Journal of Science and Technology in Civil Engineering NUCE 2020. 14 (1): 127–135

OPTIMAL PARAMETERS OF TUNED MASS DAMPERS FOR MACHINE SHAFT USING THE MAXIMUM EQUIVALENT VISCOUS RESISTANCE METHOD

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Article history:
Received 07/10/2019, Revised 21/11/2019, Accepted 26/11/2019

Abstract

The paper analyzes and determines the optimal parameters of tuned mass damper to reduce torsional vibration for the machine shaft. The research steps are as follows. First, the optimal parameters of tuned mass damper for the shafts are given by using the maximization of equivalent viscous resistance method. Second, a numerical simulation is performed for configuration of machine shaft to validate the effectiveness of the obtained analytical results. The simulation results indicate that the proposed method significantly increases the effectiveness of torsional vibration reduction. Optimal parameters include the ratio between natural frequency of tuned mass damper and the machine shaft, the ratio of the viscous coefficient of tuned mass damper. The optimal parameters found by numerical method only apply to a machine shaft with specific data. However, the optimal parameters in this paper are found as analytic and explicit to help scientists easily apply to every machine shafts when the input parameters of the machine shaft change.

Keywords: tuned mass damper; torsional vibration; optimal parameters; random excitation; equivalent viscous resistance.

https://doi.org/10.31814/stce.nuce2020-14(1)-11

1. Introduction

Under the influence of external forces, the technical constructions, the mechanical devices will generate vibrations. Vibrations can cause damage to the structure. Therefore, research harmful vibration is a matter of great concern to many scientists [1–15].

The shaft is used to transmit torque and rotation from a part to another part of the machine. During operation, the shaft will appear torsional vibration. This vibration is particularly harmful, undesirable. Reduction of the shaft vibration is an important and timely task [1–10].

A passive vibration control device attached to the shaft to reduce harmful vibration is called a tuned mass damper (TMD) [10]. Optimal parameters of the TMD to reduce the torsional vibration of the shaft by using the principle of minimum kinetic energy has been investigated in [10], the results were given by

$$\alpha_{opt}^{MKE} = \frac{1}{1+2\mu\gamma^2}; \quad \xi_{opt}^{MKE} = \gamma\sqrt{\frac{\mu}{2(1+2\mu\gamma^2)}} \tag{1}$$

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In order to develop and extend the research results in [10]. In this paper, the maximization of equivalent viscous resistance method in [12] is used for determining the optimal parameters of the TMD.

2. Shaft modelling and vibration equations

Fig. 1 shows a shaft attached with a pendulum type TMD. The symbols are summarized in Table 1.

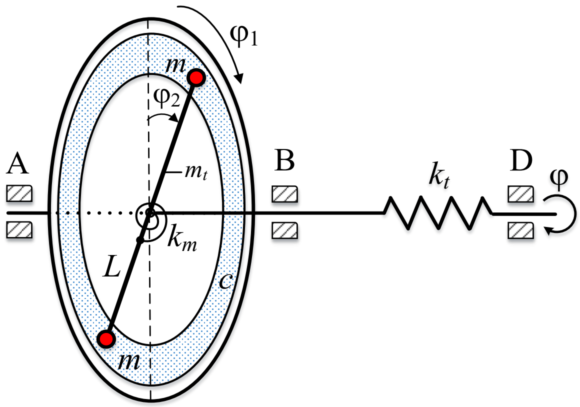

Figure 1. Shaft model with installed TMD

Table 1. Symbols used to describe the vibration of the shaft with TMD

Symbol	Description
k_t	Torsion spring coefficient of shaft
m	concentrated mass at the top of TMD
c	Damping coefficient of damper
k_m	Torsional stiffness of spring of TMD
L	Length of pendulum of TMD
m_t	Mass of pendulum rod
ρ	Radius of gyration of rotor
M	Mass of primary system
φ	Angular displacement of shaft
φ_1	Angular displacement of rotor
φ_2	Relative torsional angle between TMD and rotor
θ	Torsional vibration of primary system
θ_0	Initial condition of the torsional vibration angle

From [10], we have

$$\begin{cases} (M\rho^2 + \dfrac{2}{3}m_tL^2 + 2mL^2)\ddot{\theta} + 2(\dfrac{1}{3}m_tL^2 + mL^2)\ddot{\varphi}_2 = M(t) - k_t\theta \\ 2(\dfrac{1}{3}m_tL^2 + mL^2)\ddot{\theta} + 2(\dfrac{1}{3}m_tL^2 + mL^2)\ddot{\varphi}_2 = -k_m\varphi_2 - 2cL^2\dot{\varphi}_2 \end{cases} \tag{2}$$

where

$$\varphi_1 - \varphi = \theta \tag{3}$$

After short modification the Eqs. (2) we obtained

$$M\rho^2\ddot{\theta} + k_t\theta = k_m\varphi_2 + 2cL^2\dot{\varphi}_2 + M(t) \tag{4}$$

Hence the torque equivalent effect on the primary structure was obtained as

$$M_{eqv} = k_m\varphi_2 + 2cL^2\dot{\varphi}_2 \tag{5}$$

Eq. (5) can be used in the design of TMD.

3. Determining optimal parameters of TMD

We introduce

$$\mu = \frac{m + {}^{m_t}\!/_3}{M}; \quad \omega_d = \sqrt{\frac{k_m}{2(m + {}^{m_t}\!/_3)L^2}}; \quad \gamma = \frac{L}{\rho} \tag{6}$$

$$\xi^{MEVR} = \frac{c}{2(m + \frac{m_t}{3})\omega_d}; \quad \alpha^{MEVR} = \frac{\omega_d}{\omega_D}; \quad \omega_D = \sqrt{\frac{k_t}{M\rho^2}} \tag{7}$$

The symbols are summarized in Table 2.

Table 2. Symbols used to write the non-dimensional equations

Symbol	Description
ω_D	Natural frequency of vibration of shaft
ω_d	Natural frequency of vibration of TMD
ξ_{MEVR}	Damping ratio of TMD by using the maximization of equivalent viscous resistance method
ξ_{opt}^{MKE}	Optimal damping ratio of TMD by using the minimum kinetic energy method
ξ_{opt}^{MEVR}	Optimal damping ratio of TMD by using the maximization of equivalent viscous resistance method
μ	Ratio between mass of TMD and mass of rotor
ξ_{MEVR}	Tuning ratio of TMD by using the maximization of equivalent viscous resistance method
α_{opt}^{MEVR}	Optimal tuning ratio of TMD by using the maximization of equivalent viscous resistance method
α_{opt}^{MKE}	Optimal tuning ratio of TMD by using the minimum kinetic energy method
γ	Ratio between length of pendulum and radius of gyration of rotor
M_{eqv}	Torque equivalent effect on the primary structure

Substituting Eqs. (6)–(7) into Eqs. (2). The matrix form of Eqs. (2) are expressed as

$$\mathbf{M}^{MEVR}\ddot{\mathbf{x}}_1 + \mathbf{C}^{MEVR}\dot{\mathbf{x}}_1 + \mathbf{K}^{MEVR}\mathbf{x}_1 = \mathbf{F}^{MEVR} \tag{8}$$

where

$$\mathbf{x}_1 = \left\{ \begin{matrix} \theta & \varphi_2 \end{matrix} \right\}^T \tag{9}$$

The mass matrix, viscous matrix, stiffness matrix and excitation force vector can be derived as

$$\begin{gathered} \mathbf{M}^{MEVR} = \begin{bmatrix} 1 + 2\mu\gamma^2 & 2\mu\gamma^2 \\ 1 & 1 \end{bmatrix}; \quad \mathbf{C}^{MEVR} = \begin{bmatrix} 0 & 0 \\ 0 & 2\xi^{MEVR}\alpha^{MEVR}\omega_D \end{bmatrix}; \\ \mathbf{K}^{MEVR} = \begin{bmatrix} \omega_D^2 & 0 \\ 0 & \omega_D^2(\alpha^{MEVR})^2 \end{bmatrix}; \quad \mathbf{F}^{MEVR} = \begin{Bmatrix} \dfrac{M(t)}{M\rho^2} \\ 0 \end{Bmatrix} \end{gathered} \tag{10}$$

The state equations of Eq. (8) are expressed as

$$\dot{\mathbf{x}}_2(t) = \mathbf{B}\mathbf{x}_2(t) + \mathbf{H}_f M(t) \tag{11}$$

where

$$\mathbf{x}_2 = \left\{ \begin{matrix} \theta & \varphi_2 & \dot{\theta} & \dot{\varphi}_2 \end{matrix} \right\}^T \tag{12}$$

From Eqs. (8)–(12), the matrices $\mathbf{B}$ and $\mathbf{H}_f$ can be defined as

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_D^2 & 2\mu\gamma^2\alpha^2\omega_D^2 & 0 & 4\mu\gamma^2\xi^{MEVR}\alpha^{MEVR}\omega_D \\ \omega_D^2 & -(1+2\mu\gamma^2)(\alpha^{MEVR})^2\omega_D^2 & 0 & -2(1+2\mu\gamma^2)\xi^{MEVR}\alpha^{MEVR}\omega_D \end{bmatrix} \tag{13}$$

$$\mathbf{H}_f = \begin{bmatrix} 0 & 0 & \dfrac{1}{M\rho^2} & -\dfrac{1}{M\rho^2} \end{bmatrix}^{-1} \tag{14}$$

The quadratic torque matrix $\mathbf{P}$ is solution of the Lyapunov equation [14]

$$\mathbf{BP} + \mathbf{PB}^T + S_f \mathbf{H}_f \mathbf{H}_f^T = \mathbf{0} \tag{15}$$

where S_f is the white noise spectrum of the excitation torque.

The first step of this method is to specify these quadratic torques. Substituting Eqs. (13)–(14) into Eq. (15) and solving this equation, these quadratic torques for vibration response of shaft were obtained as

$$P_{32} = -\frac{S_f}{4\mu\gamma^2 M^2\omega_D^2\rho^4} \tag{16}$$

$$P_{33} = \frac{S_f[2(\alpha^{MEVR})^4\gamma^2\mu + (\alpha^{MEVR})^4 + 4(\alpha^{MEVR})^2(\xi^{MEVR})^2 - 2(\alpha^{MEVR})^2 + 1]}{8\mu\gamma^2\xi^{MEVR}\alpha^{MEVR}\omega_D M^2\rho^4} \tag{17}$$

$$P_{34} = \frac{S_f[(\alpha^{MEVR})^2 - 1]}{8\mu\gamma^2\xi^{MEVR}\alpha^{MEVR}\omega_D M^2\rho^4} \tag{18}$$

Substituting Eqs. (6)–(7) into Eq. (5), this becomes

$$M_{eqv} = 2(m + m_t/3)(\alpha^{MEVR})^2\omega_D^2\gamma^2\rho^2\varphi_2 + 4\xi^{MEVR}(m + m_t/3)\alpha^{MEVR}\omega_D\gamma^2\rho^2\dot{\varphi}_2 \tag{19}$$

Thus the equivalent resistance coefficient of the TMD on the primary structure was obtained as

$$c_{td} = -\frac{\left\langle M_{eqv}\dot{\theta}\right\rangle}{\left\langle \dot{\theta}^2\right\rangle} = -\frac{4\xi^{MEVR}(m + m_t/3)\alpha^{MEVR}\omega_D\gamma^2\rho^2\left\langle \dot{\varphi}_2\dot{\theta}\right\rangle + 2(m + m_t/3)(\alpha^{MEVR})^2\omega_D^2\gamma^2\rho^2\left\langle \varphi_2\dot{\theta}\right\rangle}{\left\langle \dot{\theta}^2\right\rangle} \tag{20}$$

If the primary system is excited by random moment with a white noise spectrum S_f, then the average value of Eq. (20) are the components of the matrix $\mathbf{P}$ in Eq. (15), Lyapunov equation, this means

$$c_{td} = -\frac{4\xi^{MEVR}(m + m_t/3)\alpha^{MEVR}\omega_D\gamma^2\rho^2 P_{34} + 2(m + m_t/3)(\alpha^{MEVR})^2\omega_D^2\gamma^2\rho^2 P_{32}}{P_{33}} \tag{21}$$

Substituting Eqs. (16)–(18) into Eq. (21), The c_{td} can be determined as

$$c_{td} = \frac{4(3m + m_t)\gamma^2 \xi^{MEVR} \alpha^{MEVR} \omega_D \rho^2}{3[2(\alpha^{MEVR})^4 \gamma^2 \mu + (\alpha^{MEVR})^4 + 4(\alpha^{MEVR})^2 (\xi^{MEVR})^2 - 2(\alpha^{MEVR})^2 + 1]} \tag{22}$$

Maximum conditions are expressed as

$$\left.\frac{\partial c_{td}}{\partial \alpha^{MEVR}}\right|_{\alpha_{opt}^{MEVR} = \alpha^{MEVR}} = 0 \tag{23}$$

$$\left.\frac{\partial c_{td}}{\partial \xi^{MEVR}}\right|_{\xi_{opt}^{MEVR} = \xi^{MEVR}} = 0 \tag{24}$$

Solving the system of Eqs. (22)–(24) results in optimal solutions of the TMD, as shown in Eq. (25) and Eq. (26)

$$\alpha_{opt}^{MEVR} = \alpha^{MEVR} = \frac{1}{\sqrt{(1 + 2\mu\gamma^2)}} \tag{25}$$

$$\xi_{opt}^{MEVR} = \xi^{MEVR} = \frac{\gamma\sqrt{2\mu}}{2} \tag{26}$$

From Eqs. (25-26), we obtain the optimal parameters of the TMD to reduce the torsional vibration of the shaft by using the maximization of equivalent viscous resistance method, which is different from the optimal parameters of the TMD by using the principle of minimum kinetic energy in [10]. This asserts with a shaft model with installed TMD, but applying different methods to find optimal parameters gives different analytical results.

Table 3 presents the optimal parameters obtained by the two methods according to the various mass ratios and ratio between the length of pendulum and radius of gyration of the rotor. We see that the tuning ratio of TMD is approximately 1, indicating that the optimized TMD has the natural frequency is approximately the natural frequency of the shaft. With the design of this TMD will reduce the vibration of the shaft in the best way.

Table 3. The optimal parameters of the tuned mass damper for various mass ratios and ratio between the length of pendulum and radius of gyration of the rotor

μ	γ	α_{opt}^{MKE}	α_{opt}^{MEVR}	ξ_{opt}^{MKE}	ξ_{opt}^{MEVR}
0.01	0.1	0.9998	0.9999	0.0070	0.0071
0.02	0.2	0.9984	0.9992	0.0196	0.0200
0.03	0.3	0.9946	0.9973	0.0352	0.0367
0.04	0.4	0.9874	0.9937	0.0525	0.0566
0.05	0.5	0.9756	0.9877	0.0707	0.0791
0.06	0.6	0.9586	0.9791	0.0891	0.1039
0.07	0.7	0.9358	0.9674	0.1073	0.1310
0.08	0.8	0.9071	0.9524	0.1249	0.1600
0.09	0.9	0.8728	0.9342	0.1419	0.1909
0.10	1.0	0.8333	0.9129	0.1581	0.2236

From Table 3, we again assert that the same shaft model with installed TMD is the same with the values of the various mass ratios and ratio between the length of pendulum and radius of gyration of the rotor, the optimal parameter is obtained by two methods of the principle of minimum kinetic energy and the maximization of equivalent viscous resistance method is different. However, the difference in number between the two methods of the optimal parameter found is not large.

4. Simulate vibration of the system

Numerical simulation is employed for the system by using the achieved optimal parameters of the TMD, as shown in Eq. (25) and Eq. (26). To demonstrate the above analysis, computations will be performed for a system with parameters given in Table 4.

Table 4. The input parameters for shaft and TMD

Parameter	M	ρ	k_t	m_t	M	L
Value	500 kg	1.0 m	105 Nm/rad	15 kg	10 kg	0.9 m
Parameter	μ	γ	α_{opt}^{MEVR}	ξ_{opt}^{MEVR}	c	k_m
Value	0.03	0.9	0.977	0.11	45.67 Ns/m	4634.75 Nm/rad

Plug the parameters from Table 4 into Eqs. (2). Using the Maple software to simulate system vibration, the graphs are obtained in Figs. 2–7.

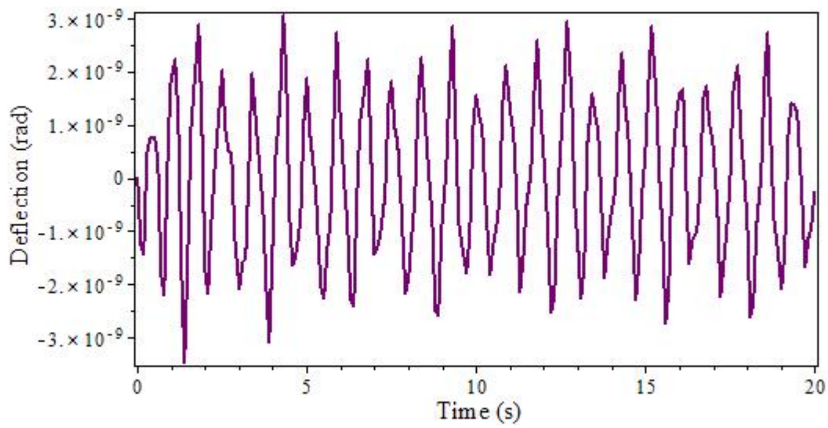

Figure 2. The vibration of the TMD with initial deflection $\theta_0 = 1.5 \times 10^{-9}$ (rad)

The torsional vibration of shaft are shown in Figs. 3, 5 and 7. Figs. 2, 4 and 6 show the vibration of the TMD. From Figs. 3, 5 and 7, we see that with the same shaft model with installed TMD with two methods are the minimum kinetic energy method (MKE) and the maximization of equivalent viscous resistance method (MEVR) finding optimal parameters for different analytical results, but the effect of reducing the vibration on the graph of the two methods are equivalent when the system is subjected to random excitation. It can be seen that the mass-spring-shaft torsional type TMD has good effect in all cases. It realized that the vibration of the shaft torsional installed the TMD has the good efficiency for damping the vibration of the system.

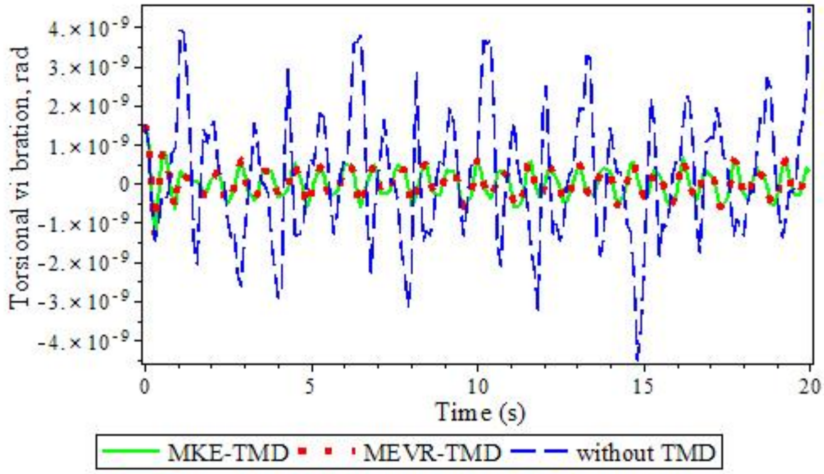

Figure 3. The vibration of the shart with initial deflection $\theta_0 = 1.5 \times 10^{-9}$ (rad)

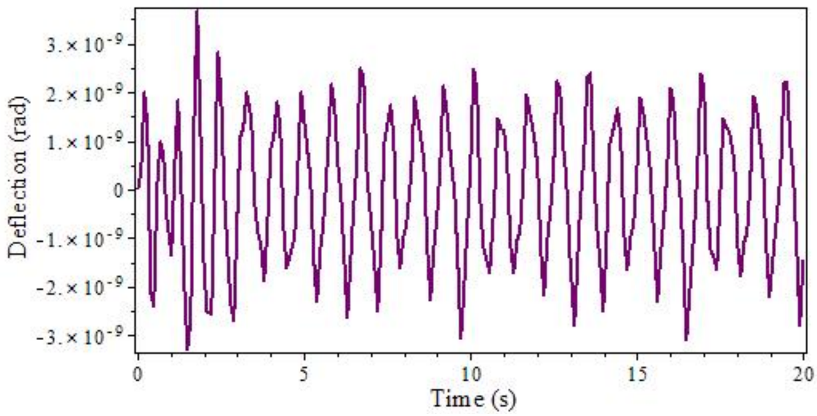

Figure 4. The vibration of the TMD with initial angular velocity $\dot{\theta}_0 = 3 \times 10^{-8}$ (rad/s)

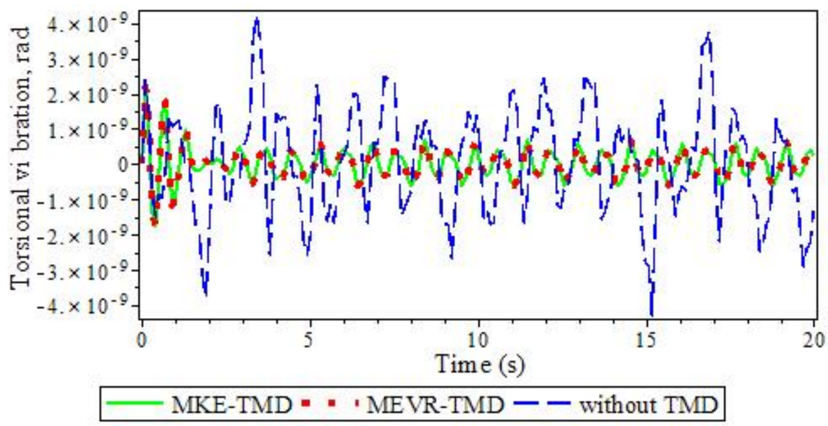

Figure 5. The vibration of the shart with initial angular velocity $\dot{\theta}_0 = 3 \times 10^{-8}$ (rad/s)

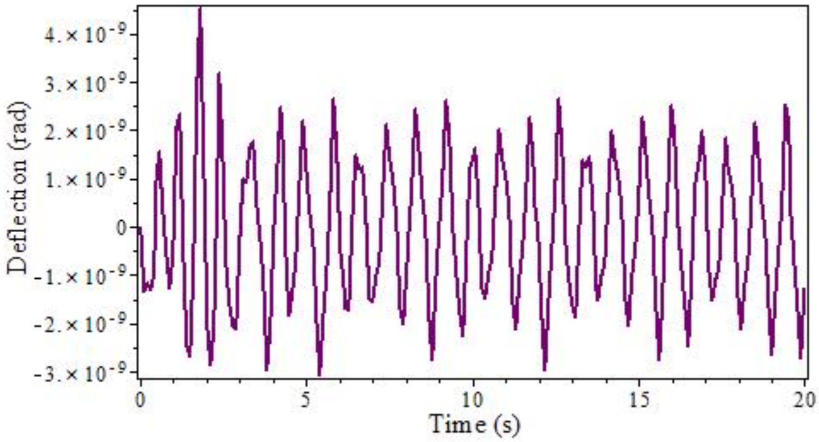

Figure 6. The vibration of the TMD with initial deflection $\theta_0 = 1.5 \times 10^{-9}$ (rad) and initial angular velocity $\dot{\theta}_0 = 3 \times 10^{-8}$ (rad/s)

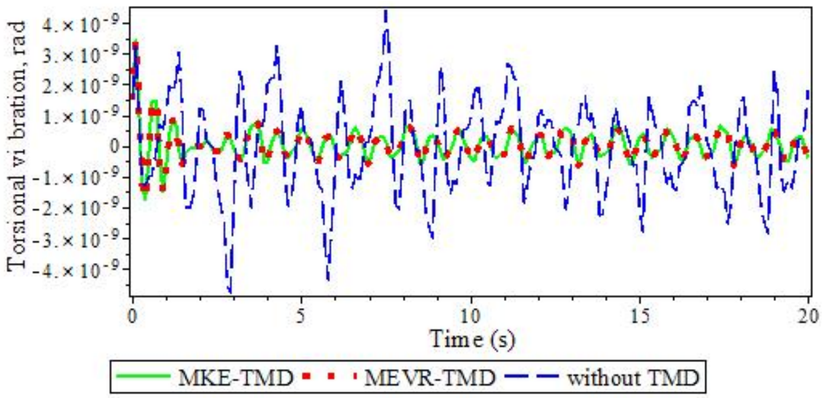

Figure 7. The vibration of the shart with initial deflection $\theta_0 = 1.5 \times 10^{-9}$ (rad) and initial angular velocity $\dot{\theta}_0 = 3 \times 10^{-8}$ (rad/s)

5. Conclusions

In this paper, maximization of equivalent viscous resistance method has been developed and examined for shaft model. The same procedure as in the conventional MEVR has been used to derive the optimum tuning and damping ratios of the device. The optimal parameters were determined in analytical form and furthermore leads to the simple explicit formulas (25), (26). The analytical results are verified by numerical simulations with a given configuration of machine shaft in some different operating conditions.

References

[1] Chinh, N. D. (2018). Shaft torsional vibration reduction using tuned-mass-damper (TMD). In *The first International Conference on Material, Machines and Methods for Sustainable Development*, volume 2, Bach Khoa Publishing House, Vietnam, 429–444.

[2] Dien, K. D., Chinh, N. D., Truong, V. X., Cuong, H. N. (2015). Research finding optimal parameters for reduction torsion oscillator shaft balancing machine method by pole. *Journal of Structural Engineering and Construction Technology*, 18(3):35–43.

[3] Dien, K. D., Chinh, N. D., Truong, V. X., Quyet, T. V., Chung, N. N. (2015). Research on specify optimal parameters of the TMD that has pendulum double form for reducing torsional vibrations of machine's shaft. *Journal of Science and Technology*, (6):15–20.

[4] Alsuwaiyan, A. S., Shaw, S. W. (2002). Performance and dynamic stability of general-path centrifugal pendulum vibration absorbers. *Journal of Sound and Vibration*, 252(5):791–815.

[5] Abouobaia, E., Bhat, R., Sedaghati, R. (2016). Development of a new torsional vibration damper incorporating conventional centrifugal pendulum absorber and magnetorheological damper. *Journal of Intelligent Material Systems and Structures*, 27(7):980–992.

[6] Chao, C.-P., Shaw, S. W., Lee, C. T. (1997). Stability of the unison response for a rotating system with multiple tautochronic pendulum vibration absorbers. *Journal of Applied Mechanics*, 64(1):149–156.

[7] Denman, H. H. (1992). Tautochronic bifilar pendulum torsion absorbers for reciprocating engines. *Journal of Sound and Vibration*, 159(2):251–277.

[8] Hosek, M., Elmali, H., Olgac, N. (1997). A tunable torsional vibration absorber: the centrifugal delayed resonator. *Journal of Sound and Vibration*, 205(2):151–165.

[9] Chinh, N. D. (2019). Optimum design of the tuned mass damper to reduce the torsional vibration of the machine shaft subjected to random excitation. *TNU Journal of Science and Technology*, 203(10):51–58.

[10] Chinh, N. D. (2019). Determination of optimal parameters of the tuned mass damper to reduce the torsional vibration of the shaft by using the principle of minimum kinetic energy. In *Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics*, volume 233, SAGE Publications Sage UK: London, England, 327–335.

[11] Van Lien, T., Khiem, N. T., Duc, N. T. (2016). Free vibration analysis of functionally graded Timoshenko beam using dynamic stiffness method. *Journal of Science and Technology in Civil Engineering (STCE)-NUCE*, 10(5):19–28.

[12] Anh, N. D., Viet, L. D. (2007). *Reducing vibration with energy dissipation device*. Publishing House for Science and Technology, Vietnam Academy of Science and Technology.

[13] Phuc, N. D., Tung, K. D., Duong, N. H. (2018). Random decrement technique for effectively determining dynamic parameters of bridges. *Journal of Science and Technology in Civil Engineering (STCE) - NUCE*, 12(2):31–35. (in Vietnamese).

[14] Warburton, G. B. (1982). Optimum absorber parameters for various combinations of response and excitation parameters. *Earthquake Engineering & Structural Dynamics*, 10(3):381–401.

[15] Hung, T. V., Viet, V. Q. (2019). A 2D model for analysis of rain-wind induced vibration of stay cables. *Journal of Science and Technology in Civil Engineering (STCE)-NUCE*, 13(2):33–47.