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GLOBAL OPTIMIZATION OF LAMINATED COMPOSITE BEAMS USING AN IMPROVED DIFFERENTIAL EVOLUTION ALGORITHM

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Abstract

Differential Evolution (DE) is an efficient and effective algorithm for solving optimization problems. In this paper, an improved version of Differential Evolution algorithm, called iDE, is introduced to solve design optimization problems of composite laminated beams. The beams used in this research are Timoshenko beam models computed based on analytical formula. The iDE is formed by modifying the mutation and the selection step of the original algorithm. Particularly, individuals involved in mutation were chosen by Roulette wheel selection via acceptant stochastic instead of the random selection. Meanwhile, in selection phase, the elitist operator is used for the selection progress instead of basic selection in the optimization process of the original DE algorithm. The proposed method is then applied to solve two problems of lightweight design optimization of the Timoshenko laminated composite beam with discrete variables. Numerical results obtained have been compared with those of the references and proved the effectiveness and efficiency of the proposed method.

Keywords: improved Differential Evolution algorithm; Timoshenko composite laminated beam; elitist operator; Roulette wheel selection; deterministic global optimization.

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1. Introduction

Composite materials have been more and more widely used in many branches of structural engineering such as aircraft, ships, bridges, buildings, automobile, etc. due to their dominate advantages in comparison with other types of materials. Composite materials have high strength-to-weight ratio, high stiffness-to-weight ratio, superior fatigue properties and high corrosion resistance [1]. Among many types of composite structures, beams have been popularly used in practical applications. Recently, many researchers have developed and proposed optimal design methods including both continuous (analytic) models and discrete (numerical) model for the composite beam structures. Valido et al. [2] used finite element analysis and sensitivity analysis model to optimize the design of various geometrically nonlinear composite laminate beam structures. Blasques et al. [3] chose fiber orientations

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and layer thicknesses as design variables to optimize the stiffness and weight of laminated composite beams using finite element approach. Liu et al. [4, 5] solve optimization problems of lightweight design of composite structures using the analytical sensitivity with frequency constraint. Qimao Liu used continuous model to analyse the sensitivity of stresses of the composite laminated beam and employed the standard gradient-based nonlinear programming algorithms to solve lightweight design problems of composite beams [6]. V. Ho-Huu et al. [7] combined finite element model and a population-based global optimization strategy to search for lightweight optimal design of discrete composite laminated beam models. T. Vo-Duy et al. [8] employed the non-dominated sorting genetic algorithm II (NSGA-II) and finite element method to solve the multi-objective optimization of laminated composite beam structures. Reis et. at. [9] optimized dimension of carbon-epoxy bars for reinforcement of wood beams using experimental and finite element analysis to achieve the maximum reinforced beam strength under bending. Roque et. at. [10] used Differential evolution optimization to find the volume fraction that maximizes the first natural frequency for a functionally graded beam with different ratios of material properties. Pham et. al. [11] combined the first order shear deformation theory-based finite element analysis with the modified Differential Evolution algorithm to optimize the weight of functionally graded beams. Nguyen et. al. [12] minimized the weight of of cellular beam under the constraints of the ultimate limit states, the serviceability limit states and the geometric limitations using the differential evolution algorithm. Cardoso et. al. [13] applied finite element technique with two-node Hermitean beam element to study design sensitivity analysis and optimal design of composite structures modelled as thin walled beams. One of the drawbacks of discrete models is that the approximate solution obtained highly depends on the mesh generation and has lower efficiency than analytical approaches of the continuous composite beam models.

In addition, optimization methods for composite beam structures can be classified into two groups, gradient-based and population-based algorithms. The gradient-based method is very fast in finding the optimal solution, but it is easy trapped in local extrema and requires the gradient information to establish the searching direction. In contrast, the population-based method can be easily implemented and can ensure the global optimum solution. In addition, it has the ability to deal with both continuous and discrete design variables, which the gradient-based approaches does not have. Among the global optimization methods, the Differential Evolution algorithm recently proposed by Storn and Price in 1997 [14] has been considered as an efficient and effective algorithm for solving optimization problems. Wang et al. [15] applied the Differential Evolution to design optimal truss structures with continuous and discrete variables. Wu and Tseng [16] solve the COP of the truss structures using a multi-population Differential Evolution with a penalty-based, self-adaptive strategy. Le-Anh et al. [17] used an adjusted Differential Evolution algorithm combining with smoothed triangular plate elements for static analysis and frequency optimization of folded laminated composite plates. Ho-Huu et al. [18] proposed a new version of the Differential Evolution algorithm to optimize the shape and size of truss with discrete variables. However, using the method in finding the global optimum solution still gets highly computational cost. Therefore, it is necessary to develop many other techniques to modify the algorithm and increase its effectiveness.

Based on all the above considerations, in this paper, an improved version of Differential Evolution algorithm is introduced for dealing with optimization problems of composite laminated beam, which is continuous Timoshenko beam model. The improved Differential Evolution is the original algorithm with two modifications in mutation phase and selection phase. In particular, in mutation phase, the individuals are chosen based on Roulette wheel selection via acceptant stochastic instead of the random selection. In selection phase, the elitist operator is used for the selection progress instead of basic selection. Numerical results obtained are verified with others in the literature to manifest the accuracy and the efficiency of the proposed method.

2. Optimization problem formulation

The mathematical model of a lightweight optimization problem of Timoshenko composite beam can be described as follows:

$$\begin{aligned}&\text{Find } \mathbf{d} = [b, h]^T \\ &\text{minimize } \mathbf{Weight}(\mathbf{d}) \\ &\text{s.t.} \quad \sigma_{\text{Tsai–Wu}} < 1 \\ &\qquad f_{del} < 1 \\ &\qquad r_{disp} = w_0 - \overline{w}_0 \leq 0\end{aligned} \tag{1}$$

where **Weight**(**d**) is the objective function; $\mathbf{d} = [b, h]$ is the vector of design variables; b, h are respectively the width and the height of the beam; $\sigma_{\text{Tsai–Wu}}, f_{del}, r_{disp}$ are strength failure function, delamination failure function and stiffness failure function, respectively.

3. Methodology for solving optimization problem of composite laminated beam

3.1. Exact analytical displacement and stress of Timoshenko composite beam

Consider a segment of composite laminated beam with N layers and the fiber orientations of layers are θ_i $(i = 1, ..., N)$. The positions of layers are denoted by z_i $(i = 1, ..., N)$. The beam has rectangular cross section with the width b and the length h as depicted in Fig. 1. The beam segment dx is subjected to the transversal force as shown in Fig. 2.

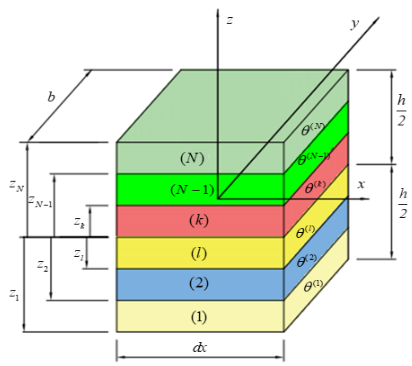

Figure 1. Composite laminated beam model

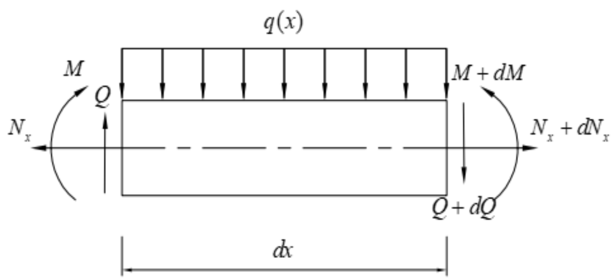

Figure 2. Free-body diagram

The displacement fields of the composite laminated beam calculated analytically based on the first-order shear deformation theory (also called Timoshenko beam theory) are:

$$u_0(x) = -\overline{B}\left(\frac{q_0}{6}x^3 + \frac{1}{2}C_1x^2 + 4C_4x + C_5\right) \tag{2}$$

$$w_0(x) = -\overline{A}\frac{q_0}{24}x^4 - \frac{1}{6}\overline{A}C_1x^3 - \left(\overline{C}\frac{q_0}{2} + \frac{1}{2}\overline{A}C_2\right)x^2 + C_6x + C_7 \tag{3}$$

$$\phi(x) = \overline{A}\left(\frac{q_0}{6}x^3 + \frac{1}{2}C_1x^2 + 4C_2x + C_3\right) \tag{4}$$

where $C_i (i = 1, ..., 7)$ are indefinite integration constants determined by using the boundary conditions of the composite laminated beams as shown in the following section.

$$\overline{A} = \frac{A_{11}}{b(B_{11}^2 - A_{11}D_{11})}, \quad \overline{B} = \frac{B_{11}}{b(B_{11}^2 - A_{11}D_{11})}, \quad \overline{C} = \frac{1}{bKA_{55}} \tag{5}$$

where $A_{11}, B_{11}, D_{11}, A_{55}$ are respectively extensional stiffness, bending-extensional coupling stiffness, bending stiffness and extensional stiffness of the composite laminate. K is the shear correction factor with the value of 5/6.

The stress fields of the composite laminated beam include the plane stress components and the shear stress components. According to the coordinate system between the materials (123) and the beam/laminate (xyz) as depicted in Fig. 3, in which the fiber orientation coincides with the 1-axis, the plane stress components are expressed as follows

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{pmatrix} = \mathbf{T}^{(k)}\overline{\mathbf{Q}}^{(k)} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}, \quad z_{k+1} \le z \le z_k \tag{6}$$

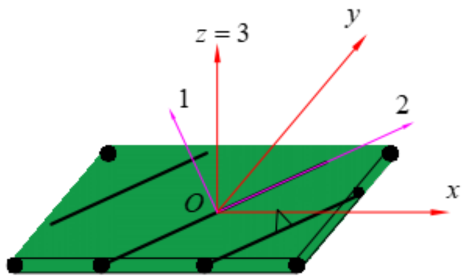

Figure 3. The material and laminate coordinate system

where the strain components $\varepsilon_y = 0, \gamma_{xy} = 0$, and

$$\varepsilon_x = -\overline{B}\left(\frac{q_0}{3}x^2 + C_1 x + C_4\right) + z\overline{A}\left(\frac{q_0}{2}x^2 + C_1 x + C_2\right) \tag{7}$$

$T^{(k)}$ is the coordinate transformation matrix and $\overline{Q}^{(k)}$ is the matrix of material stiffness coefficients

$$\mathbf{T}^{(k)} = \begin{bmatrix} \cos^2\theta^{(k)} & \sin^2\theta^{(k)} & 2\sin\theta^{(k)}\cos\theta^{(k)} \\ \sin^2\theta^{(k)} & \cos^2\theta^{(k)} & -2\sin\theta^{(k)}\cos\theta^{(k)} \\ -\sin\theta^{(k)}\cos\theta^{(k)} & \sin\theta^{(k)}\cos\theta^{(k)} & \cos^2\theta^{(k)} - \sin^2\theta^{(k)} \end{bmatrix} \tag{8}$$

$$\overline{\mathbf{Q}}^{(k)} = \begin{bmatrix} \overline{Q}_{11}^{(k)} & \overline{Q}_{12}^{(k)} & \overline{Q}_{61}^{(k)} \\ \overline{Q}_{21}^{(k)} & \overline{Q}_{22}^{(k)} & \overline{Q}_{26}^{(k)} \\ \overline{Q}_{16}^{(k)} & \overline{Q}_{26}^{(k)} & \overline{Q}_{66}^{(k)} \end{bmatrix} \tag{9}$$

The shear stress components in the material coordinate systems are

$$\begin{pmatrix} \tau_{23} \\ \tau_{13} \end{pmatrix} = \mathbf{T}_s^{(k)}\overline{\mathbf{Q}}_s^{(k)} \begin{pmatrix} \gamma_{yz} \\ \gamma_{xz} \end{pmatrix}, \quad z_{k+1} \le z \le z_k \tag{10}$$

where the shear strain components $\gamma_{yz} = 0$ and

$$\gamma_{xz} = \overline{A}\left(\frac{1_0}{6}x^3 + \frac{1}{2}C_1x^2 + C_2x + C_3\right) - \overline{A}\frac{q_0}{6}x^3 - \frac{1}{2}\overline{A}C_1x^2 - (\overline{C}q_0 + \overline{A}C_2)x + C_6 \tag{11}$$

The coordinate transformation matrix $T_s^{(k)}$ and the matrix of stiffness coefficients $\overline{Q}_s^{(k)}$ can be described as

$$\mathbf{T}_s^{(k)} = \begin{bmatrix} \sin\theta^{(k)} & \cos\theta^{(k)} \\ \cos\theta^{(k)} & -\sin\theta^{(k)} \end{bmatrix} \tag{12}$$

$$\overline{\mathbf{Q}}_s^{(k)} = \begin{bmatrix} \overline{Q}_{44}^{(k)} & \overline{Q}_{45}^{(k)} \\ \overline{Q}_{45}^{(k)} & \overline{Q}_{55}^{(k)} \end{bmatrix} \tag{13}$$

In the above equations, $\overline{Q}_{ij}^{(k)}$ is the stiffness coefficients of the k^{th} lamina in the laminate coordinate system. More detail related to the formulation of Timoshenko composite laminated beam including boundary conditions are clearly described in [6].

3.2. Brief introduction of the Improved Differential Evolution algorithm

a. Basic Differential Evolution Algorithm

The original differential evolution algorithm firstly proposed by Storn and Price [14] and consists of four main phases as follows:

Phase 1: Initialization

Creating an initial population, containing NP individuals, by randomly sampling from the search space

$$x_{i,j} = x_{i,j}^l + rand[0,1] \times (x_{i,j}^u - x_{i,j}^l), i = 1,2,...,NP; j = 1,2,...,D \tag{14}$$

where $x_{i,j}^l$ and $x_{i,j}^u$ are the lower and upper bounds of $x_{i,j}^l$, respectively; $rand[0,1]$ is a uniformly distributed random number in [0, 1]; D is the number of design variables; NP is the size of the population.

Phase 2: Mutation

Generate a new mutant vector $\mathbf{v}_i$ from each current individual $\mathbf{x}_i$ based on mutation operation 'DE/rand/1'

$$\mathbf{v}_i = \mathbf{x}_{r_1} + F \times (\mathbf{x}_{r_2} - \mathbf{x}_{r_3}) \tag{15}$$

where integer r_1, r_2, r_3 are randomly selected from $1, 2, \ldots, NP$ such that $r_1 \neq r_2 \neq r_3 \neq i$; the scale factor F is randomly chosen within [0, 1].

Phase 3: Crossover

Create a trial vector $\mathbf{u}_i$ by replacing some elements of the mutant vector $\mathbf{v}_i$ via crossover operation.

$$u_{i,j} = \begin{cases} v_{i,j} & \text{if } rand[0,1] \leq CR \text{ or } j = j_{rand} \\ x_{i,j} & \text{otherwise} \end{cases} \tag{16}$$

where $u_{i,j}$ is the j^{th} component of the trial vector $u_i, i \in \{1,2,...,NP\}; j \in \{1,2,...,D\}$; j_{rand} is an integer randomly generated from 1 to D; and CR is the crossover control parameter.

Phase 4: Selection

Compare the trial vector $\mathbf{u}_i$ with the target vector $\mathbf{x}_i$. One with lower objective function value will survive in the next generation

$$\mathbf{x}_i = \begin{cases} \mathbf{u}_i & \text{if } f(\mathbf{u}_i) \leq f(\mathbf{x}_i) \\ \mathbf{x}_i & \text{otherwise} \end{cases} \tag{17}$$

b. Improved Differential Evolution Algorithm

To improve the convergence speed of the algorithm, the *Mutation phase* and the *Selection phase* are modified as follow:

In the **mutation phase**, parent vectors are chosen randomly from the current population. This may make the DE be slow at exploitation of the solution. Therefore, the individuals participating in mutation should be chosen following a priority based on their fitness. By doing this, good information of parents in offspring will be stored for later use, and hence will help to increase the convergence speed. To store good information in offspring populations, the individuals is chosen based on Roulette wheel selection proposed by Lipowski and Lipowska [19] via acceptant stochastic instead of the random selection. To do this, each member in the current population is assigned a selection probability, which is proportional to its fitness value compared with the fitness value of the best individual, and calculated as follows:

$$p_i = \frac{f_i}{f_{\max}}, \quad i = 1, 2, ..., NP \tag{18}$$

where p_i and f_i are, respectively, the selection probability and fitness value of the i^{th} individual; $f_{\max}$ is the largest fitness value of the best individual in the whole population in the current generation.

In the **selection phase**, the elitist selection technique introduced by Padhye et al. [20] is used for the selection progress instead of basic selection as in the conventional DE. In the elitist process, the children population **C** consisting of trial vectors is combined with parent population **P** of target vectors to create a combined population **Q**. Then, best individuals are chosen from the combined population **Q** to construct the population for the next generation. By doing so, the best individuals of the whole population are always saved for the next generation.

4. Numerical examples

In this paper, the width and the depth of the beam are chosen as the design variables to obtain the lightweight designs of the beams. Consider the design optimization model of the composite beam taking into account the constraint of the stiffness failure criterion, strength failure criterion and delamination failure criterion [6]:

$$\begin{aligned}
&\text{Find } \mathbf{d} = [b, h]^T \\
&\text{Minimize } W(\mathbf{d}) \\
&\text{Subject to } g_j = \left(\frac{\sigma_1^2}{X_t X_c} - \frac{\sigma_1 \sigma_2}{\sqrt{X_t X_c Y_t Y_c}} + \frac{\sigma_2^2}{Y_t Y_c} + \frac{\tau_{12}^2}{S^2} + \frac{X_c - X_t}{X_t X_c}\sigma_1 + \frac{Y_c - Y_t}{Y_t Y_c}\sigma_2 - 1 \right)_j < 0 \\
&f_j = \left(\frac{\tau_{13}^2}{S_{13}^2} + \frac{\tau_{23}^2}{S_{23}^2} - 1 \right)_j < 0 \\
&r = w_0(\alpha L) - \overline{w}_0 \leq 0 \\
&\underline{b} \leq b \leq \overline{b} \\
&\underline{h} \leq h \leq \overline{h}
\end{aligned}$$

where $W(\mathbf{d})$ is the mass of the composite laminated beam. g, f and r are respectively strength failure function, delamination failure function and stiffness failure function. $\underline{b}$ and $\overline{b}$ are the lower and upper bound of the width of the beam. $\underline{h}$ and $\overline{h}$ are the lower and upper bound of the depth of the beam, respectively. αL determines the location in x-direction where the deflection of the beam is monitored. α is different for various types of boundary conditions: Pined-Pined (PP): $\alpha = 1/2$, Fixed-Fixed (FF): $\alpha = 1/2$, Fixed-Pined (FP): $\alpha = 505/873$ and Cantilivered (CL): $\alpha = 1$. $w_0(\alpha L)$ is the deflection of the beam at the position αL. $\overline{w}_0$ is the limits on the deflection of the beam. The subscript ($j = 1, 2, \ldots, N_m$) indicates the j^{th} monitored point in the set N_m monitored points of the strength and delamination. X_t and X_c are the tensile strength and compressive strength along the 1-axis of the material coordinate system, respectively. Y_t and Y_c are the tensile strength and compressive strength along the 2-axis of the material coordinate system, respectively. S is the shear strength on the plane

102 of the material coordinate system. S_{12} and S_{23} are the shear strength on the plane 103 and 203 of the material coordinate system. In this paper, $S_{12} = S_{23}$.

4.1. Optimal design with variables: b and h

Consider the composite beams with the material properties given in Table 1. The beams have $N = 8$ layers with symmetric fiber orientations of $[0/90/45/-45]_s$. The span of the composite laminated beams are $L = 7.2$ m. The beams are subjected to the uniform distributed loading $q_0 = 10^5$ N/m and are considered under various types of constraint including PP, FF, FP and CL. The initial design of the composite laminated beams is $b = 0.3$ m and $h = 0.48$ m (the thickness of each layer is 0.06), mass $W = 1597$ kg. The lower and upper boundary of the design variables are $0.1 \text{ m} \le b \le 2$ m, $0.2 \text{ m} \le h \le 2$ m.

Table 1. Material properties of lamina

Property	T300/5208
E_1 (GPa)	136.00
$E_2 = E_3$ (GPa)	9.80
$G_{12} = G_{13}$ (GPa)	4.70
G_{23} (GPa)	5.20
$\nu_{12} = \nu_{13}$	0.28
ν_{23}	0.15
ρ(kg/m^3)	1540
X_t (Mpa)	1550
X_c (Mpa)	1090
Y_t (Mpa)	59
Y_c(Mpa)	59
S (Mpa)	75
$S_{13} = S_{13}$ (Mpa)	75

The optimization design problems are solved by using three different population-based algorithms including Jaya, DE, iDE and one gradient-based algorithm from Liu's work with different types of boundary conditions (P-P, F-F, F-P and C-L). The initial parameters used for iDE including the number of population $NP = 30$; the scaling factor F of 0.4 and the crossover control parameter CR of 0.7. The numerical results are presented in Table 2. As shown in the table, the optimal mass obtained from iDE are agreed well with other solutions. However, the iDE algorithm consumed least time to achieve the optimal solution in compared with other approaches. Among the four methods, the SQP (implemented by *fmincon* promt in Matlab) algorithm used in Liu's work reached the optimal solution very fast but it could be stuck in the local optimum. The iDE method also outperforms other global optimization methods DE and Jaya. In particularly, for the case of P-P condition, the computational time of iDE is less than that of Jaya and DE 15% and 35%, respectively. For the case of C-L condition, these numbers are 6.5% and 43%, respectively. The number of average function count is also reduced up to maximum 40% when using the iDE method instead of the DE for the case of C-L conditions. The iDE is also faster than Jaya approach in reaching the optimal solutions in all case considered. From the above analyses, iDE can be considered as the most effective and the efficient algorithm.

4.2. Optimal design with variables: b and t_i

In this section, the depth of the composite laminated beam (h) is divided into thicknesses of the layers of the beam to optimize. This is implemented with the intention of improving the optimal design of the composite laminated beam and achieving lighter weight for the beam. The design variables in this case are the thicknesses of each layer, denoted by $[t_1, t_2, t_3, t_4]_s$ and can be considered as the discrete design variables. The results obtained are presented in Table 3. It can be seen that the optimal masses obtained by all the population-based optimization methods with discrete design variables are just equal to half of that derived from Liu's work using the SQP algorithm with continuous design variables. And once again, the iDE method dominates the other population-based methods in both the

Table 2. Comparison of optimal design with continuous design variables

BC	Optimal results	Liu [6]	Jaya	DE	iDE
P-P	Mass	909.2634	909.2634	909.2634	909.2634
	$[h, b]$	[0.1000, 0.8200]	[0.1000, 0.8200]	[0.1000, 0.8200]	[0.1000, 0.8200]
	Worst mass	-	909.2638	909.2640	909.2642
	Mean mass	-	909.2636	909.2636	909.2637
	Std.	-	0.0001	0.0001	0.0002
	Average f-count	7	1181	1660	1075
	CPU time (s)	45 [0.54]*	0.82	1.06	0.69
F-F	Mass	560.7427	560.7428	560.7428	560.7428
	$[h, b]$	[0.1000, 0.5057]	[0.1000, 0.5057]	[0.1000, 0.5057]	[0.1000, 0.5057]
	Worst mass	-	560.7430	560.7430	560.7433
	Mean mass	-	560.7428	560.7429	560.7429
	Std.	-	0.0001	0.0001	0.0001
	Average f-count	9	1144	1654	1084
	CPU time (s)	6 [0.17]	0.84	1.15	0.73
F-P	Mass	706.5145	706.5145	706.5145	706.5145
	$[h, b]$	[0.1000, 0.6372]	[0.1000, 0.6372]	[0.1000, 0.6372]	[0.1000, 0.6372]
	Worst mass	-	706.5151	706.5148	706.5150
	Mean mass	-	706.5146	706.5146	706.5147
	Std.	-	0.0001	0.0001	0.0001
	Average f-count	32	1153	1641	1061
	CPU time (s)	11 [0.11]	0.77	1.07	0.67
C-L	Mass	2065	2064.9646	2064.9645	2064.9645
	$[h, b]$	[0.1000, 1.8623]	[0.1000, 1.8623]	[0.1000, 1.8623]	[0.1000, 1.8623]
	Worst mass	-	2064.9654	2064.9658	2064.9658
	Std.	-	0.0002	0.0003	0.0003
	Average f-count	16	1039	1591	954
	CPU time (s)	38 [0.13]	0.31	0.51	0.29

[]*: CPU time in this study by using SQP algorithm in *fmincon* Matlab.

number of function count and the CPU time. The results from Table 3 also show that the optimization with discrete design variables is much more effective than solving the problem with continuous design variables.

Regarding the performance of the algorithms, it can be seen from Table 3 that iDE dominates other methods in both the computational time and the number of structural analyses. This can also be seen in Fig. 4, where the convergence curves obtained by each method for the P-P boundary condition are illustrated.

5. Conclusions

In this paper, a new effective and efficient method, called iDE, has been introduced and applied to handle the optimization problem of Timoshenko composite laminated beam . This method was formed by modifying the mutation step and selection step in the optimization process of the original

Table 3. Comparison of optimal design with discrete design variables

BC	Optimal results	Liu [6]	Jaya	DE	iDE
P-P	Mass	909.2634	410.256	410.256	410.256
	b	0.1000	0.100	0.100	0.100
	$[t_1, t_2, t_3, t_4]_s$	$[0.1025, 0.1025, 0.1025, 0.1025]_s$	$[0.190, 0.060, 0.060, 0.060]_s$	$[0.190, 0.060, 0.060, 0.060]_s$	$[0.190, 0.060, 0.060, 0.060]_s$
	Worst mass	-	415.800	415.800	415.800
	Mean mass	-	411.088	410.533	414.341
	Average f-count	7	3132	3375	2225
	CPU time (s)	45	2.24	2.75	1.81
F-F	Mass	560.7427	260.568	260.568	260.568
	b	0.1000	0.100	0.100	0.100
	$[t_1, t_2, t_3, t_4]_s$	$[0.0632, 0.0632, 0.0632, 0.0632]_s$	$[0.100, 0.045, 0.040, 0.050]_s$	$[0.105, 0.040, 0.040, 0.050]_s$	$[0.085, 0.050, 0.050, 0.055]_s$
	Worst mass	-	260.568	260.568	266.112
	Mean mass	-	260.568	260.568	260.860
	Std.	-	0.000	0.000	1.272
	Average f-count	9	2534	2675	2325
	CPU time (s)	6	1.81	2.18	1.90
F-P	Mass	706.5145	327.096	327.096	327.096
	b	0.1000	0.100	0.100	0.100
	$[t_1, t_2, t_3, t_4]_s$	$[0.0796, 0.0796, 0.0796, 0.0796]_s$	$[0.130, 0.050, 0.050, 0.065]_s$	$[0.130, 0.060, 0.055, 0.050]_s$	$[0.130, 0.055, 0.060, 0.050]_s$
	Worst mass	-	327.096	327.096	327.096
	Mean mass	-	327.096	327.096	327.096
	Std.	-	0.000	0.000	0.000
	Average f-count	32	2499	2650	2225
	CPU time (s)	11	1.83	2.17	1.83
C-L	Mass	2065	942.480	942.480	942.480
	b	0.1000	0.100	0.100	0.100
	$[t_1, t_2, t_3, t_4]_s$	$[0.2328, 0.2328, 0.2328, 0.2328]_s$	$[0.425, 0.140, 0.145, 0.140]_s$	$[0.430, 0.140, 0.140, 0.140]_s$	$[0.400, 0.145, 0.150, 0.160]_s$
	Worst mass	-	942.480	942.480	948.024
	Mean mass	-	942.480	942.480	945.415
	Std.	-	0.000	0.000	2.852
	Average f-count	16	3129	3500	2350
	CPU time (s)	38	1.16	1.48	1.01

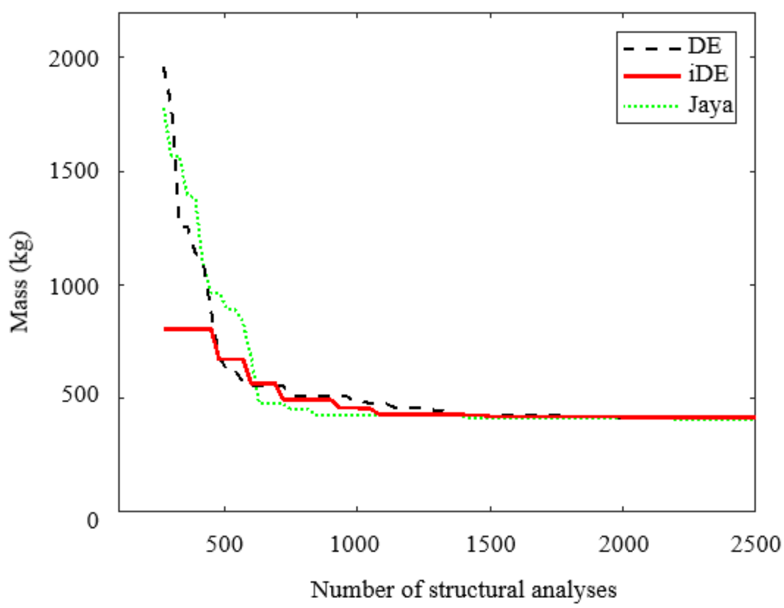

Figure 4. Convergence curves of DE, IDE, Jaya for the beam with P-P condition

DE algorithm by using Roulette wheel selection and elitist operation technique, respectively. This work has some novelties as follows:

1. The proposed iDE algorithm has been first-time applied to optimize the Timoshenko composite beam with stress constraint functions computed from exact analytical formula.

2. The depth of the composite laminated beam (h) is divided into thicknesses of the layers of the beam to optimize. This helped to improve the optimal design of the composite laminated beam and optimal weight achieved much better than that of Liu's work [6].

The results obtained showed that the iDE outperformed the comparison methods in reaching the global optimal solutions in both the number of function count and the CPU time.

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