LARGE DISPLACEMENT ELASTIC STATIC ANALYSIS OF SEMI-RIGID PLANAR STEEL FRAMES BY COROTATIONAL EULER–BERNOULLI FINITE ELEMENT

Nguyen Van Hai[a], Le Van Binh[a], Doan Ngoc Tinh Nghiem[a], Ngo Huu Cuong[a,*]

[a]*Faculty of Civil Engineering, University of Technology, Vietnam National University Ho Chi Minh City, 268 Ly Thuong Kiet street, District 10, Ho Chi Minh City, Vietnam*

Article history:
Received 04/03/2019, Revised 22/04/2019, Accepted 22/04/2019

Abstract

A corotational finite element for large-displacement elastic analysis of semi-rigid planar steel frames is proposed in this paper. Two zero-length rotational springs are attached to the ends of the Euler-Bernoulli element formulated in corotational context to simulate the flexibility of the beam-to-column connections and then the equilibrium equations of the hybrid element, including the stiffness matrix which contains the stiffness terms of the rotational springs, are established based on the static condensation procedure. The linear and Kishi-Chen three-parameter power models are applied in modelling the moment-rotation relation of beam-column connections. The arc-length nonlinear algorithm combined with the sign of displacement internal product are used to predict the equilibrium paths of the system under static load. The analysis results are compared to previous studies to verify the accuracy and effectiveness of the proposed element and the applied nonlinear procedure.

Keywords: corotational context; Euler-Bernoulli element; large displacement; semi-rigid connection; steel frame; static analysis.

https://doi.org/10.31814/stce.nuce2019-13(2)-03

1. Introduction

In structural nonlinear analysis, there are two main finite element formulations depending on the way of updating the system kinematics during the analysis process such as the Lagrangian and corotational models. Among these models, the latest developed corotational approach is more simple and effective than the Lagrangian type in the prediction of the large displacement behaviour of the structures.

Recent studies based on the corotational formulation for large displacement analysis are briefly presented as follows. Battini [1] proposed the Bernoulli and Timoshenko beam elements for large displacement analysis of the 2D and 3D structure under static load with the consideration of material nonlinearity via von Mises criterion with isotropic hardening at numerical integration points. Yaw et al. [2] proposed the meshfree formulation for large displacement and material nonlinear analysis of two-dimensional continua under static load by using maximum-entropy basic functions. Le et al. [3] derived the elastic force vector and tangent stiffness matrix as well as the inertia terms by using the cubic interpolation function for lateral displacement for dynamic nonlinear analysis of 2D arches

*Corresponding author. *E-mail address:* ngohuucuong@hcmut.edu.vn (Cuong, N. H.)

and frames. Doan-Ngoc et al. [4] proposed the beam-column elements for second-order plastic-hinge analysis of planar steel frames by using the approximate seventh-order polynomial function for the beam-column deflection solutions.

The actual behaviour of the real beam-to-column connections is basically semi-rigid. This connection flexibility affects the response and ultimate strength of the steel frames significantly and therefore needs be considered in the frame analysis for practical design. So far, many studies have been done to predict the large displacement response of semi-rigid frames under static and dynamic loads. However, most of them are related to Lagrangian type formulation, such as the studies of Chan and Zhou [5], So and Chan [6], Tin-Loi and Misa [7], Park and Lee [8], Ngo-Huu et al. [9], Saritas and Koseoglu [10], etc. In this study, a corotational finite element is formulated by using the approximate third-order and first-order Hermitian polynomial functions for lateral deflection and axial deformation, respectively, for large displacement analysis of planar steel frames under static load. An effective strain is applied to avoid membrane locking as discussed by Crisfield [11]. The semi-rigid connection is modelled as rotational springs attached at the ends of corotational element to simulate the moment-rotation relation. Then, the static condensation algorithm is applied to eliminate the internal degrees of freedom between element ends and rotational springs at the same positions. As the result, a new element stiffness matrix considering the connection flexibility is formulated with the same size as normal finite element. The linear rotational spring or the Kishi-Chen three-parameter power model (Lui and Chen [12]) is used to describe the beam-to-column flexibility. The arc-length nonlinear algorithm is combined with the sign of displacement internal product proposed by Posada [13] in order to solve the nonlinear equilibrium systems. The analysis results are compared to the previous studies to verify the accuracy and effectiveness of the proposed element.

2. Finite element formulation

2.1. Corotational finite element

The original undeformed and current deformed configurations of the element in the global coordinate system (X, Y) are shown in Fig. 1. A local coordinate system (X_L, Y_L) is attached to the element at the left node and it continuously moves with the element.

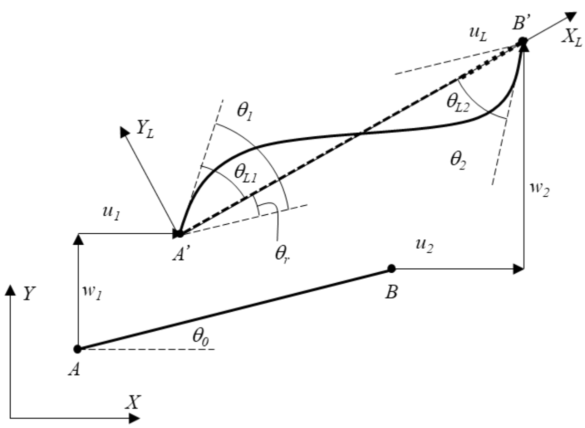


Figure 1. Kinematic model of corotational element

The global displacement vector is defined by

$$d = \begin{bmatrix} u_1 & w_1 & q_1 & u_2 & w_2 & q_2 \end{bmatrix}^T \tag{1}$$

The local displacement vector is defined by

$$d_L = \begin{bmatrix} u_L & q_{L1} & q_{L2} \end{bmatrix}^T \tag{2}$$

The vectors of global and local internal force are respectively given by

$$f = \begin{bmatrix} N_1 & Q_1 & M_1 & N_2 & Q_2 & M_2 \end{bmatrix}^T \tag{3}$$

$$f_L = \begin{bmatrix} N_L & M_{L1} & M_{L2} \end{bmatrix}^T \tag{4}$$

The components of d_L are computed by

$$u_L = l - l_0, \qquad \theta_{L1} = \theta_1 - \theta_r, \qquad \theta_{L2} = \theta_2 - \theta_r \tag{5}$$

where l_0 and l are original and current length of the element respectively and θ_r is the rigid rotation. By equating the virtual work in both local and global coordinate system, the relation between the local internal force vector f_L and global one f is obtained as follows

$$f = B^T f_L \tag{6}$$

where $B = \dfrac{\partial d_L}{\partial d}$ is the corotational transformation matrix.

The global tangent stiffness matrix is obtained through differentiation of the internal force vector f, $\delta f = K\delta d$ in combination with Eq. (6) [2], as follows

$$K = B^T K_L B + A_1 N_L + A_2 \left(M_{L1} + M_{L2}\right) \tag{7}$$

where

$$K_L = \frac{\partial f_L}{\partial d_L} \tag{8}$$

$$A_1 = \frac{\partial^2 u_L}{\partial d^2} \tag{9}$$

$$A_2 = \frac{\partial^2 \theta_r}{\partial d^2} \tag{10}$$

According to Crisfield [11], an effective strain ε_{ef} is applied to avoid membrane locking. In Euler-Bernoulli assumption, the strain ε is defined as

$$\varepsilon = \varepsilon_{ef} - y\kappa = \frac{1}{2}\int_L \left[\frac{\partial u}{\partial \xi} + \frac{1}{2}\left(\frac{\partial w}{\partial \xi}\right)^2\right] d\xi - y\kappa \tag{11}$$

where u and w are the axial and lateral displacements using a linear interpolation function and cubic one, respectively.

The principle of virtual work is used to calculate the local internal forces as follows

$$V = \int_V \sigma\delta\varepsilon dV = N_L\delta u_L + M_{L1}\delta\theta_{L1} + M_{L2}\delta\theta_{L2} \tag{12}$$

The components of f_L are calculated from Eq. (12). Then, the local tangent stiffness matrix is determinated from Eq. (8) and the global one is easily determined from Eq. (7). For elastic analysis, the Gauss quadrature with two Gauss points is exact enough to calculate the numerical values of f_L, K_L and K.

2.2. Hybrid corotational element

The initial corotational finite element has to satisfy the equilibrium equation $\underset{6\times6}{K}\ \underset{6\times1}{d} = \underset{6\times1}{P}$. Because K is the global tangent stiffness matrix, both of d and P must be formed in global coordinate system. The nodal load vector in the global coordinate system is

$$P = TP' \tag{13}$$

where T is the transformation matrix and P' is nodal load vector in the local coordinate system

$$P' = \left\{ \begin{matrix} P'_1 & V'_1 & M'_1 & P'_2 & V'_2 & M'_2 \end{matrix} \right\}^T \tag{14}$$

In semi-rigid beam-to-column connection, only rotational deformation is considered due to negligible axial and shear strains. An assembly procedure is described in Fig. 2. The semi-rigid connections are modelled as a zero-length rotational springs attached to nodes A and B of the element. The equilibrium equation at element level $\underset{8\times8}{K^*}\ \underset{8\times1}{d^*} = \underset{8\times1}{f^*}$ has 8 degrees of freedom. Then, a static condensation algorithm proposed by Wilson [14] is used to eliminate the first and second degrees of freedom. As a result, a 6-DOFs hybrid element is formulated as normal finite element. The hybrid element significantly reduces the computational cost because the rotational displacements at nodes A and B are not included in the global stiffness matrix. However, an updated displacement procedure at nodes A and B must be required at each nonlinear solution iteration to find the rigid rotations of semi-rigid connection.

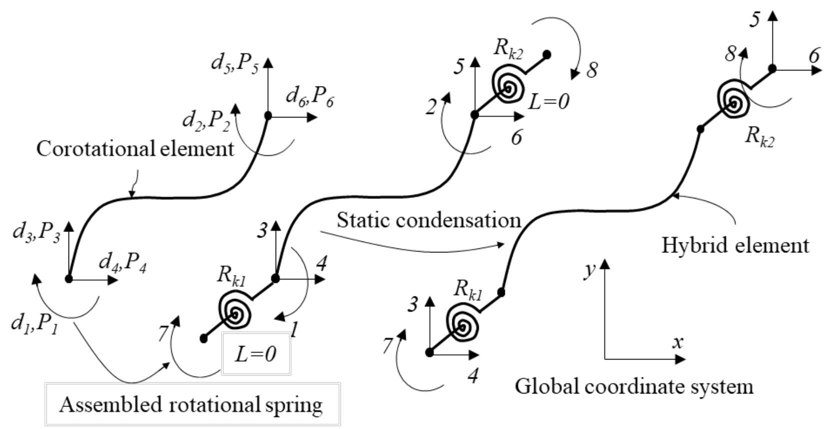


Figure 2. Formulation of hybrid corotational element

2.3. Algorithm of nonlinear equation solution

At each iteration loop, the out of balance vector is defined as

$$R_j^{i-1} = F_{in}{}_j^{i-1} - \lambda_j^{i-1} F_{ex} \tag{15}$$

where F_{in} is the internal force vector which is assembled from vector f, F_{ex} is the reference load vector and λ is the load factor. In order to find the equilibrium path of system at snapback and snapthrough point, the spherical arc-length nonlinear algorithm is used in combination with the scalar product criterion proposed by Posada [13]. The sign of incremental load factor $\Delta\lambda_j^1$ at the first iteration of each incremental load level is

$$\Delta\lambda_j^1 = \pm\frac{\Delta s_j}{\sqrt{\left(\delta\hat{u}_j^1\right)^T\left(\delta\hat{u}_j^1\right)}} \tag{16}$$

$$\text{sign}\left(\Delta\lambda_j^1\right) = \text{sign}\left(\left(\{\Delta u\}_{j-1}^{\text{satisfied}}\right)^T\{\delta\hat{u}\}_j^1\right) \tag{17}$$

where $\Delta\lambda_j^1$ and $\{\Delta u\}_{j-1}^{\text{satisfied}}$ are the incremental load factor at jth loadstep and the previous converged incremental displacement vector, $\delta\hat{u}_j^1 = K_j^0 F_{ex}$ is the current tangential displacement vector.

3. Numerical examples

A structural analysis program written in MATLAB programming language is developed to predict the large displacement responses of rigid and semi-rigid planar members and frames under static load based on the above-mentioned algorithm. Its accuracy is verified through following numerical examples.

3.1. Pinned-fixed square diamond frame

The geometric and material properties of the diamond frame and its equivalent system are shown in The geometric and material properties of the diamond frame and its equivalent system are shown in Fig. 3. The variations of the analysis results with different number of proposed elements in modeling each member shown in Fig. 4 indicate that the analysis result is converged by the use of three proposed elements per member. It can be seen that the results using three proposed elements per member are almost identical to Mattiasson's elliptic integral solution [15] in two cases of tensile and compressive loads as shown in Fig. 5.

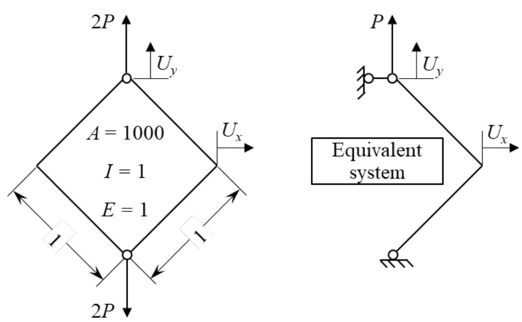


Figure 3. Diamond frame

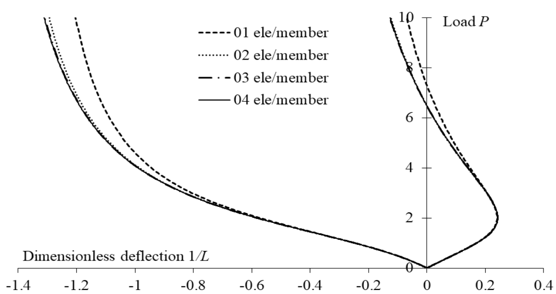


Figure 4. Analysis results using different number of proposed element per member

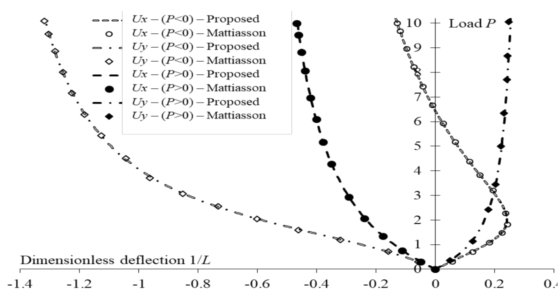


Figure 5. Load-deflection curves of diamond frame

3.2. Lee's frame

The geometric and material properties of Lee's frame are shown in Fig. 6. Park and Lee [8] used ten linearized finite elements while Le et al. [2] used twenty Timoshenko corotational elements in analysis. The equilibrium path of the frame with three proposed elements per member (Fig. 7) converges in good agreement with the results obtained by Park and Lee [8] and Battini [1] as shown in Fig. 8. The analysis results also show that the developed program can handle the critical points as snap-back and snap-through and draw entire load-displacement curve with the least number of elements in comparison to the above-mentioned authors.

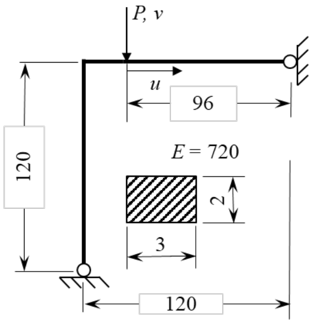


Figure 6. Lee's frame

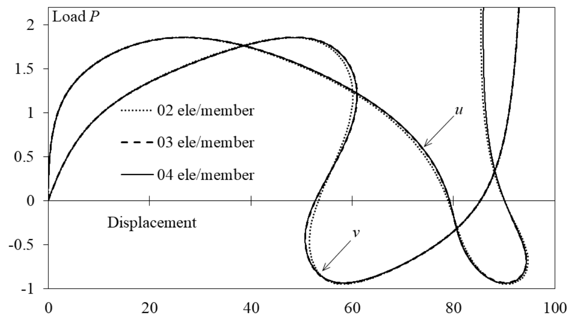


Figure 7. Load-displacement curves with different number of elements

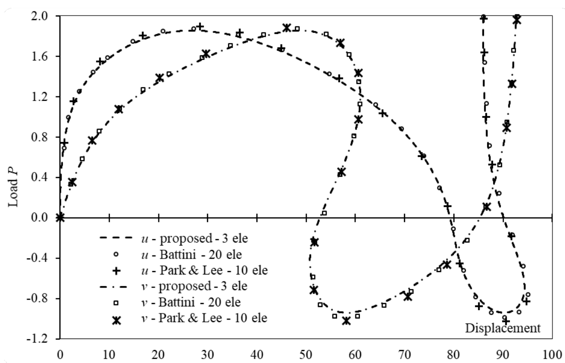


Figure 8. Displacement at point load

3.3. Eccentrically loaded column with linear semi-rigid connection

An eccentrically loaded column with geometric and material properties shown in Fig. 9 was analysed by So and Chan [6] using 3-node element which is established by fourth-order polynomial function for lateral displacement v and the minimum residual displacement algorithm. The convergence of the equilibrium path according to number of proposed elements is shown in Fig. 10. It can be seen that the column must be modelled at least three proposed elements in two cases in order to have the results identical to those of So and Chan [6] using two fourth-order elements as shown in Fig. 11.

3.4. Cantilever beam with a semi-rigid connection

A cantilever beam subjected to a point load at free end shown in Fig. 12(b) was studied by Aristizábal-Ochoa [16] using the classical algorithm of Elastica and the corresponding elliptical functions. Kishi-Chen three-parameter power model is applied in modelling semi-rigid behaviour of end

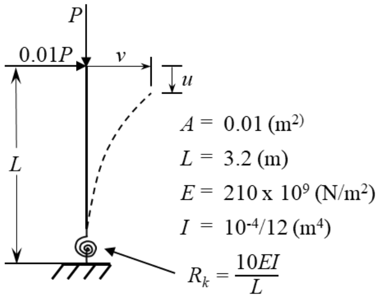


Figure 9. Eccentrically loaded column

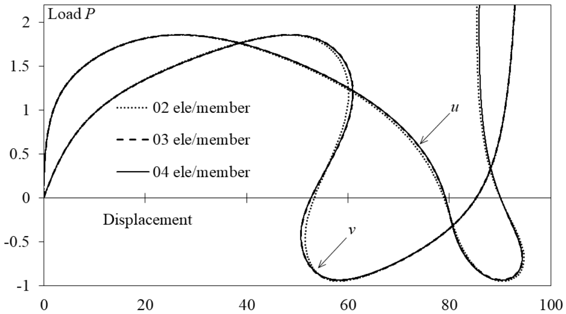


Figure 10. Convergence of the equilibrium path according to number of proposed elements

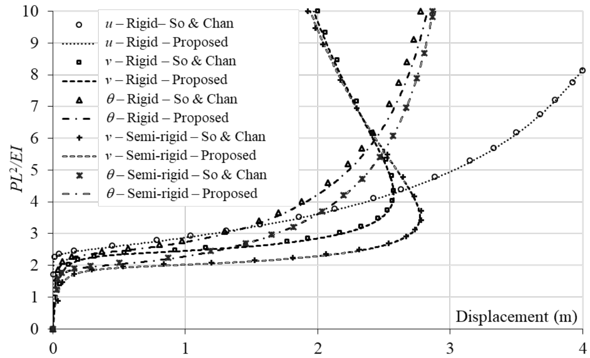
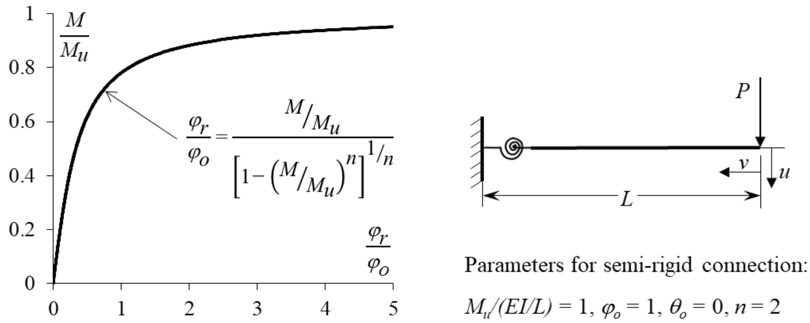


Figure 11. Displacements at free end

connection shown in Fig. 12(a). The analysis results with eight proposed elements per member show good convergence with Aristizábal-Ochoa's solution as shown in Fig. 13.



(a) Semi-rigid connection model (b) Structural model

Figure 12. Cantilever beam with semi-rigid connection

3.5. Williams' toggle frame

The Williams' toggle frame shown in Fig. 14 was analysed with three cases of different support conditions: (1) rigid connection; (2) linear semi-rigid connection; (3) hinge connection. In the first case, the analysis results of the proposed program well converge to Williams' analytical solution [17]

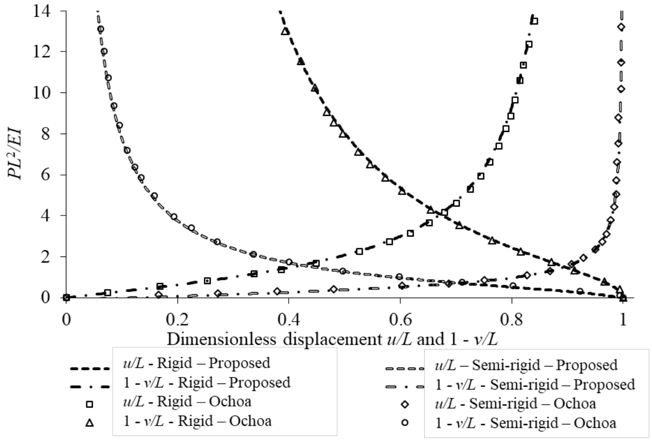


Figure 13. Displacements at free end

until the deflection ratio (δ/h) of about 1.2 by using only two proposed elements per member as shown in Fig. 15. For all three cases, the analysis results with two proposed elements per member coincide with the ones obtained by Tin-Loi and Misa [7] as shown in Fig. 16.

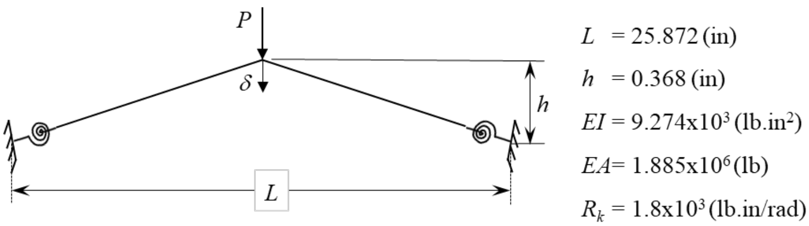


Figure 14. Williams' toggle frame

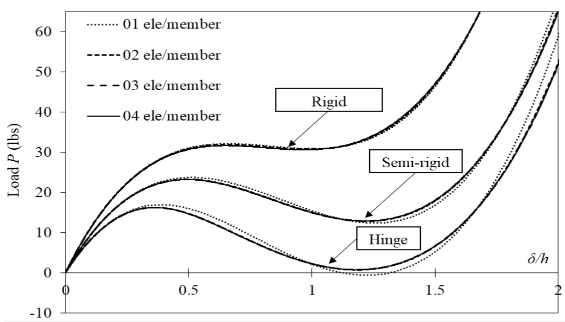


Figure 15. Load-deflection curves according to number of elements

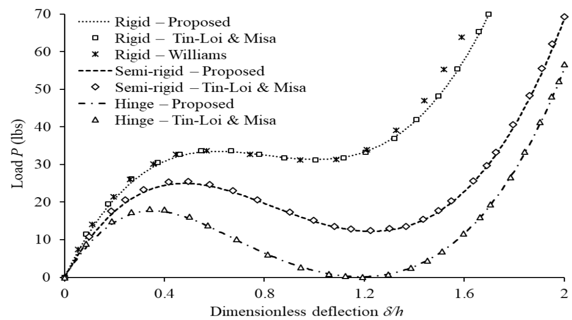


Figure 16. P-δ relation curves

4. Conclusions

A hybrid corotational finite element for large-displacement elastic analysis of semi-rigid planar steel frames is presented in this study. The semi-rigid connections are modelled by zero-length rotational springs with linear or nonlinear behaviour of moment-rotation relation. A Matlab computer program using arc-length method combined the sign of displacement internal product is developed to solve nonlinear equilibrium equation system. The results of numerical examples prove that the proposed hybrid element can accurately predict the large displacement behaviour of semi-rigid planar steel frames subjected to static load.

Acknowledgements

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 107.01-2016.34.

References

[1] Battini, J.-M. (2002). Co-rotational beam elements in instability problems. PhD thesis, KTH, Stockholm, Sweden.

[2] Yaw, L. L., Sukumar, N., Kunnath, S. K. (2009). Meshfree co-rotational formulation for two-dimensional continua. *International Journal for Numerical Methods in Engineering*, 79(8):979–1003.

[3] Le, T.-N., Battini, J.-M., Hjiaj, M. (2011). Efficient formulation for dynamics of corotational 2D beams. *Computational Mechanics*, 48(2):153–161.

[4] Doan-Ngoc, T.-N., Dang, X.-L., Chu, Q.-T., Balling, R. J., Ngo-Huu, C. (2016). Second-order plastic-hinge analysis of planar steel frames using corotational beam-column element. *Journal of Constructional Steel Research*, 121:413–426.

[5] Chan, S. L., Zhou, Z. H. (1994). Pointwise equilibrating polynomial element for nonlinear analysis of frames. *Journal of structural engineering*, 120(6):1703–1717.

[6] So, A. K. W., Chan, S. L. (1995). Buckling and geometrically nonlinear-analysis of frames using one element member-reply. *Journal of Constructional Steel Research*, 32(2):227–230.

[7] Tin-Loi, F., Misa, J. S. (1996). Large displacement elastoplastic analysis of semirigid steel frames. *International Journal for Numerical Methods in Engineering*, 39(5):741–762.

[8] Park, M. S., Lee, B. C. (1996). Geometrically non-linear and elastoplastic three-dimensional shear flexible beam element of von-Mises-type hardening material. *International Journal for Numerical Methods in Engineering*, 39(3):383–408.

[9] Ngo-Huu, C., Nguyen, P.-C., Kim, S.-E. (2012). Second-order plastic-hinge analysis of space semi-rigid steel frames. *Thin-Walled Structures*, 60:98–104.

[10] Saritas, A., Koseoglu, A. (2015). Distributed inelasticity planar frame element with localized semi-rigid connections for nonlinear analysis of steel structures. *International Journal of Mechanical Sciences*, 96: 216–231.

[11] De Borst, R., Crisfield, M. A., Remmers, J. J. C., Verhoosel, C. V. (2012). *Nonlinear finite element analysis of solids and structures*. John Wiley & Sons.

[12] Lui, E. M., Chen, W.-F. (1986). Analysis and behaviour of flexibly-jointed frames. *Engineering Structures*, 8(2):107–118.

[13] Posada, L. M. (2007). Stability analysis of two-dimensional truss structures. Master's thesis, University of Stuttgart, Germany.

[14] Wilson, E. L. (1974). The static condensation algorithm. *International Journal for Numerical Methods in Engineering*, 8(1):198–203.

[15] Mattiasson, K. (1981). Numerical results from large deflection beam and frame problems analysed by means of elliptic integrals. *International Journal for Numerical Methods in Engineering*, 17(1):145–153.

[16] Aristizábal-Ochoa, J. D. ı. o. (2004). Large deflection stability of slender beam-columns with semirigid connections: Elastica approach. *Journal of Engineering Mechanics*, 130(3):274–282.

[17] Williams, F. W. (1964). An approach to the non-linear behaviour of the members of a rigid jointed plane framework with finite deflections. *The Quarterly Journal of Mechanics and Applied Mathematics*, 17(4): 451–469.