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WEIGHT OPTIMIZATION OF COMPOSITE CELLULAR BEAM BASED ON THE DIFFERENTIAL EVOLUTION ALGORITHM

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Abstract

In this study, the differential evolution algorithm is used for solving the optimum design problem of composite cellular beams. The design variables are hot rolled profile from which the cellular beam will be produced as well as opening size and its spacing. The objective function is the minimum weight of cellular beam while the design constraints include satisfying the ultimate limit states, the serviceability limit states and the geometric limitations. The design method adopted in this study is based on EN 1994-1-1. Furthermore, a parametric study is conducted to evaluate the influence of beams spacing to the weight of floor beam system. As a result, an optimal spacing of composite cellular beams is proposed.

Keywords: composite beam; cellular beam; web opening; steel beam optimization; differential evolution.

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1. Introduction

Cellular beams are steel sections with repeating circular openings in the web of beam. Cellular beams are usually fabricated by cutting a hot rolled H-shape profile web in a half circular pattern along its centerline. The two obtained Tee sections are shifted and re-welded as shown in Fig. 1. This technique increases the overall beam depth, as a result, the moment of inertia and the section modulus are increased while reducing the overall weight of the beam. Advantages of cellular beam include long span capability, light weight and the ability to pass heating, ventilation and air conditioning (HVAC) systems through the openings. Cellular beams can span up to 40 meters without intermediate supporting columns when using as roof beams. They can also be used as floor beams in buildings. In that case, cellular beams are usually designed as a composite beam when headed studs are provided to connect concrete slab to the top flange of steel beam. Due to combining the great compressive strength of the concrete slab and the tensile strength of the bottom Tee, the flexural resistance of the composite cellular beam (CCB) significantly increases. Because of its advantages, cellular beams have been widely used over the world.

The design of a cellular beam requires the selection of a hot rolled profile from which the cellular beam is to be produced, the selection of circular opening diameter and the spacing between two adjacent openings. As both the hot rolled profile and the opening dimensions can be varied, it is too difficult for engineers to determine the most economical solution. Many researchers have used mathematical algorithm to solve this optimum design problem. In [1], M. P. Saka et al. introduced a method to optimize the weight of steel cellular beams using the harmony search (HS) algorithm and

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the particle swarm (PS) algorithm. A. Kaveh and F. Shokohi presented a cost optimization method of the castellated steel beam based on the charged system search (CSS) algorithm [2, 3]. N. D. Lagaros described an optimum design method of steel structure with web openings using the evolution strategies (ES) algorithm [4]. Vu Anh-Tuan applied the differential evolution (DE) algorithm to optimize several kinds of structure such as steel portal frame [5] or steel-concrete composite beam [6]. In comparison with other algorithms, DE is simple to implement, easy to use, reliable and fast [7, 8].

It can be noted that while the optimal design problem of steel cellular beams has been studied [1–4], there still remains a need to optimize the cellular beam considering the effect of composite action and this gives the motivation for the current study. In this study, the DE algorithm is used for weight optimization of the simple supported composite cellular beams. The paper is organized as follows: The optimization problem is defined in Section 2. In Section 3, the DE algorithm is described in more details. A numerical example is conducted to evaluate the efficiency of the algorithm. Additionally, a parametric study is performed in Section 4 to find the optimal spacing between two composite cellular beams. Finally, conclusions are presented in Section 5.

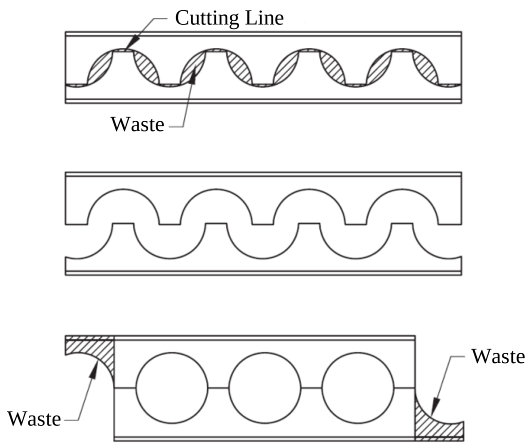

Figure 1. Manufacturing of cellular beams

2. Optimization problem definition

In general, the optimization problem requires identifying the design variables, the design constants, the objective function and the design constraints.

2.1. Design variables and design constants

Considering a simply supported composite cellular beam. In practice, there are many dimensional parameters that should be designed. In this study, three design variables are used for finding the optimal results including the hot rolled H-shape profile, the circular opening diameter and the spacing between openings. The hot rolled profile variable is defined through the sequence number of the profile in the standard steel section list. The bounds on variables are summarized in Table 1. The remaining parameters like materials, span, loads are considered as design constants in the optimization problem.

Table 1. Design variables

Variable	Hot rolled profile	Opening diameter h_o (mm)	Opening spacing s_o (mm)
Variable boundaries	Universal Beam (UB) section list	200 ÷ 900	200 ÷ 900

Once the steel beam profile is selected, then the dimensions of the flange and the thickness of the web become available. The overall depth of cellular beam is determined from the depth of original steel beam, the diameter and the spacing of opening as following (Fig. 2):

$$h = h_{sb} + \frac{h_o}{2} - loss \tag{1}$$

$$loss = \frac{h_o}{2} - \sqrt{\left(\frac{h_o}{2}\right)^2 - \left(\frac{s - h_o}{2}\right)^2} \tag{2}$$

where h_{sb} is the depth of original hot rolled profile.

2.2. *Objective function*

In this study, the weight of the steel cellular beam is chosen as objective function. The weight of the steel cellular beam can be expressed as follows:

$$W = \rho\left(AL - n_o t_w \frac{\pi h_o^2}{4}\right) \tag{3}$$

where W denotes the weight of steel cellular beam; ρ is the density of steel; A is the gross cross-section area of the cellular beam; L is the span of the cellular beam; n_o is the total number of openings.

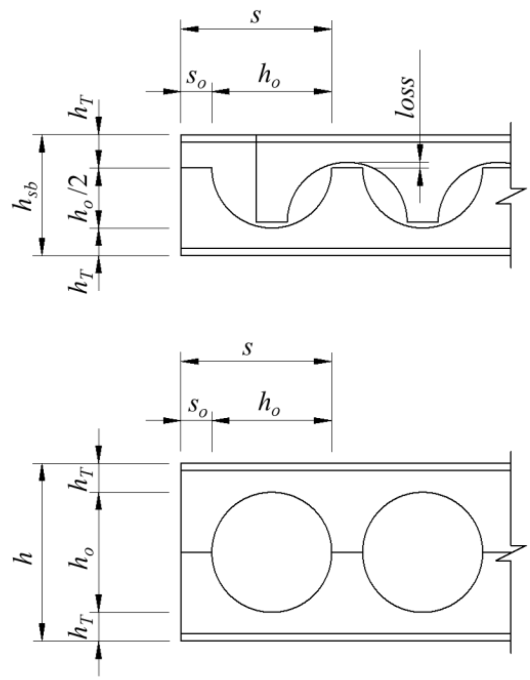

Figure 2. Geometrical parameters of a cellular beam

2.3. *Design constraints*

The design constraints include ultimate limit state (ULS) constraints, serviceability limit state (SLS) constraints and geometrical limitation constraints. For the ULS constraints, the following limit states that should be considered when designing CCB as: global flexural strength, shear strength, Vierendeel bending strength of Tees, web-post shear strength and web-post buckling strength. The SLS constraints are such that the maximum deflection should be lesser than or equal the allowable deflection of floor beam.

The full design method for the composite beam with large web openings was described in [9], in which, all equations are consistent with EN 1994-1-1 [10]. In this paper, the equations are presented in simplified form for only composite cellular beams.

a. Global flexural capacity

As described in previous study [9], the circular openings in the web of beam may be treated as an equivalent rectangular opening with effective length and height are taken as: $l_e = 0.45h_o$ and $h_e = 0.9h_o$ where h_o is the diameter of openings (Fig. 3).

The plastic bending resistance of the composite beam at the opening position is given by [9]:

$$M_{o,Rd} = N_{bT,Rd}\left(h_{eff} + z_t + h_s - 0.5z_c\right) - \left(N_{bT,Rd} - N_{c,Rt}\right)(z_t + h_s - 0.5z_c) \tag{4}$$

where $N_{bT,Rd} = A_T f_y/\gamma_{M0}$ is the tensile resistance of the bottom Tee; $N_{c,Rd}$ is the compressive resistance of composite slab; h_{eff} is the effective depth of the steel section between centroid of the Tees;

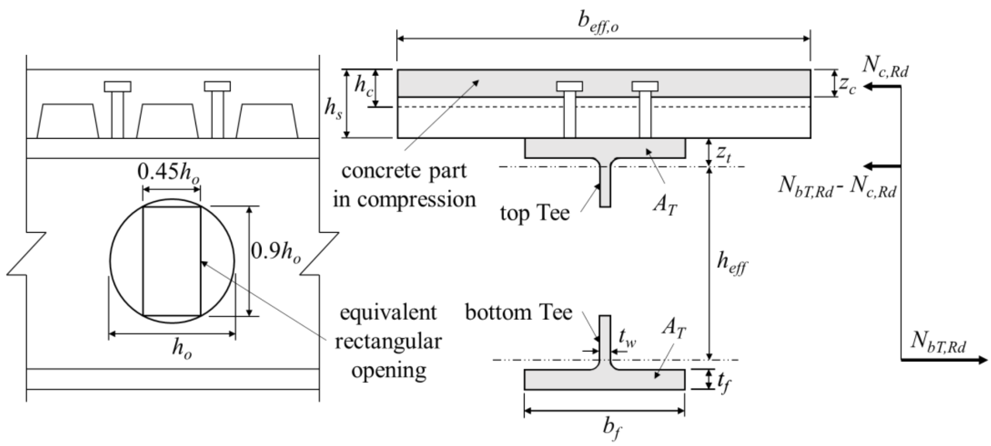

Figure 3. Forces at opening section

z_t is the distance from the centroid of the top Tee to the outer edge of the flange; h_s is the total depth of slab; z_c is the depth of concrete part in compression, A_T is the cross sectional area of the Tee; f_y is the yield strength of steel and γ_{M0} is the partial factor for resistance of structural steel.

The compressive resistance of composite slab is the smaller value of concrete compressive resistance and shear resistance of headed stud connectors between the support and the opening:

$$N_{c,Rd} = \min\left(0.85 f_{ck}\, b_{eff,o}\, h_c/\gamma_c; n_{sc}\, P_{Rd}\right) \tag{5}$$

where f_{ck} is the characteristic compressive cylinder strength of concrete; $b_{eff,o}$ is the effective slab width at opening which will be defined in Eq. (6); h_c is the depth of concrete above decking;γ_C is partial factor for concrete; n_{sc} is the number of shear connector provided in the length x from the position of the considered opening to the nearest support; and P_{Rd} is the shear resistance of one shear connector.

The effective slab width at an opening depends on the distance from the position of considered opening to the nearest support. It can be expressed as following:

$$\begin{aligned} b_{eff,o} &= 3L/16 + x/4 \leq B \quad && \text{if } x \leq L/4 \\ b_{eff,o} &= L/4 \leq B && \text{if } x > L/4 \end{aligned} \tag{6}$$

The maximum design moment should not exceed the plastic bending resistance of the CCB. It can be expressed as follows:

$$M_{Ed} \leq M_{o,Rd} \tag{7}$$

b. Shear capacity

The maximum shear force should be lesser than the sum of the shear resistance of the top and bottom Tees which is determined as following [3]:

$$V_{Ed} \leq V_{pl,Rd} = 0.9\left(A_{w,t} + A_{w,bT}\right)\left(f_y/\sqrt{3}\right)/\gamma_{M0} \tag{8}$$

where $A_{w,tT}; A_{w,bT}$ are the area of the webs of the top and bottom Tee respectively.

c. Vierendeel bending

The Vierendeel moment due to transfer of shear across the opening must be less than the combination of the Vierendeel bending resistances of the Tees with the bending resistance due to local composite action between the top Tee and the slab. This may be expressed as:

$$2M_{bT,NV,Rd} + 2M_{tT,NV,Rd} + M_{vc,Rd} \geq V_{Ed} l_e \tag{9}$$

where $M_{bT,NV,Rd}$; $M_{tT,NV,Rd}$ are the reduced Vierendeel bending resistances of the Tees in presence of axial and shear force; $M_{vc,Rd}$ is the local composite bending resistance and it may be ignored; V_{Ed} is the design shear force.

The bending resistance of the Tees depends on the class of the web of Tees. The plastic bending resistance can be considered when the web of the Tees is Class 1 or 2. When the web is Class 3 or 4, the elastic bending resistance must be used. For this classification, the effective length of equivalent rectangular opening may be taken as $l_{o,eff} = 0.7h_o$ [9].

d. Web-post resistance

In case of symmetric section, there is not any moment at the mid-height of the web-post because the shear force resisted by the top and bottom Tees are equal. The condition to check web-post shear and buckling resistance can be expressed as following [3, 9]:

$$V_{wp,Rd} = \frac{(0.9 s_o t_w) f_y \big/ \sqrt{3}}{\gamma_{M0}} \geq V_{wp,Ed} = \max \left\{ \begin{array}{c} \dfrac{V_{Ed} s}{\left(h_{eff} + z_t + h_s - 0.5h_c\right)} \\ \dfrac{V_{Ed} s - \Delta N_{cs,Rd} \left(z_t + h_s - 0.5h_c\right)}{h_{eff}} \end{array} \right. \tag{10}$$

$$N_{wp,Rd} = \chi \frac{s_o t_w f_y}{\gamma_{M1}} \geq N_{wp,Ed} = V_{wp,Ed} \tag{11}$$

where s_o is the edge-to-edge spacing of adjacent openings; s is the center-to-center spacing of adjacent openings; $\Delta N_{cs,Rd}$ is the increase in compression resistance of the slab due to shear connectors between two adjacent openings; χ is the reduced factor for buckling of the web-post which is determined from buckling curve "b" based on EN 1993-1-1 Clause 6.3.1.2 [11] with the buckling length is $l_w = 0.5\sqrt{\left(s_o^2 + h_o^2\right)}$, and γ_{M1} is the partial factor for resistance of member to instability.

e. Deflection

The additional deflection due to the openings may be determined approximately by the formula [9]:

$$\delta_{add} = 0.47 n_o (h_o/h)^2 \left(h/L\right) \delta_b \tag{12}$$

where n_o is the number of openings along the beam; h is the depth of steel cellular beam; L is the span of the beam; and δ_b is the deflection of unperforated beam.

The total deflection of CCB is obtained from the sum of three components: the deflection in the construction stage, the deflection in the composite stage and the additional deflection due to the openings. The deflection constraints are expressed as following:

$$\begin{array}{l} \delta_1 \leq [\delta] = L/360 \\ \delta_{total} \leq [\delta] = L/250 \end{array} \tag{13}$$

where δ_1 is the deflection of steel cellular beam in construction stage, δ_{total} is the total deflection of composite cellular beam in the composite stage.

f. Geometric limitations

Furthermore, when designing the CCB, diameter and spacing of its openings should be checked the geometric limits as following [9]: $h_o \leq 0.8h$; $h_T \geq t_f + 30$ mm; $s_o \geq 0.4h_o$; $1.08 \leq s/h_o \leq 1.5$ and $1.25 \leq h/h_o \leq 1.75$.

3. Optimization method

In structural optimization, most problems are difficult to find the mathematical result due to a huge number of discrete variables and complex constraints. Evolutionary algorithms are an effective approach for solving the structural optimization problem in which the DE algorithm is one of the most common methods of evolutionary algorithms.

3.1. Differential evolution algorithm

DE was first introduced by [7]. Like other population-based methods, DE generates new trial candidates to find better solutions. To produce the trial vector, DE perturbs existing vector with the scaled difference of two randomly selected population vectors. The procedure of DE consists of four steps as following:

1. Initialization: generate randomly initial population which contains Np individuals, each individual is an D-dimensional vector that represents a candidate solution

$$x_{ij} = x_j^L + rnd(0,1) \times \left(x_j^U - x_j^L\right), i = (1, Np), j = (1, D) \tag{14}$$

where Np is the size of population; D is the number of design variables; x_{ij} is the j^{th} component of individual X_i; x_j^L and x_j^U are the lower and upper bounds of x_j.

2. Mutation: for each of the Np individuals chosen as the base vector, create a mutant vector by adding a scaled difference vector to the base vector as:

$$V_i = X_{r1} + F \times (X_{r2} - X_{r3}) \tag{15}$$

where V_i is a mutant vector; $X_{r1}; X_{r2}; X_{r3}$ are three different, randomly chosen vectors; $r_1 \neq r_2 \neq r_3$ are randomly selected from $(1, Np)$; F is the scale factor.

3. Crossover: trial vector U_i is created by crossing each vector X_i with a mutant vector V_i:

$$u_{ij} = \begin{cases} v_{ij} & \text{if } rnd(0,1) \leq Cr \\ x_{ij} & \text{otherwise} \end{cases} \tag{16}$$

where v_{ij} is j^{th} component of the mutant vector V_i; x_{ij} is j^{th} component of the vector X_i; Cr is the crossover probability.

4. Selection: vector U_i and vector X_i are compared, the better vector is kept for the next generation.

$$X_i^{new} = \begin{cases} U_i & \text{if } f(U_i) \leq f(X_i) \\ X_i & \text{otherwise} \end{cases} \tag{17}$$

where $f(U_i)$ and $f(X_i)$ are the objective function values.

Optimization process is repeated until pre-assigned number of generations is reached.

3.2. Numerical example

A simply supported composite cellular beam with a span of 10000 mm is selected to optimize. The design constants are following: spacing of beams $B = 2500$ mm; steel grade S235; composite slab $h_s = 120$ mm with concrete class C25/30; depth of decking profile $h_p = 58$ mm; headed studs with diameter $d_s = 19$ mm; height $h_{sc} = 100$ mm; number of studs per rib $n_r = 1$; super dead load $SDL = 1.5$ kN/m^2; imposed load $LL = 3.5$ kN/m^2.

Using the DE algorithm to optimize the weight of steel cellular beam. The optimal shape of the steel cellular beam is presented in Table 2 and Fig. 4. The full history of optimization process during 50 generations is plotted in Figs. 5 and 6.

Based on Fig. 5, it can be obtained that the optimal solution is found at 39$^{\text{th}}$ generation. The ratio between spacing and diameter of openings equals $s/h_o = 1.382$ and the ratio between diameter of openings and overall depth of steel cellular beam equals $h_0/h = 0.607$. The ratios of optimal shape found by the DE algorithm are matched with previous research [12].

Table 2. Optimal design result

Hot rolled profile	Opening diameter h_o (mm)	Opening spacing s (mm)	Overall depth of cellular beam h (mm)	Weight of steel cellular beam W (kg)
UB 406 $\times$ 140 $\times$ 46	340	470	560	435

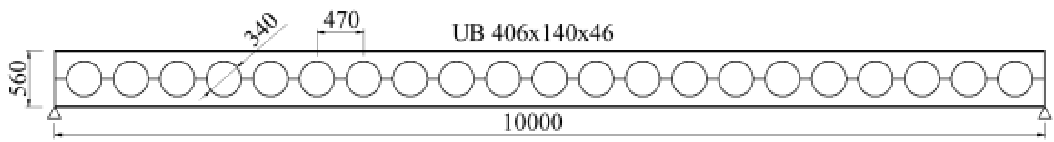

Figure 4. Optimal shape of steel cellular beam

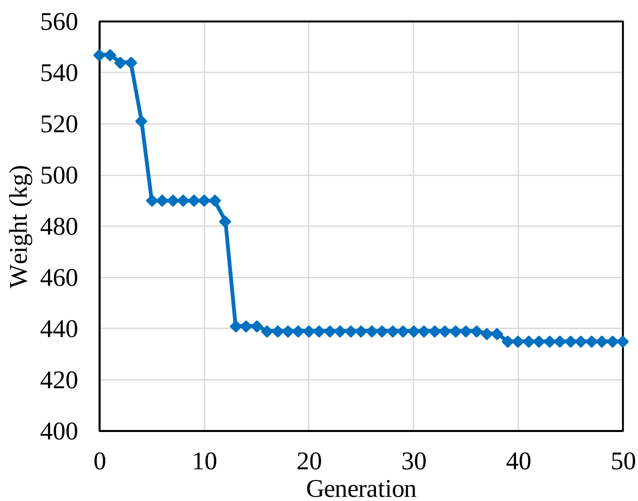

Figure 5. History of optimization process

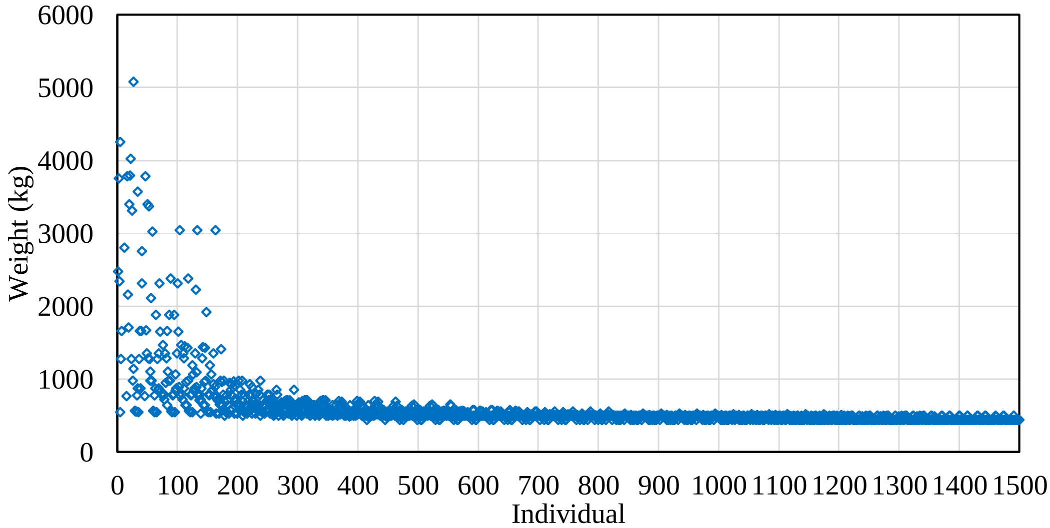

Figure 6. Weight of individuals during 50 generations

4. Parametric study

Considering a floor system with dimensions 10000 mm × 12000 mm as shown in Fig. 7. One-way composite slab subjects a super dead load $SDL = 1.5$ kN/m^2 and an imposed load $LL = 3.5$ kN/m^2. Cellular beams are used as secondary beam and they are carried by main steel beams. Constant data: concrete class C25/30; depth of decking profile $h_p = 58$ mm; diameter of studs $d_s = 19$ mm; number of studs per rib $n_r = 1$.

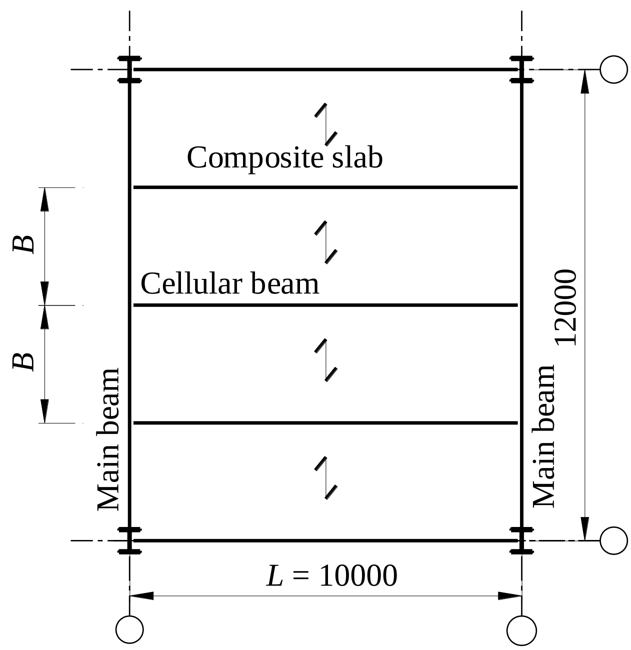

Figure 7. Composite floor system using cellular beams

A parametric study with 12 cases is carried out to find the optimal spacing between CCBs. The parameters included steel grade, beam spacing and slab thickness are presented in Table 3. Using the algorithm that was developed in Section 3 to determine the most economical solution of each case. The results of the parametric study are summarized in Table 4 and Fig. 8.

Table 3. Parameters

No.	Steel grade f_y (N/mm^2)	Beam spacing B (mm)	Number of cellular beams n (pcs)	Slab thickness h_s (mm)
1		2000	7	120
2	S235	2400	6	120
3	(235)	3000	5	150
4		4000	4	200
5		2000	7	120
6	S275	2400	6	120
7	(275)	3000	5	150
8		4000	4	200
9		2000	7	120
10	S355	2400	6	120
11	(355)	3000	5	150
12		4000	4	200

Table 4. Parametric study results

No.	Hot rolled profile	Opening diameter h_o (mm)	Opening spacing s (mm)	Total weight of cellular beams W (kg)
1	UB 406 × 140 × 39	360	470	2527
2	UB 406 × 140 × 46	350	470	2598
3	UB 457 × 152 × 60	350	620	2820
4	UB 533 × 210 × 82	320	620	3024
5	UB 406 × 140 × 39	400	520	2513
6	UB 406 × 140 × 39	330	430	2178
7	UB 457 × 152 × 52	320	580	2450
8	UB 457 × 152 × 67	310	520	2576
9	UB 356 × 127 × 33	360	470	2128
10	UB 356 × 127 × 33	320	430	1842
11	UB 406 × 140 × 39	310	440	1850
12	UB 457 × 152 × 60	360	510	2288

As can be seen in Fig. 8, the total weight of cellular beams made from steel grade S235 is minimum when the beams spacing equals 2000 mm. The weight increases while expanding spacing of beams. In contrast, for both steel grade S275 and S355, the sum of cellular beams weight decreases in the range from B = 2000 mm to B = 2400 mm. The weight rebounds after hitting the lowest point at B = 2400 mm. Overall, the optimal spacing of CCBs ranges from 2000 to 2500 mm.

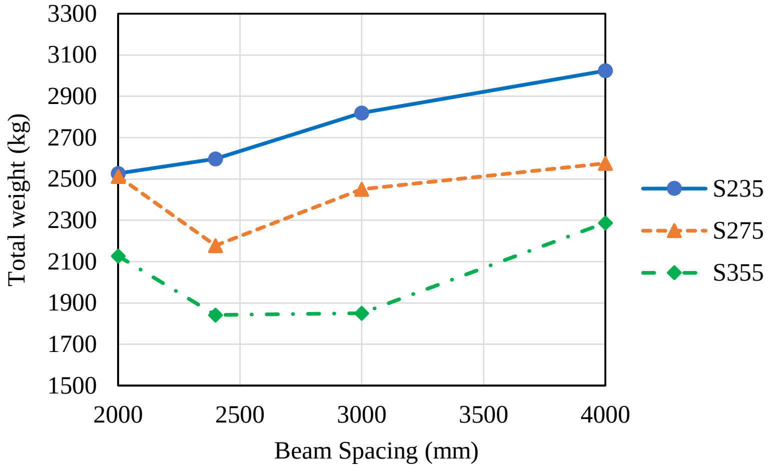

Figure 8. Relation between spacing and weight of cellular beams

5. Conclusion

In this paper, the differential evolution algorithm is utilized to find the optimal solution of composite cellular beams. The objective function is the minimum weight of steel cellular beam while the sequence number of profile, the opening diameter and the spacing of openings are considered as design variables. In practical design, using the differential evolution algorithm permits designers to find the best solution regardless their experience. Furthermore, a parametric study of floor system is carried out and the result indicates that the composite cellular beams should be arranged with spacing from 2000 to 2500 mm.

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