

# APPLICATION OF HILBERT HUANG TRANSFORM TO IDENTIFY THE NATURAL FREQUENCIES OF STEEL FRAME

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## **Abstract**

Identification of dynamic structural characteristics such as natural frequency based on measured vibration responses at site is one of the important steps in the structural investigation work of loading test and structural health monitoring. During the service stage, the dynamic characteristics can be changed due to the stiffness reduction and nonlinear of structure under extrema excitation. Therefore, the identification of instantaneous parameters of structure is very necessary in detecting and monitoring the structural deterioration continuously by the time. This paper presents the data processing method using Hilbert Huang Transform (HHT) algorithm in determining a time-dependent frequencies of steel frame. The vibration experiment for steel frame was conducted in this study. The acceleration responses in time domain were recorded and analyzed by HHT to determine the fundamental natural frequencies of frame. The instantaneous frequencies in the time domain of steel frame are the main finding in this research. The traditional data processing method using a Fast Fourier Transform (FFT) was conducted and compared with the HHT method.

**Keywords:** Hilbert Huang Transform (HHT); steel frame; Fast Fourier Transform (FFT); natural frequency.

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## **1. Introduction**

Currently, a dynamic response analysis is a new efficient approach based on the site measurement responses of structure using a signal processing technique to evaluate the behavior of structure more realistically. The main signal processing technique includes Fast Fourier Transform (FFT), the Wavelet Transform (WT), and the Hilbert Huang Transform (HHT) in identifying dynamic characteristics of structure [1, 2]. However, as commented by Huang and Shen [3] the Fourier spectral analysis is restricted to linear systems and signal data should be either periodic or stationary and only presents the results in the energy-frequency space. Wavelet Transform (WT) allows a energy-time-frequency resolution for non-stationary data, however it is established only based on the complete theory and is suitable for linear signal data. More advantageously, HHT is a powerful technique for analyzing data from nonlinear and non-stationary processes and its theoretical base is empirical. It has been used extensively in the maintenance or structural health monitoring (SHM). There are a lot of theoretical and practical studies that have been developed to this method for a wide variety of structures such as

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bridges, high buildings, offshore drilling platforms ... [4–6]. Mahato et al. [7] applied HHT to identify the modal parameters of a reinforced concrete framed building and Kunwar et al. [8] and Reddy et al. [9] proposed the damage detection for bridge structure using the HHT technique.

In Vietnam, dynamic response analysis is used to determine the fundamental natural frequencies of the components such as beams, stay cables, suspension cables of bridges. In the offshore platform structure, the dynamic response analysis has almost not conducted to identify the dynamic characteristics. Trung [10] in 2010 also presented a study to determine the fundamental natural frequency of a steel frame using the Fast Fourier Transform (FFT) based on acceleration measurement data, however the FFT algorithm was limited in determining the time-dependent frequency. Therefore, this study was conducted to evaluate the method of determining variation of the natural frequency in the time domain for the steel frame structure using the Hilbert Huang Transform algorithm.

## 2. Signal processing methodology

### 2.1. Fast Fourier Transform (FFT)

The Fourier series is made up of sines and cosines; the Fourier transform is a generalization of the Fourier series, and made up of exponentials and complex numbers. The Fourier analysis indicates wide applications in mathematics and engineering, used in modeling diverse physical phenomena.

Fast Fourier Transform (FFT) is an algorithm using the discrete Fourier transform (DFT). The discrete Fourier transform is to be converted from the specific types of sequences of functions into other types of representations.

$$x(t) \xrightarrow{FT} X(F) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi Ft} dt \quad (1)$$

where  $x(t)$  is the function in time domain (s);  $FT$  is the fast Fourier Transform;  $X(F)$  is the function in frequency domain.

By capturing the peak of power spectrum at the phase of 90 degrees, the natural frequencies of the structure can be determined, as shown in Fig. 1. The peak performs the resonance state (the state where the frequency of the external force and the natural frequency of the structure are equal). The phase difference is 90 degrees when measured by the displacement responses and in the case of the velocity data, while it is 180 degrees later than this by 90 degrees, [11].

### 2.2. Huang Hilbert transformation

Hilbert Huang Transformation (HHT) is an algorithm developed by Huang from the Hilbert transform. The HHT method is a combination of two steps: (1) Decomposing the data into different simple

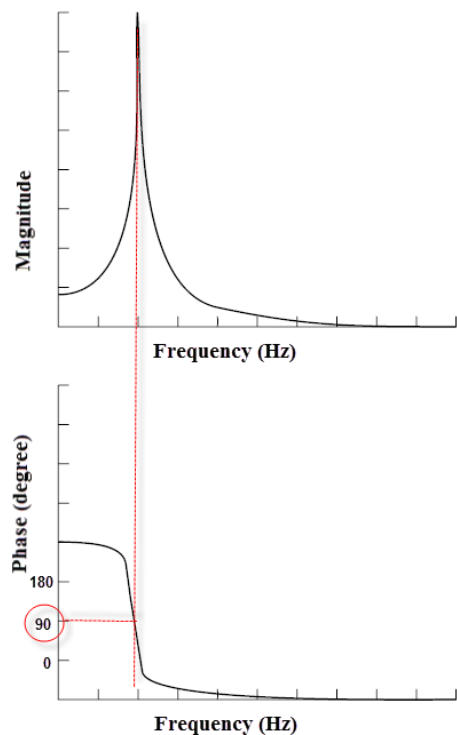


Figure 1. Identification method of natural frequency based on [11]

intrinsic modes of oscillation (Empirical Mode Decomposition, EMD); (2) Performing Hilbert transform to each simple mode (Hilbert Spectrum Analysis, HAS).

Because the Hilbert transform is limited to narrow band passed signals, it is not suitable for actual oscillation data in long band passed signal with a frequency range of nonlinear and non-stationary. Therefore, Huang et al. [1] introduced a new sifting process method denoted as an empirical model decomposition (EMD). The decomposition is based on the simple assumption that any data set consists of different simple intrinsic modes of oscillations. Each of these oscillation modes, an intrinsic mode function (IMF), is defined by the following conditions: (1) Over the entire data set, the number of extrema and the number of zero-crossings must be equal or differ at most by one; (2) At any points, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. An IMF represents a simple oscillation mode which is similar to a component in the simple harmonic function, but it is more general because the amplitude and frequency are a function of time.

Any function can be decomposed as follows:

1. Identify all the local extrema, and then connect all the local maxima by a cubic spline as the upper envelope.
2. Repeat the procedure for the minima to produce the lower envelope. The upper and lower envelopes should cover all the data.

If the mean is designated as  $m_1$  and the difference between the data and  $m_1$  is the first component  $h_1$ , then:

$$h_1 = x(t) - m_1 \quad (2)$$

where  $h_1$  is an IMF, as shown in Fig. 2. The mean  $m_1$  is given by the sum of local extrema connected by the cubic spline.

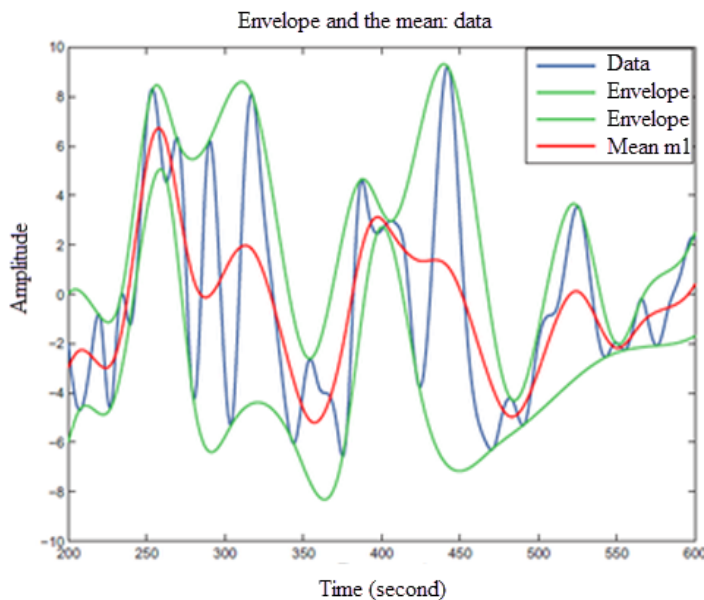


Figure 2. The data (blue) upper and lower envelopes (green) defined by the local maxima and minima, respectively, and the mean value of the upper and lower envelopes given in red, [2]

To treat  $h_1$  as a new set of data, a new mean is computed:

$$h_1 - m_{11} = h_{11} \quad (3)$$

After repeating the sifting process up to  $k$  times,  $kh_1$  becomes the IMF, that is:

$$h_{1(k-1)} - m_{1k} = h_{1k} \quad (4)$$

Let  $h_{1k} = c_1$ , the first IMF  $c_1$  from the data should contain the finest scale or the shortest period component of the data. Now  $c_1$  can be separated from the rest of the data by:

$$x(t) - c_1 = r_1 \quad (5)$$

Since  $r_1$  is the residue, it contains information on a longer period component; it is now treated as the new data and subjected to the same sifting process. The procedure is repeated for all subsequent  $r_i$  and the result is:

$$r_1 - c_2 = r_2 \quad \dots \quad r_{n-1} - c_n = r_n \quad (6)$$

where  $c_2$  is now the second IMF of the data.

Any oscillation signals can be decomposed into  $n$ -empirical modes and a residue  $r_n$ :

$$x(t) = \sum_{i=1}^n c_i(t) + r_n(t) \quad (7)$$

where  $c_i$  is the  $i$ th IMF component, and  $r_n$  is the  $n^{\text{th}}$  residue.

Each simple harmonic IMF is suitable to apply Hilbert transform in computing the instantaneous frequency. The Hilbert Transform representation of the  $i^{\text{th}}$  IMF component,  $c_i$ , is written as:

$$h_i(t) = \frac{1}{\pi} PV \int \frac{c_i(\tau)}{t - \tau} \quad (8)$$

where  $PV$  denotes the Cauchy principal value, and its analytic signal,  $z_i(t)$ , is defined as:

$$z_i(t) = c_i(t) + h_i(t)i = a_i(t)e^{i\theta_i(t)} \quad (9)$$

where  $a_i(t) = \sqrt{c_i(t)^2 + h_i(t)^2}$  is the instantaneous amplitude, which describes the envelope of  $c_i(t)$  versus time; and the instantaneous phase of  $c_i(t)$  versus time is defined as

$$\theta_i(t) = \arctan \left[ \frac{h_i(t)}{c_i(t)} \right] \quad (10)$$

The residue  $r_n$  has been left out on purpose, the original data  $x(t)$  can be expressed as the real part in the following form:

$$x(t) \cong \sum_{i=1}^n c_i(t) = RP \sum_{i=1}^n a_i(t) e^{i\theta_i(t)} = RP \sum_{i=1}^n [c_i(t) + h_i(t)i] \quad (11)$$

In addition, the instantaneous frequency,  $\omega_i(t)$ , for each IMF component can be defined as follows:

$$\omega_i(t) = \frac{d\theta_i(t)}{dt} \quad (12)$$

Eqs. (10) and (11) give both the amplitude  $a_i(t)$  and frequency  $\omega_i(t)$ , of each component as functions of time.



### 3. Vibration test

#### 3.1. Physical model

The physical model of the steel frame structure is made in the factory. The first and second floors are of the same height - 0.6 m, and the total height of the structure is 1.2 m. The width of the floors is 0.35 m. The steel section is a rectangular tube of  $0.13 \text{ m} \times 0.13 \text{ m}$  in shape and 1.1 mm in thickness. The welded joints are joined together, the four columns are tightly joined to the heavy concrete slab as shown in Fig. 3.

#### 3.2. Equipments

Accelerometer sensors used in this experiment is 4507-B-004, manufactured in Denmark, as shown in Fig. 4. The features of these sensors are designed with a small volume, in contrast to high sensitivity and can be measured in three directions and tested under high temperature conditions, see Fig. 3. Pulse LabShop ver.10.3 software was used in this experiment to process the response data of steel frame, shown in Fig. 5.

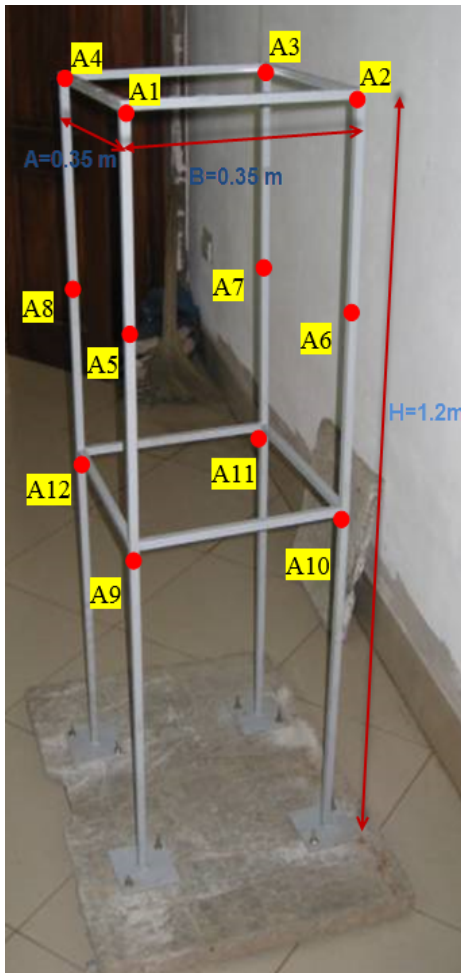


Figure 3. Physical Steel model

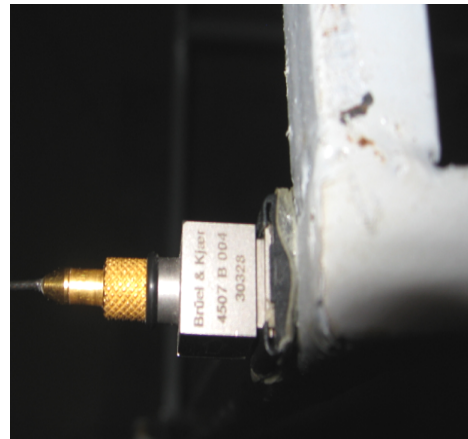


Figure 4. Accelerometer sensor



Figure 5. Software using in experiment  
PULSE LabShop version 10.3

Main specifications are: the sensitivity of  $10 \text{ mV/ms}^{-2}$ ; the measuring range of  $700 \text{ ms}^{-2}$ ; the frequency range of  $0.3 - 6 \text{ kHz}$ ; the external resistance of  $< 2\Omega$ ; the temperature coefficient of sensitivity of  $0.09\% \text{ }^{\circ}\text{C}$ .

The accelerometers were installed at positions A1 and A5, as shown in Fig. 4. Trung [10] proposed that the natural oscillation responses in any points on the structure contained all of natural vibration modes of structure including the natural frequency of component and the frequency of excitation force. However, if the structure is excited in the higher amplitude then the identification of the natural frequencies becomes more clearly. In this study, the point A1 was selected as a rigid joint with a large response amplitude, meanwhile the point A5 located at the middle of the beam A1A9 was in the smaller amplitude and this point might contain more natural vibration of beam A1A9, except for natural mode of whole structure. Therefore, A1 and A5 points were selected enough for the identification and assessment of natural fundamental frequencies of steel frame structure based on the vibration amplitude of response from excitation

### 3.3. Measurement methodology

The steps for measuring the structural vibration response are shown in Fig. 6. In this reasearch, the excitation force was hammered at the A4 point, the vibration response signals from the accelerometers were recorded at the specified points. The responses were processed by using the FFT or HTT algorithm to identify the natural frequencies of structure.

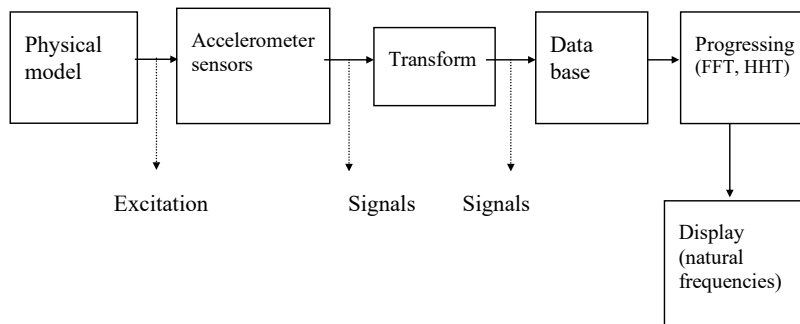


Figure 6. Testing progress of dynamic response measurement

## 4. Results and discussions

### 4.1. Fast Fourier Transform

Figs. 7 and 8 show the acceleration response time histories at the A1 and A5 points. The data was recorded at the physical model. The duration is specified of about 16 seconds based on the oscillation amplitude which gradually reduces to zero.

The FFT algorithm is used to identify the fundamental frequencies of the steel frame structure by capturing the peaks of power spectrum density in a combination with the angle phase variation [11], see Figs. 9 and 10.

From Fig. 9 of A1, it can be seen that the resonant frequencies are around 15.63 and 22.1 Hz, respectively. These similar values are also observed in Fig. 10 of A5. Table 1 shows the results of comparing the natural frequencies determined from the FFT method with the numerical method (obtaining the results from the author's previous study, [10]).

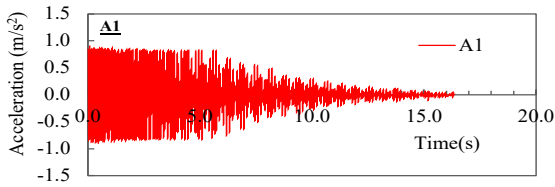


Figure 7. Acceleration response time history at A1

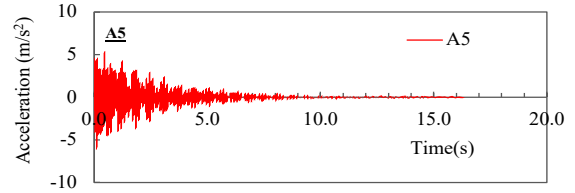


Figure 8. Acceleration response time history at A5

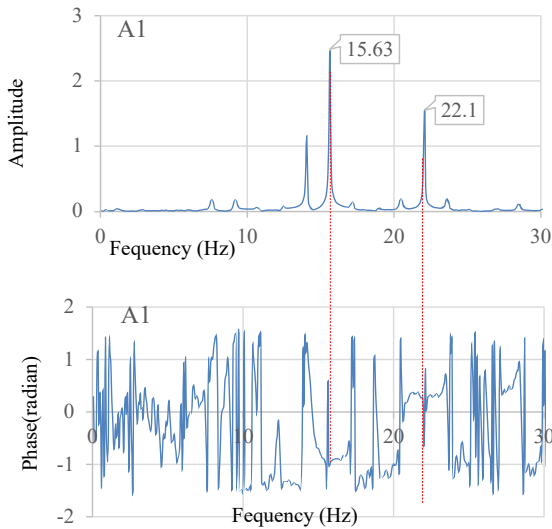


Figure 9. Identification of natural frequency using FFT at A1

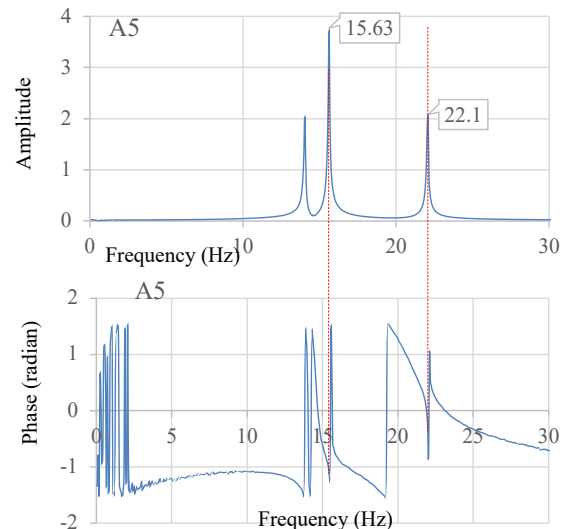


Figure 10. Identification of natural frequency using FFT at A5

Table 1. Identified natural frequencies by FFT method

No	Symbol	Determined frequencies by FEM (Hz) [6]	Identified frequency by FFT (Hz)	Identified frequency by HHT (Hz)	Note
1	$f_1 = f_2$	16.25	15.65	15.72	The first and second frequency
2	$f_3$	21.24	22.64	20.85	The third frequency

#### 4.2. Hilbert Huang Transform (HHT)

Fig. 11 shows the function obtained from the acceleration responses at A1, which includes eight IMFs simple oscillation modes from  $c_1$  to  $c_8$ . The data function from  $c_1$  to  $c_8$  was processed and arranged in a high-to-low frequency range.

From these simple data, the instantaneous frequencies were obtained by using the Hilbert transform. Fig. 12 shows the instantaneous frequency which was determined by the IMF  $c_{11}$  and was almost constant of 21.31 Hz during the vibration time. This frequency is approximately the same as the third natural frequency determined by FEM. Due to the excitation force is not large enough to explore the nonlinear behavior of structure, the variation of frequency in the time domain is much insignificant. Fig. 13 also presents the identified frequency of IMF  $c_2$  and its values, which varies around 15.66 Hz.

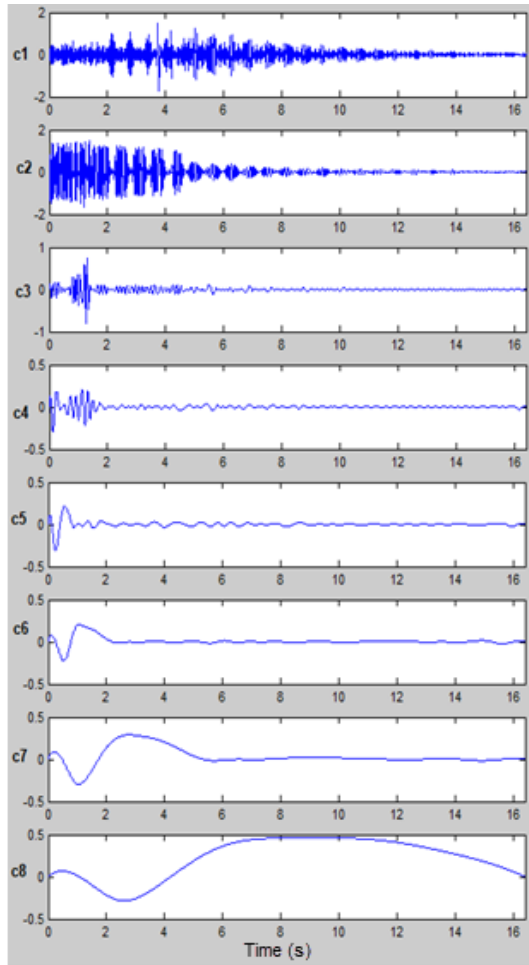


Figure 11. Intrinsic mode function from  $c_1$  to  $c_8$  (IMF1-IMF8) obtained at A1

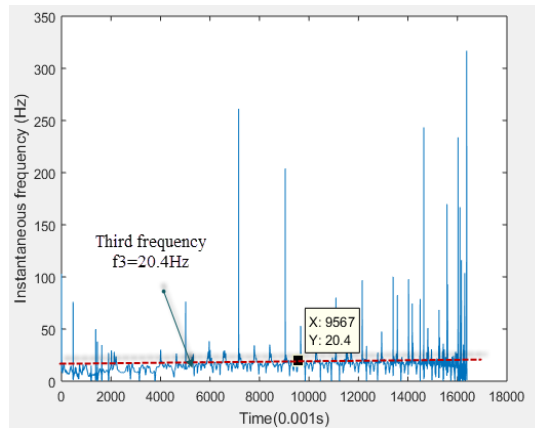


Figure 12. Instantaneous frequency of  $c_1$  (IMF1)

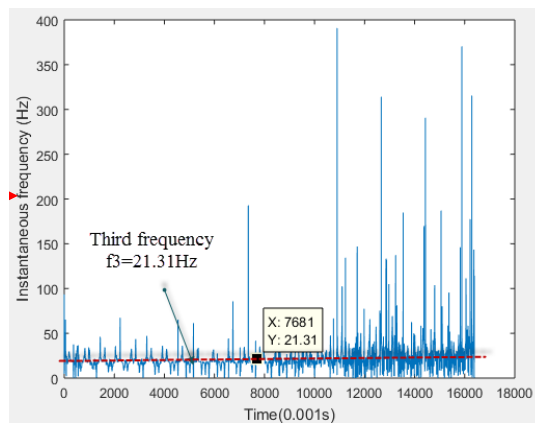


Figure 13. Instantaneous frequency of  $c_2$  (IMF2)

The obtained frequency is approximately the same as the first and second frequency in the FEM and FFT method, as shown in Table 1.

Fig. 14 also presents the function obtained from the acceleration responses at A5, which includes eleven IMFs simple oscillation modes from  $c_1$  to  $c_{11}$ . The instantaneous frequency of the electric power determined by the IMF  $c_2$  was almost constant of 52.09 Hz, as shown in Fig. 15. Figs. 16 and 17 show the instantaneous frequency determined by  $c_3$  and  $c_4$  and constantly keep the values by 20.4 and 15.78 Hz during the vibration time, respectively. These frequencies are approximately the same as these natural frequencies determined by FEM, FFT as shown in Table 1 and at A1.

The number of intrinsic mode function at A5 was processed much more than at A1; however, the instantaneous frequencies were identified by the acceleration response data from both the A1 and A4 points. The difference between them is not much significant. As a result, the HHT method could help identify the time-dependent frequency (instantaneous frequency) of the steel frame.

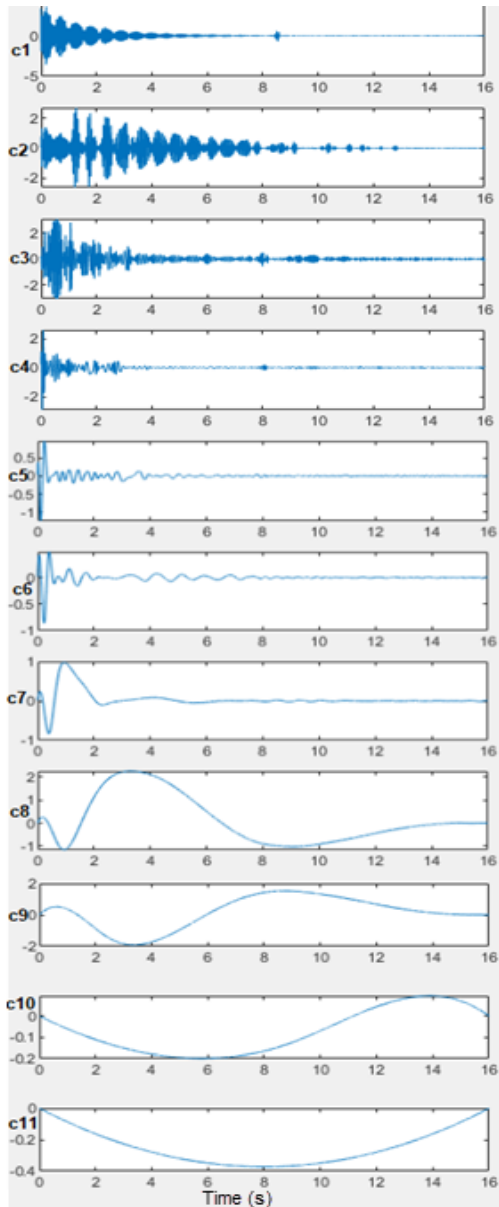


Figure 14. Intrinsic mode function from  $c_1$  to  $c_{11}$  (IMF1-IMF11) obtained at A5

## 5. Conclusions

The dynamic response measurement using Hilbert Huang Transform (HHT) to identify the dynamic characteristic of the steel frame was conducted in this study. There are some main findings as follows:

- (1) Identification of natural frequencies of the steel frame using the HHT method is almost the same as these in the FFT and FEM method.
- (2) HHT method could perform fundamental natural frequencies of the steel frame as a time-dependent function. The instantaneous frequencies are well-used to perform a variation of frequency

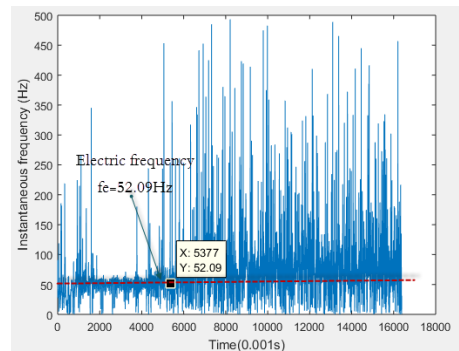


Figure 15. Instantaneous frequency of  $c_2$

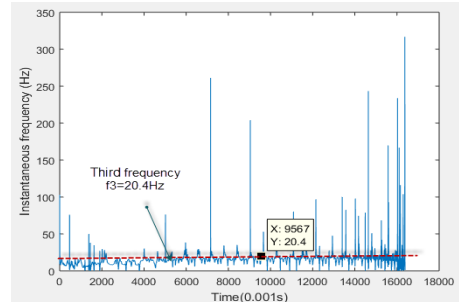


Figure 16. Instantaneous frequency of  $c_3$

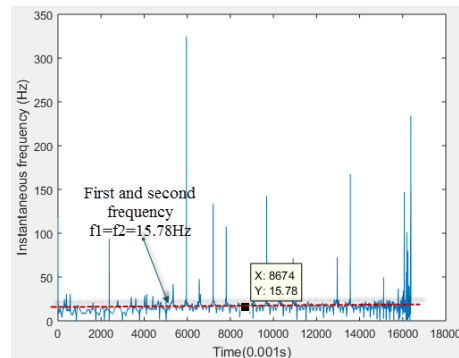


Figure 17. Instantaneous frequency of  $c_4$

during and after the excitation. The effect of nonlinear properties and soil-structure interaction on the variation of the global stiffness of the steel structure will be in the next research.

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