were tested. All beams were of the same rectangular cross-section: 150 mm in width and 500 mm in depth. The length of the beams was 2000 mm with a clear span of 1600 mm.

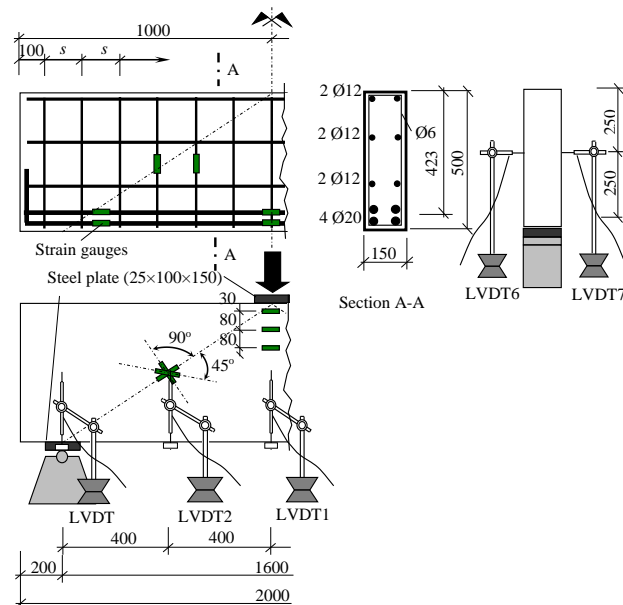


Figure 1. Details of the tested beams and test arrangement

The beams were divided into three groups designated as A, B, and C with different stirrup ratios. The stirrup ratio ρ_{sw} of Group A, B, and C was 0.25, 0.15, and 0.1%, respectively. These ratios corresponded to the stirrup spacing of 150, 260, and 400 mm, respectively. Each group included one control beam without fibers and three SFRC concrete beams having different fiber amounts m_f [30, 45, and 65 kg/m³ (Table 1)]. All the beams had the same longitudinal tensile reinforcement ratio $\rho_s = 1.77\%$. Details of the tested beams are shown in Fig. 1.

Table 1. Test specimens and material mechanical properties

Group	Specimens	$b \times h \times L$ (mm)	$f_{c,cube}$ (MPa)	$f_{sp,cube}$ (MPa)	f_y (MPa)	f_{yw} (MPa)	m_f (kg/m ³)	V_f (%)	ρ_{sw} (%)
A	D0-0.25	150×500×2000	30.5	3.1	420	260	0	0	0.25
	D0.4-0.25		40.0	3.8	420	260	30	0.40	0.25
	D0.6-0.25		42.0	4.3	420	260	45	0.60	0.25
	D0.85-0.25		39.5	4.6	420	260	65	0.85	0.25
B	D0-0.15	150×500×2000	30.5	3.1	420	260	0	0	0.15
	D0.4-0.15		40.0	3.8	420	260	30	0.40	0.15
	D0.6-0.15		42.0	4.3	420	260	45	0.60	0.15
	D0.85-0.15		39.5	4.6	420	260	65	0.85	0.15
C	D0-0.1	150×500×2000	30.5	3.1	420	260	0	0	0.10
	D0.4-0.1		40.0	3.8	420	260	30	0.40	0.10
	D0.6-0.1		42.0	4.3	420	260	45	0.60	0.10
	D0.85-0.1		39.5	4.6	420	260	65	0.85	0.10

where $f_{c,cube}$ and $f_{sp,cube}$ are respectively the compressive strength and splitting tensile strength of the concrete cube; f_{yw} and f_y are respectively the yield stress of the stirrups and longitudinal reinforcement; m_f is the weight of the total fibers in a unit concrete volume; V_f is the fiber volume fraction (the total fiber volume in a unit concrete volume), $= 100m_f/7850$; and ρ_{sw} is the stirrup ratio.

The tested beams were cast at the same time from the same batch of concrete. Adequate compaction was achieved by needle vibrators. Observed concrete slumps of Group A, B, and C specimens were from 90 mm to 120 mm. All the tested beams were constructed and cured under the same conditions and tested after achieving a curing period of 28 days.

2.2. Materials

The tested beams were made using the concrete containing cement PC40 (440 kg/m³), natural sand (0-4 mm, 640 kg/m³), coarse aggregate (20-25 mm, 1250 kg/m³), water (176 l/m³), and plasticizer (5 l/m³). Cube specimens (150×150×150 mm) were used to determine the compressive $f_{c,cube}$ and splitting tensile strengths $f_{sp,cube}$ of concrete. Hooked-end steel fibers were used in the test program with the tensile strength of 1100 MPa and the elastic modulus of 200 GPa. The length and diameter of the individual steel fiber were in turn 60 and 0.75 mm. The average tested concrete strengths $f_{c,cube}$ and $f_{sp,cube}$ obtained from the tests were summarized in Table 1, in which m_f (kg/m³) is the total fiber weight in a unit volume of concrete, V_f (%) is the total fiber volume in a unit volume of concrete [$V_f(\%) = 100m_f/7850$].

2.3. Testing procedure and instrumentation

All the beams were tested under three-point bending as illustrated in Fig. 1. Five linear variable differential transformers (LVDTs) were used to determine the deflection at mid-span, 1/4 span, and the supports of the beams. Another two LVDTs were installed opposite to each other, perpendicular to the web at 1/4 span of the beams to measure the out-of-plane deflection. Six electrical strain gauges were bonded on two longitudinal tensile rebars to measure their strain at mid-span and shear-span near the supports. Four strain gauges were installed on the stirrups to measure their strain. The concrete compressive strain was measured by three strain gauges installed along the beam's height at mid-span. To assess the strain capacity of the concrete diagonal compressive strut in shear-span, a pair of multi-axial strain gauges was bonded in the web to measure its diagonal compressive and tensile strain. The beams were tested by using a 1000 kN capacity hydraulic testing machine using a load-manipulated regime. The loading increment was 10 kN up to failure, which equals the loading rate of 15 kN per minute as per the previous studies [9, 15]. At each loading increment, the beam's displacement, strains of concrete and rebars, and crack development and propagation were recorded. All of the instrumentation's locations are shown in Fig. 1.

Rebars of 20 mm diameter were used as longitudinal tensile reinforcement in the beams. Rebars of 12 mm diameter were used for the top and distributed rebars while rebars with 6 mm in diameter were used for stirrups. The mechanical properties of steel reinforcement were determined by tensile tests. The average tested yield strength and ultimate tensile strength of longitudinal tensile rebars and stirrups were 420 MPa and 550 MPa, and 260 MPa and 460 MPa, respectively. The modulus of elasticity of the steel reinforcements E_s was 210 GPa.

3. Test results and discussion

3.1. Failure Patterns

All the tested beams failed in shear combined with concrete crushing at the loading point. The crack patterns of the beams are illustrated in Fig. 2. The crack formation was initiated with vertical cracks (flexural cracks) at mid-span at loading levels equal to approximately 15-20% of the failure load. As the load increased, the existing vertical cracks developed gradually in width and length, accompanied by some new flexural cracks propagating towards the supports. At the loading level of 30-45% of the failure load, the inclined cracks begun in the beam web and developed towards the loading point and the support as the load increased. At a loading level of 70% of the failure load, the flexural cracks stopped developing; however, the inclined cracks opened quickly, which led to concrete crushing at the loading point and the beam's collapse. The failure mode of the SFRC deep beams in this study can be classified as the shear compression failure.

Failure modes of the beams containing no fibers were quite brittle and sudden (Fig. 2(a)), in accordance with the well-known fact about a traditional RC beam failing in shear, while the SFRC beams failed in more ductile modes with smaller crack width (Figs. 2(b), 2(c), and 2(d)). Moreover, the inclined cracks in the beams having higher stirrup ratios distributed more uniformly and had smaller widths in comparison with those of the beams with smaller stirrup ratios. The steel fibers hardly affected the inclination of the cracks but the stirrup ratio had considerable influence on it. As the stirrup ratio increased, the angle of the major inclined cracks with respect to the beam's longitudinal axis increased from 33° to 43°.

It should be noted that the initiation of the inclined cracks in the beams is governed by concrete tensile strength rather than discrete stirrups. Once principal tensile stress exceeds the tensile strength

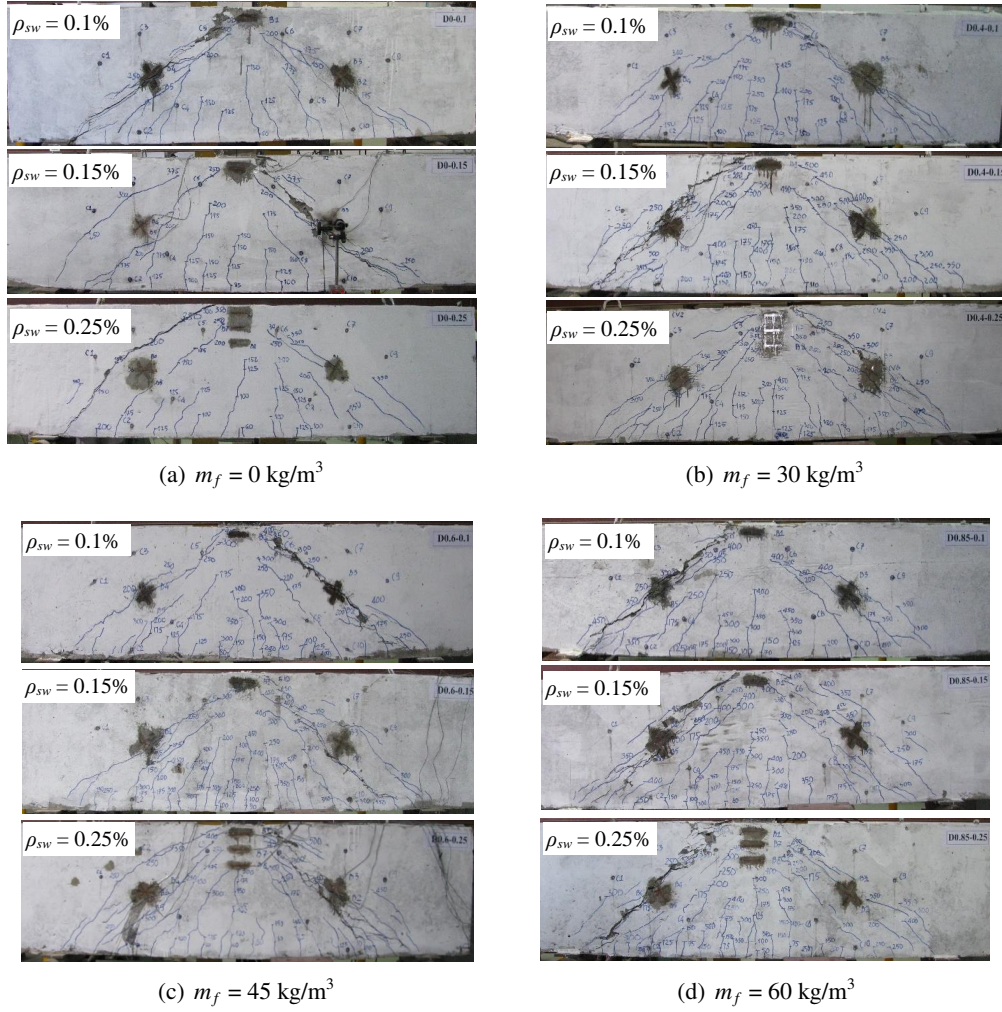


Figure 2. Crack patterns at failure of the specimens

of concrete, the inclined cracks occur regardless of the presence of stirrups. On the other hand, it is evident that the steel fibers increased considerably the tensile strength of concrete (Table 1). Consequently, the steel fibers delayed the initiation of the first shear crack in the SFRC beams as shown via the load level corresponding to the first appearance of shear cracks ($P_{cr,sh}$) of the SFRC beams being higher than that of the beams without fibers (Table 2). Also, due to the bridging effect, the steel fibers helped to redistribute the tensile stress in the beam web which eventuated in the distribution and width of the inclined cracks in the SFRC deep beams being more uniformly and smaller than those in the beams without fibers. Nevertheless, once the inclined cracks have been initiated and opened large enough, the role of concrete decreases quickly. Hence, the transfer of principal tensile stress, as well as crack control, is mainly undertaken by stirrups. As a result, the cracks of the beams with a higher stirrup ratio distributed more uniformly and had a smaller width in comparison with those of the beams with a lower stirrup ratio.

Table 2. Experimental results

Group	Specimens	$P_{cr,fl}$ (kN)	$P_{cr,sh}$ (kN)	$P_{u,test}$ (kN)	$\varepsilon_{s,u}$ (‰)	$\varepsilon_{sw,u}$ (‰)	$\varepsilon_{cu,mid}$ (‰)	$\varepsilon_{cu,inc}$ (‰)	$\varepsilon_{ct,inc}$ (‰)	w_u (mm)	δ_u (mm)
A	D0-0.25	60	150	390	3.17	4.63	3.22	4.10	0.059	3.7	5.06
	D0.4-0.25	80	175	550	5.11	7.23	3.49	2.16	0.410	3.1	5.87
	D0.6-0.25	80	185	570	5.61	5.16	3.81	3.04	0.590	3.0	7.03
	D0.85-0.25	85	195	590	5.92	4.92	4.09	2.88	0.697	2.8	7.35
B	D0-0.15	60	150	375	3.35	4.85	1.21	1.08	0.055	3.6	4.73
	D0.4-0.15	85	180	500	4.33	4.72	4.36	0.66	0.383	3.0	5.44
	D0.6-0.15	90	185	520	4.51	5.94	3.89	2.37	0.452	2.9	5.93
	D0.85-0.15	90	190	530	4.61	5.31	2.95	2.64	0.674	2.8	6.41
C	D0-0.1	60	150	330	2.84	5.48	3.68	3.11	0.052	3.5	4.43
	D0.4-0.1	80	175	410	3.76	7.20	1.65	2.86	0.375	2.9	4.45
	D0.6-0.1	90	185	460	3.99	6.41	2.81	-	0.458	2.8	4.65
	D0.85-0.1	90	190	510	4.09	7.82	3.66	1.90	0.680	2.8	5.13

Notes: the first number in the name of the specimens indicates fiber volume and the second number shows the stirrup ratio; $P_{cr,fl}$, $P_{cr,sh}$ and $P_{u,test}$ are respectively the flexural cracking, shear cracking and the ultimate force of the beams; $\varepsilon_{sw,u}$ and $\varepsilon_{s,u}$ are respectively the ultimate strain of stirrups and longitudinal reinforcement; $\varepsilon_{cu,mid}$ is the ultimate concrete compressive strain at midspan; $\varepsilon_{cu,inc}$ is the inclined compressive strain of the web concrete in the shear span at ultimate; $\varepsilon_{ct,inc}$ is the inclined-cracking strain of the web concrete in the shear span (the inclined tensile strain at $P_{cr,sh}$); w_u is the maximum crack width; and δ_u is the maximum displacement of the beams.

3.2. Load-displacement response and shear cracking behavior

The load-deflection responses of the tested beams are shown in Fig. 3 and their ultimate deflection is presented in Table 2. The behavior of the beams is characterized by small deformation. In general, the behavior of all the beams can be divided into two stages: prior to shear cracking (corresponding to loading level approximately 45–55% of the failure load of the beam without fibers) and post-shear cracking. In the first stage, only beam stiffness was slightly improved with the use of steel fibers, and hence the behavior of the tested beams was nearly the same. In the second stage, the stiffness of the SFRC beams was significantly different from the control beams without fibers. The SFRC beams behaved stiffer than the control beams, which eventuated a reduction in the displacement of the SFRC beams compared to the control beams under the same loads. As the load increased and approached the failure load, this trend was clearer. The smaller displacement of SFRC beams could be attributed to the bridging effect of steel fibers. After cracks appear in the beam, the fibers bridge the cracks and commence carrying tensile stresses in the tension side of the beam. This mechanism delays the crack development, reduces the crack width, and provides tensile loading resistance to the SFRC concrete which is not the case for plain concrete. As a result, the post-cracking stiffness of the SFRC beam is enhanced and thus the deflection of the SFRC beam is smaller than that of the beam without fibers.

For the beams in Group A ($\rho_{sw} = 0.25\%$), the use of 30 kg/m^3 fibers reduced significantly the ultimate deflection of the beams by up to 44%. However, as the fiber amount used in the beams increased to 45 or 65 kg/m^3 , the reduction remained virtually the same. Similarly, for the beams in Group B ($\rho_{sw} = 0.15\%$) and C ($\rho_{sw} = 0.10\%$), the reduction in the ultimate deflection is approximately 40%.

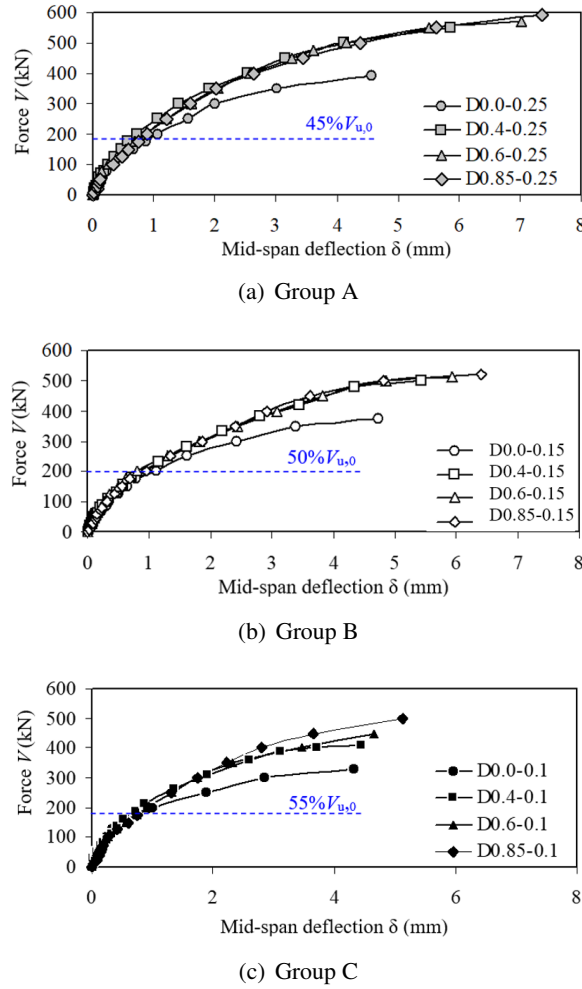
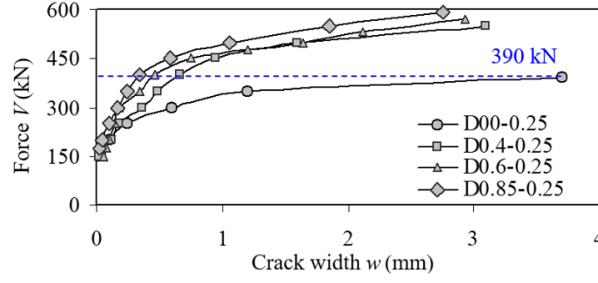


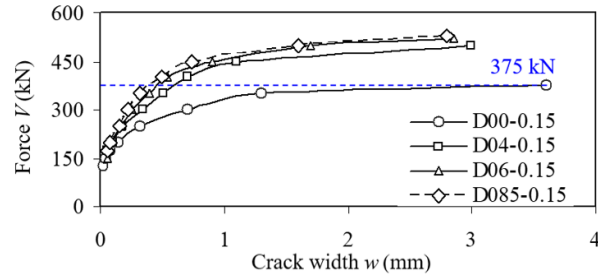
Figure 3. Load-deflection diagrams of the tested beams

The significant reduction in deflection of the SFRC beams compared to the control beams without fibers can be explained by the smaller crack width of the former beams. Test results show that the steel fibers reduced significantly the crack width of the SFRC beams with regard to the control beams (Fig. 4). At the same loading level, the cracks in the beams with fibers had a much smaller width in comparison with the ones in the beams without fibers. The smaller crack width resulted in larger stiffness of the SFRC beams and therefore smaller deflection of the beams. Regarding the aspect of deflection reduction, the test results also indicate that the most effective fiber amount in the beams with stirrups should be about 30 kg/m^3 . The use of a higher fiber amount did not lead to significantly higher effectiveness of the fibers in the improvement of the beam's stiffness.

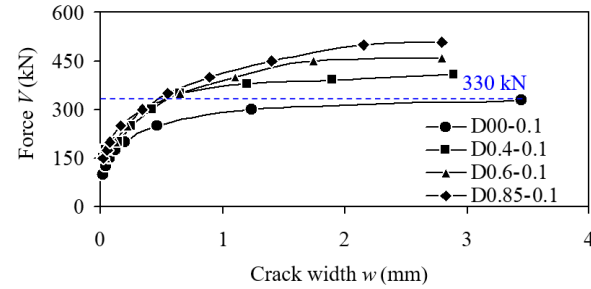
Increasing the stirrup ratio also reduced the beam's deflection in the post-shear cracking stage but less effective than increasing the fiber amount. It is interesting to note that the effectiveness of the stirrups in reducing the beam's deflection decreased as the fiber amount increased. For the beams with 0 and 30 kg/m^3 fibers, an increase in the stirrup ratio from 0.1 to 0.25% reduced the beam's deflection by approximately 42%. For the beams with 45 and 65 kg/m^3 fibers, the corresponding reduction was 29% and 14%, respectively.



(a) Group A



(b) Group B



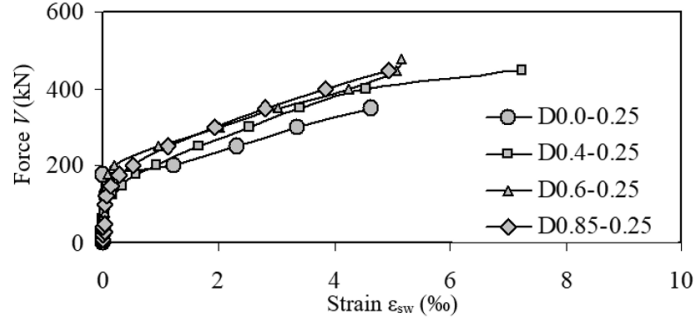
(c) Group C

Figure 4. Typical load-crack width diagrams of the tested beams

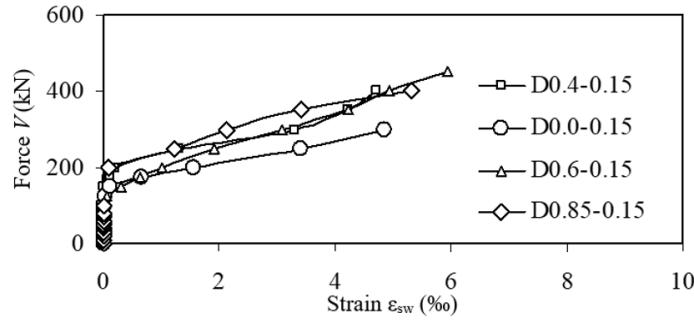
The shear cracking behavior of the tested beams is illustrated in Fig. 4 and the maximum crack width is presented in Table 2. The results clearly show that the crack width of the SFRC beams was much smaller than the one of the beams without fibers. For Group A, at the failure load (390 kN) of the beam without fibers (D0.0-0.25), the SFRC beams had crack width 6 to 11 times smaller compared to the beam without fibers. Similarly, for Group B, at the failure load (375 kN) of the beam without fibers (D0.0-0.15), the SFRC beams had crack width 6 to 9 times smaller compared to the beam without fibers. The corresponding number for Group C was 6 to 8 times. These results confirm the efficiency of the steel fibers in crack control. This good crack control mechanism of the fibers can be explained by their well-known crack-bridging effect.

3.3. Strains of stirrups, longitudinal reinforcement and concrete

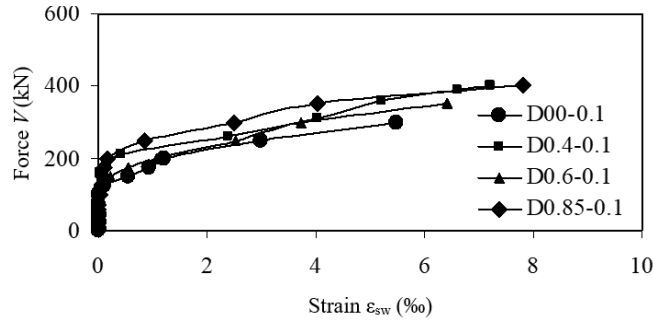
The load-stirrup strain relationships are plotted in Fig. 5 and the maximum stirrup strain is summarized in Table 2. The behavior of the stirrups was divided clearly into two stages: before and after



(a) Group A



(b) Group B



(c) Group C

Figure 5. Load-stirrup strain diagrams of the tested beams

the initiation of the first shear crack. In the stage before the initiation of the first shear crack (approximately 32 to 45% of the ultimate load), the stirrup strain was very small, showing clearly that the stirrups did not work much in this stage. In the stage after initiation of the shear crack to failure, the strain of the stirrups increased rapidly. The strain of stirrups in the SFRC beams was much smaller than the one of the corresponding control beams. For the beams in Group A, the control beam without fibers (D0.0-0.25) had the maximum stirrup strain equal to 4.63% at the loading level of 350 kN. At this loading level, the stirrup strains of the beams with 30, 45, and 65 kg/m³ fibers were only 3.41, 3.04, and 2.80%, which were equal to the reduction of 26, 34, and 39.5%, respectively. For the beams in Groups B and C, the corresponding reduction ranged from 32 to 56% and from 26 to 54%, respectively. These results show that the use of the steel fibers reduced significantly tensile strain in

the stirrups of the tested beams and the reduction increased as the fiber amount increased. Moreover, the maximum stirrup strain of the tested beams was from 4.63 to 7.82‰, indicating that the stirrups yielded substantially.

The maximum strain of the tensile longitudinal reinforcement of the tested beams at mid-span ranged from 2.84 to 5.92%. These results indicate that the rebars yielded. It is worth noting that the ultimate strain of the rebars in SFRC beams was much higher than that of the beams without fibers. This can be explained by the presence of the fibers in the concrete that increased the loading capacity of the beams, resulting in the higher tensile stress/strain in the rebars of the SFRC beams.

Moreover, the diagonal-cracking tensile strain of concrete ($\varepsilon_{ct,inc}$) was improved by the use of the steel fibers as shown in Table 2, where $\varepsilon_{ct,inc}$ is the diagonal strain of the web concrete at shear span at the loading level corresponding to the first appearance of shear cracks ($P_{cr,sh}$). $\varepsilon_{ct,inc}$ of the SFRC beams was 7 to 13 times higher than $\varepsilon_{ct,inc}$ of the beams without fibers and the increase was proportional to the fiber amount. Finally, the ultimate compressive strain of the concrete at mid-span ranged from 1.21 to 4.09‰.

3.4. Shear cracking force and shear resistance of tested beams

The experimental shear cracking force $P_{cr,sh}$ and ultimate shear loading capacity $P_{u,test}$ of the tested beams are summarized in Table 2. The results showed that the SFRC beams had higher shear cracking force in comparison with the beams without fibers. The use of the fibers increased the shear cracking resistance of the tested beams by 16.7-30.0% (Fig. 6) in comparison with beams without fibers. Regarding shear capacity, the results indicate that the fibers improved significantly the shear capacity of the beams. The use of 30 to 65 kg/m³ fibers increased the shear capacity of the SFRC beams by 24.2-54.5% in comparison with the beams without fibers. The shear capacity of the beams increased as the fiber amount increased (Fig. 7). It can also be seen from Fig. 7 that apart from Group C (with a low stirrup ratio $\rho_{sw} = 0.1\%$), the increase rate in the shear capacity of the beams declined when raising the fiber amount, particularly when the fiber amount rose from 30 kg/m³ to 45 kg/m³. This meant that the efficacy of the steel fibers in improving the shear capacity of the deep beams decreased when applying a high amount of the fibers. One of the possible reasons for this decrease in the efficacy could be attributed to the fiber balling phenomenon in the process of making concrete when a high fiber amount was used, which resulted in the inconsistency of the concrete quality. The consolidation of the fibrous concrete may be further deterred in beams with a high level of reinforcement congestion (e.g. high stirrup ratios), thereby leading to an added inconsistency of the

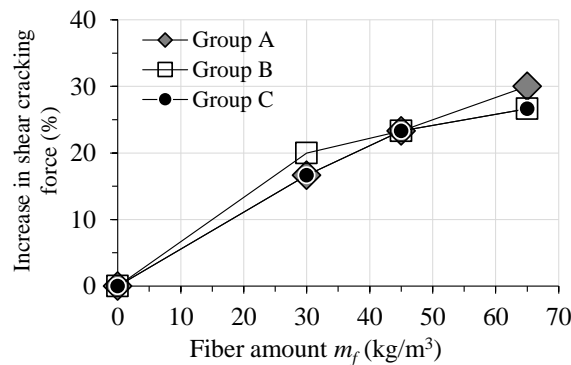


Figure 6. Increase in shear cracking force of the tested beams against fiber amount

concrete quality. Also, it is believed that with a lower stirrup ratio in Group C, the shear contribution of stirrups was less, and thus the effect of steel fibers in increasing shear resistance of the deep beams in Group C was higher as compared to that in Groups A and B. However, further researches are worth being conducted to provide more understanding of this phenomenon. Additionally, the efficacy of the steel fibers appeared to rise when the stirrup ratio increased in the beams with the fiber amount not higher than 45 kg/m^3 as shown in Fig. 7. However, the opposite trend was witnessed when the fiber amount was higher than 45 kg/m^3 as the efficacy of the fibers decreased when the stirrup ratio rose (Fig. 7). These observations prove that an interaction between the stirrup ratio and fiber amount in an RC deep beam exists. Thus, deploying an unsuitable combination of stirrup ratios and steel fiber amounts in a deep beam can reduce the efficacy of the fibers.

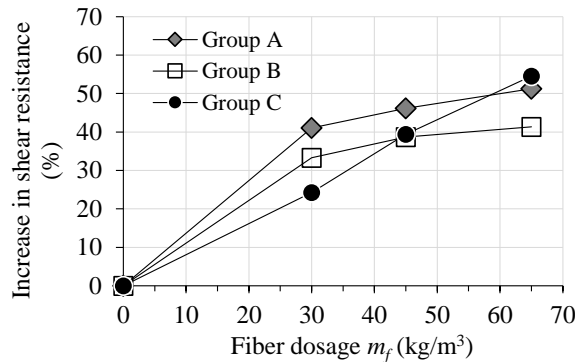


Figure 7. Increase in shear capacity of the tested beams against fiber amount

Lastly, the stirrup ratio affected considerably the shear cracking force as well as the shear capacity of the beams. An increase of the stirrup ratio from 0.1 to 0.25% led to an increase in the shear capacity of the beams without fibers by approximately 18% and of the SFRC beams by about 34%.

4. Conclusions

Based on the results obtained from this study, the following conclusions can be drawn:

1. The use of the steel fibers in the tested deep beams subjected to shear loads has proven the following benefits:

- Increase in the shear cracking resistance (up to 30%);
- Increase in the shear capacity (up to 54.5%);
- Reduction in the crack width (up to 11 times);
- Reduction in the deflection (up to 44%);
- Increase in the concrete diagonal-cracking tensile strain (up to 13 times).

2. The use of unsuitable fiber amount and stirrup ratio can reduce the efficiency of the steel fibers in a deep beam because there is an interaction between the fibers and stirrups. The test results from this study indicate that increasing the stirrup ratio in a deep beam using a high steel fiber amount can reduce the efficacy of the fibers in improving the beam's shear capacity. The most cost-effective amount of the steel fibers for the deep beam's configuration used in this study could be around 30 to 45 kg/m^3 .

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