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A FAST FUZZY FINITE ELEMENT APPROACH FOR LATERALLY LOADED PILE IN LAYERED SOILS

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Abstract

A fuzzy finite element approach for static analysis of laterally loaded pile in multi-layer soil with uncertain properties is presented. The finite element (FE) formulation is established using a beam-on-two-parameter foundation model. Based on the developed FE model, uncertainty propagation of the soil parameters to the pile response is evaluated by mean of the α-cut strategy combined with a response surface based optimization technique. First order Taylor's expansion representing the pile responses is used to find the binary combinations of the fuzzy variables that result in extreme responses at an α-level. The exact values of the extreme responses are then determined by direct FE analysis at the found binary combinations of the fuzzy variables. The proposed approach is shown to be accurate and computationally efficient.

Keywords: laterally-loaded pile; uncertainty; fuzzy finite element analysis; α-cut strategy; response surface method; optimization.

1. Introduction

Piles subjected to lateral loadings can be found in many civil engineering structures such as offshore platforms, bridge piers, and high-rise buildings. For the design of pile foundations of such structures, special attention needs to be concentrated not only on the bearing capacity but also on the behavior (horizontal displacement, stress) of the piles under lateral loading conditions. The deterministic analysis of lateral loading behavior of piles is complicated and in general requires a numerical solution procedure (e.g., the finite difference method, finite element method).

On the other hand, uncertainty is often present in the input data, especially in geotechnical engineering data. These uncertainties can be accounted for by using probabilistic methods, e.g., methods proposed in [1–6]. However, very often the input data fall into the category of non-statistical uncertainty. The reason for this uncertainty is that the made observations could be best categorized with linguistic variables (e.g., the soil may be described with linguistic variables such as "very soft," "soft," or "stiff"; "loose", "dense", or "very dense"), or that only a limited number of samples are available

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and a particular soil properties are unknown or vary from one location to another location. These types of uncertainties can be appropriately represented in the mathematical model as fuzziness [7].

In recent years, non-probabilistic FE methods based on fuzzy set theory have been introduced to the analysis of uncertain structural systems. The fuzzy FE methods have been applied for both static and dynamic analysis of various structures [8–11]. In this paper, an efficient fuzzy FE approach is developed to analyze the response of laterally loaded pile in multi-layer soils. It is assumed that only rough estimates of the soil parameters are available and these are modeled as fuzzy values. The analysis of the pile-soil interaction is based on a "Beam-on-two-parameter-linear-elastic-foundation" FE model. The fuzzy pile response is estimated by a response surface based optimization technique using first order Taylor's expansion of the pile response. The accuracy and computational efficiency of the proposed approach are illustrated in a numerical example.

2. General fuzzy structural analysis

2.1. Fuzzy model of uncertainty

Among practical engineering problems, randomness and fuzziness are associated with the model parameters (e.g. material properties, geometrical dimensions, loads). These uncertainties can be modeled in form of fuzzy sets [7]. According to [7] a fuzzy set is defined as $\tilde{X} = (X, \mu_X)$ with X is a set and $\mu_X \to [0, 1]$ is called the membership function. Corresponding to each element $x \in X$, the value $\mu_X(x)$ is called membership level of x; $\mu_X(x)$ defines the level of x belonging to the fuzzy set $\tilde{X}$. The value 0 states that x does not belong to $\tilde{X}$; the value 1 means that x definitely belongs to $\tilde{X}$; the value in interval 0 to 1 shows that the level of x belonging to $\tilde{X}$ is uncertain.

The α-cut, X_α of the fuzzy set $\tilde{X}$ is a set of elements $x \in X$ with the membership level $\mu_X(x) \geq \alpha$:

$$X_\alpha = \{x \in X : \mu_X(x) \geq \alpha\}, \quad \alpha \in [0, 1] \tag{1}$$

Fig. 1 illustrates the membership function and an α-cut of a triangular fuzzy set.

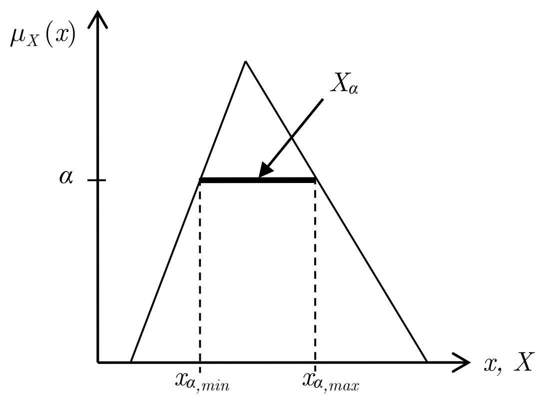


Figure 1. Membership function and the α-cut of a fuzzy set

2.2. The α-level optimization

Consider a model output y given by $y = f(x_1, x_2, \ldots, x_n)$ with x_i being n fuzzy input variables, $x_i \in X_i : \mu_{X_i}(x) \to [0, 1]$. The function $f(\cdot)$ can be any function or numerical model, e.g. the finite element model. Through the mapping function $f(\cdot)$, the output y is also a fuzzy quantity represented by its output fuzzy set $\tilde{Y} = \{y \in Y : \mu_Y(y) \to [0, 1]\}$. A practical mean to determine the membership function of y, $\mu_Y(y)$, is the α-cut strategy [8]. Here, the fuzzy input variables are discretized into m levels, $\alpha_k, (k = 1, 2, \ldots, m)$. Corresponding to each level α_k, we have crisp sets of values of inputs, $X_{i,\alpha_k} \subset X_i$. The output interval of y corresponding to level α_k (the α_k-cut Y_{α_k} of $\tilde{Y}$) is then determined by interval analysis of the input sets X_{i,α_k} through the mapping model $f(\cdot)$. Thus, a discrete approximation of the membership function of the output can be obtained by repeating the interval analysis on a finite number of α_k-levels. Fig. 2 illustrates the fuzzy analysis using the α-cut strategy for a function of two input variables.

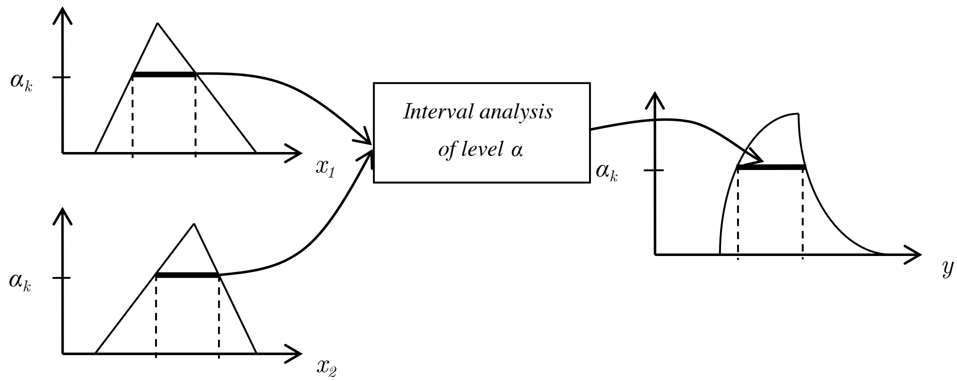


Figure 2. Illustration of fuzzy analysis by α-cut strategy

The smallest and largest values (the extreme values) of the α-cut Y_{α_k} define two points of the membership function of the fuzzy output, $\tilde{Y}$. The exact extreme values of the α-cut Y_{α_k} are often determined by solving two optimization problems, which are referred as the α-level optimization [12]:

$$\begin{aligned} y_{\alpha_k,\min} &= \min_{x_i \in X_{i,\alpha_k}} \left(f(x_1, x_2, \ldots, x_n) \right) \\ y_{\alpha_k,\max} &= \max_{x_i \in X_{i,\alpha_k}} \left(f(x_1, x_2, \ldots, x_n) \right) \end{aligned} \tag{2}$$

The solution for the optimization problems of Eq. (2) can be numerical demanding. In order to reduce the computational burden, researchers have focused on efficient procedures to reduce the number of function evaluations in performing these optimization problems [8, 10, 11].

This paper introduces a fast solution for the above optimization problems based on a response surface method, which is applicable for the fuzzy analysis of laterally loaded piles with uncertain soil parameters. The methodology is presented in the followings.

3. Fuzzy finite element analysis of laterally loaded pile

3.1. Model of analysis

Consider a vertical pile embed in a soil deposit containing n layers, with the thickness of layer i given by H_i (Fig. 3(a)). The top of the pile is on the ground surface and the bottom end of the pile is considered embedded in the n-th layer. Each soil layer is assumed to behave as a linear, elastic material with the compressive resistance parameter k_i and shear resistance parameter t_i. The pile is subjected to a lateral force F_0 and a moment M_0 at the pile top. The pile behaves as an Euler–Bernoulli (EB) beam with length L_p and a constant flexural rigidity EI. The governing differential equation for pile deflection w_i within any layer i is given in [13]:

$$EI\frac{d^4w_i}{dz^4} + k_iw_i - 2t_i\frac{d^2w_i}{dz^2} = 0 \tag{3}$$

Eq. (3) is exactly the same as the equation for the "Beam-on-two-parameter-linear-elastic-foundation" model introduced by Vlasov and Leont'ev [14]. The use of linear elastic analysis in the laterally loaded pile problem, especially in the prediction of deformations at working stress levels, has become a widely accepted model in geotechnical engineering. Also in the real problem where nonlinear stress-strain relationships for the soil must be used, linear elastic solution provides the framework for the analysis, in which the elastic properties of the soil will be changed with the changing deformation of the soil mass (e.g., the "p–y" method [15]).

In this paper, this Beam-on-linear-elastic-foundation model is the basis for the finite element formulation of the laterally loaded pile problem which will be presented in the next section.

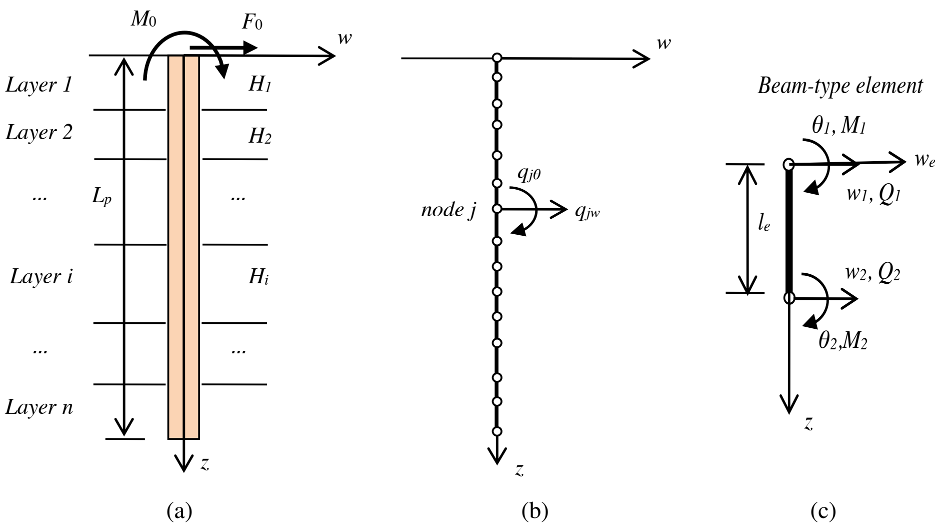


Figure 3. (a) A laterally-loaded pile in a layered soil; (b) FE discretization; (c) Beam-type element

3.2. Finite element modeling

The pile is divided into m finite elements and to each j-th node of their interconnection, two degrees of freedom are allowed: q_{jw} - the deflection and $q_{j\theta}$ - the rotation of cross section with positive direction as in Fig. 3(b). Element of EB-beam type is chosen for each pile element with length l_e and two nodes, one at each end. The element is connected to other elements only at the nodes. To each element, two degrees of freedom are allowed at both ends: deflection, w_1 and rotation, θ_1, and w_2, θ_2 respectively, positive in the system of local axes as shown in Fig. 3(c). The element nodal displacement vector $\{q\}_e$ and the element nodal force vector $\{r\}_e$ with respect to the system of local axes are defined:

$$\{q\}_e = \{w_1\ \theta_1\ w_2\ \theta_2\}^T\,, \quad \{r\}_e = \{Q_1\ M_1\ Q_2\ M_2\}^T \tag{4}$$

It is noted that Q_1 and Q_2 from (4) include shear force in the pile section and also shear force in the soil.

The equilibrium equation of an element has the form:

$$[k]_e \{q\}_e = \{r\}_e \tag{5}$$

In Eq. (5) $[k]_e = [k]_b + [k]_w + [k]_t$ represents the stiffness matrix of one-dimension finite element of pile on two-parameter elastic foundations. The terms of $[k]_b$, $[k]_w$, $[k]_t$ matrices have been established in [16] as:

$$[k]_b = \frac{EI}{l_e^3}\begin{bmatrix} 12 & -6l_e & -12 & -6l_e \\ -6l_e & 4l_e^2 & 6l_e & 2l_e^2 \\ -12 & 6l_e & 12 & 6l_e \\ -6l_e & 2l_e^2 & 6l_e & 4l_e^2 \end{bmatrix} \tag{6}$$

$$[k]_w = \frac{kl_e}{420}\begin{bmatrix} 156 & -22l_e & 54 & 13l_e \\ -22l_e & 4l_e^2 & -13l_e & -3l_e^2 \\ 54 & -13l_e & 156 & 22l_e \\ 13l_e & -3l_e^2 & -3l_e^2 & 4l_e^2 \end{bmatrix} \tag{7}$$

$$[k]_t = \frac{2t}{30l_e}\begin{bmatrix} 36 & -3l_e & -36 & -3l_e \\ -3l_e & 4l_e^2 & 3l_e & -l_e^2 \\ -36 & 3l_e & 36 & 3l_e \\ -3l_e & -l_e^2 & 3l_e & 4l_e^2 \end{bmatrix} \tag{8}$$

The system equation is obtained by assembly of all elements, implementation of boundary conditions, and introduction of loads.

3.3. Proposed fuzzy analysis

Assume that a pile response y is monotonic with respects to the fuzzy soil parameters a_i, $i = 1, 2, \ldots, n$, (here a_i can be compressive parameters or shear parameters). A first order Taylor's expansion of y at the soil parameter value $(a_1^0, a_2^0, \ldots, a_n^0)$ given by

$$y(a_1, a_2, .., a_n) \simeq y(a_1^0, a_2^0, \ldots, a_n^0) + \sum_{i=1}^{n} \dot{y}_i^0 (a_i - a_i^0) \tag{9}$$

where $\dot{y}_i^0$ is the partial derivative of y with respect to the parameter a_i, taken at $(a_1^0, a_2^0, \ldots, a_n^0)$.

The extreme values of y at an α-level can be determined then as

$$\begin{aligned} y_{\min} &= y(a_1^0, a_2^0, \ldots, a_n^0) + \sum_{i=1}^{n} \min\left\{\dot{y}_i^0 (a_i - a_i^0)\right\} \\ y_{\max} &= y(a_1^0, a_2^0, \ldots, a_n^0) + \sum_{i=1}^{n} \max\left\{\dot{y}_i^0 (a_i - a_i^0)\right\} \end{aligned} \tag{10}$$

or for monotonic function,

$$\begin{aligned} y_{\min} &= y(a_1^0, a_2^0, \ldots, a_n^0) + \sum_{i=1}^{n} \min\left\{\dot{y}_i^0 (a_{i,\min} - a_i^0), \dot{y}_i^0 (a_{i,\max} - a_i^0)\right\} \\ y_{\max} &= y(a_1^0, a_2^0, \ldots, a_n^0) + \sum_{i=1}^{n} \max\left\{\dot{y}_i^0 (a_{i,\min} - a_i^0), \dot{y}_i^0 (a_{i,\max} - a_i^0)\right\} \end{aligned} \tag{11}$$

where $a_{i,\min}$ and $a_{i,\max}$ are the lower and upper bound of a_i, respectively, corresponding to that α-level. Since Eq. (9) is only an approximation of the actual response, the extreme values obtained by (11) do not represent the real bounds of the response. To calculate the exact bounds of y, we directly evaluate y using FE analysis at the binary combinations of the fuzzy parameter values that result in the extreme responses of (11).

Furthermore, the partial derivative $\dot{y}_i^0$ is approximated as:

$$\dot{y}_i^0 \simeq \frac{y(a_1^0, a_2^0, \ldots, a_i^0 + \delta a_i, \ldots, a_n^0) - y(a_1^0, a_2^0, \ldots, a_i^0 - \delta a_i, \ldots, a_n^0)}{2\delta a_i} \tag{12}$$

where δa_i is a small variation of a_i, taken as $0.001 a_i^0$ in this study. The determination of $\dot{y}_i^0$ is carried out once for each a_i, with $(a_1^0, a_2^0, \ldots, a_n^0)$ to be the value of the fuzzy variable a_i having the membership of 1. Thus, the proposed approach requires $2(n+m)+1$ model analysis to approximate the fuzzy membership function of a pile response, where m is the number of discretized membership levels.

The flowchart of the proposed fuzzy analysis is presented in Fig. 4.

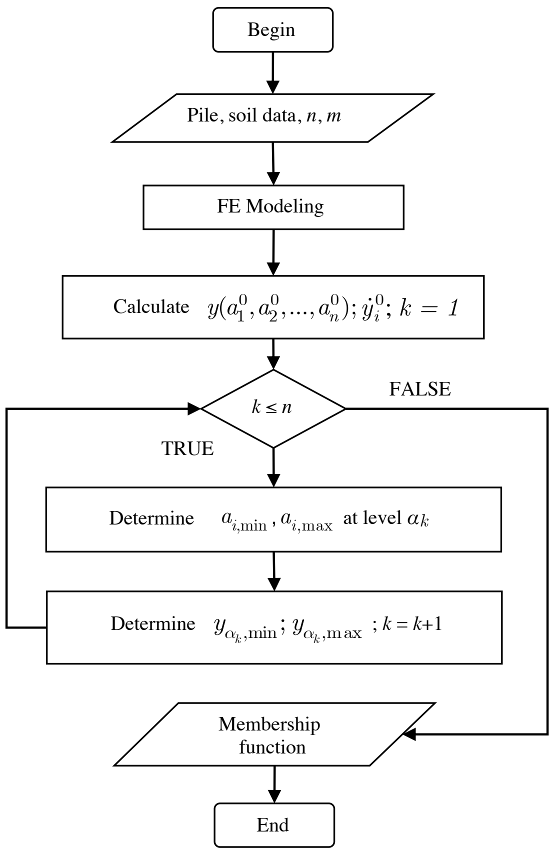
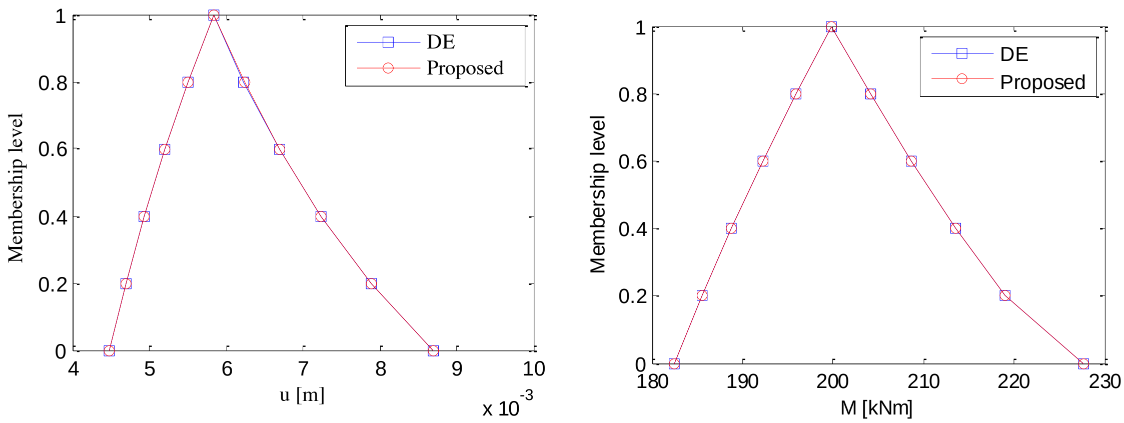


Figure 4. Flowchart of the proposed fuzzy analysis for pile

4. Application

To verify the above approach, a laterally-loaded pile taken from [17] is analyzed. The pile of length L_p = 20 m, cross-section radius r_p = 0.3 m and modulus E_p = 25 × 106 kN/m^2 is subjected to a lateral force $F_0 = 300$ kN and a moment $M_0 = 100$ kNm at the pile head. The soil deposit has four layers with $H_1 = H_2 = H_3 = 5$ m, and $H_4 = \infty$. The soil properties are uncertain and given by triangular fuzzy numbers: $k_1 = (33.6, 56.0, 78.4)$ MPa, $k_2 = (84.0, 140.0, 196.0)$ MPa, $k_3 = (93.0, 155.0, 217.0)$ MPa and $k_4 = (120.0, 200.0, 280.0)$ MPa, and $t_1 = (6.6, 11.0, 15.4)$ MN, $t_2 = (16.8, 28.0, 39.2)$ MN, $t_3 = (24.0, 40.0, 56.0)$ MN and $t_4 = (36.0, 60.0, 84.0)$ MN. Each fuzzy parameter has the relative variation at different levels of membership with respect to the main value at the membership of 1 not exceed 40%.

A finite-element model of forty elements with equal length 0.5 m is used for the analysis. Using five membership levels, the estimated membership functions of the top deflection and the maximum bending moment in the pile are shown in Fig. 5(a) and Fig. 5(b), respectively. The corresponding membership functions obtained by direct optimization using differential evolution (DE) [18] are also plotted in Fig. 5 for comparison. Moreover, the values of these membership functions at each membership level are listed in Table 1.



(a) (b)

Figure 5. Membership function: (a) Top displacement; (b) Maximum bending moment

Table 1. Results of the fuzzy analysis for the pile

$\mu_Y(y)$	Top displacement (min;max) [m]		Max. bending moment (min;max) [kNm]	
	DE	Proposed	DE	Proposed
1.0	0.0058	0.0058	199.8863	199.8863
0.8	(0.0055; 0.0062)	(0.0055; 0.0062)	(195.9505; 204.1065)	(195.9505; 204.1065)
0.6	(0.0052; 0.0067)	(0.0052; 0.0067)	(192.2638; 208.6543)	(192.2638; 208.6544)
0.4	(0.0049; 0.0072)	(0.0049; 0.0072)	(188.7972; 213.5837)	(188.7972; 213.5838)
0.2	(0.0047; 0.0079)	(0.0047; 0.0079)	(185.5262; 218.9637)	(185.5261; 218.9637)
0.0	(0.0045; 0.0087)	(0.0045; 0.0087)	(182.4303; 227.6920)	(182.4300; 227.6922)

It is seen that the results obtained by the proposed approach and those provided by direct optimization are almost identical. In this example, the membership functions of the pile responses are approximated with five membership levels. To obtain sufficient good results DE requires more than

1000 FE analyses, while the proposed approach needs only $2(8 + 5) + 1 = 27$ FE analyses to produce exact results. This clearly demonstrates the computational efficiency of the proposed approach.

5. Conclusion

This paper presents a fuzzy finite element analysis approach for the laterally-loaded pile in multi-layered soils. The pile is idealized as a one-dimensional beam and the soil as two-parameter elastic foundation model. A fast α-level optimization procedure is developed using a response surface methodology based on the first order Taylor's expansion of the pile response. The procedure is validated by an example of a pile in 4-layer soil with fuzziness in soil parameters. Numerical results show that the obtained fuzzy pile responses agree well with those obtained by direct optimization. The advantage of the approach is that it does not require a large number of finite-element analyses as often found in direct optimization strategy.

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References

[1] Fan, H. and Liang, R. (2012). Application of Monte Carlo simulation to laterally loaded piles. In *Geo-Congress 2012: State of the Art and Practice in Geotechnical Engineering*, 376–384.

[2] Fan, H. and Liang, R. (2013). Performance-based reliability analysis of laterally loaded drilled shafts. *Journal of Geotechnical and Geoenvironmental Engineering*, 139(12):2020–2027.

[3] Fan, H. and Liang, R. (2013). Reliability-based design of laterally loaded piles considering soil spatial variability. In *Foundation Engineering in the Face of Uncertainty: Honoring Fred H. Kulhawy*, 475–486.

[4] Chan, C. L. and Low, B. K. (2009). Reliability analysis of laterally loaded piles involving nonlinear soil and pile behavior. *Journal of Geotechnical and Geoenvironmental Engineering*, 135(3):431–443.

[5] Tandjiria, V., Teh, C. I., and Low, B. K. (2000). Reliability analysis of laterally loaded piles using response surface methods. *Structural Safety*, 22(4):335–355.

[6] Chan, C. L. and Low, B. K. (2012). Probabilistic analysis of laterally loaded piles using response surface and neural network approaches. *Computers and Geotechnics*, 43:101–110.

[7] Zadeh, L. A. (1965). Fuzzy set. *Information Control*, 8(1):338–353.

[8] Moens, D. and Hanss, M. (2011). Non-probabilistic finite element analysis for parametric uncertainty treatment in applied mechanics: Recent advances. *Finite Elements in Analysis and Design*, 47(1):4–16.

[9] Huynh, L. X. and Duy, L. C. (2013). An algorithm for solving basic equations of the finite element method with membership functions of fuzzy input parameters. *Journal of Science and Technology in Civil Engineering*, 6(3):45–53.

[10] Tuan, N. H., Huynh, L. X., and Anh, P. H. (2015). A fuzzy finite element algorithm based on response surface method for free vibration analysis of structure. *Vietnam Journal of Mechanics*, 37(1):17–27.

[11] Tuan, N. H., Huynh, L. X., and Anh, P. H. (2014). Fuzzy structural analysis subjected to harmonic forces using the improvement response surface method and assessing the safety level. *Journal of Science and Technology in Civil Engineering (STCE)-NUCE*, 8(3):12–21.

[12] Möller, B., Graf, W., and Beer, M. (2000). Fuzzy structural analysis using α-level optimization. *Computational Mechanics*, 26(6):547–565.

[13] Basu, D. and Salgado, R. (2007). Elastic analysis of laterally loaded pile in multi-layered soil. *Geomechanics and Geoengineering: An International Journal*, 2(3):183–196.

[14] Vlasov, V. Z. and Leont'ev, N. N. (1966). *Beams, plates and shells on elastic foundations*. Israel Program for Scientific Translations, Jerusalem.

[15] Reese, L. C. and Van Impe, W. F. (2010). *Single piles and pile groups under lateral loading*. A.A. Balkema: Rotterdam, Netherlands.

[16] Anh, P. H. (2014). Fuzzy analysis of laterally-loaded pile in layered soil. *Vietnam Journal of Mechanics*, 36(3):173–183.

[17] Basu, D., Salgado, R., and Prezzi, M. (2008). *Analysis of laterally loaded piles in multilayered soil deposits*. Joint Transportation Research Program, Department of Transportation and Purdue University, West Lafayette, Indiana.

[18] Anh, P. H., Thanh, N. X., and Hung, N. V. (2014). Fuzzy structural analysis using improved differential evolutionary optimization. In *Proceedings of the International Conference on Engineering Mechanics and Automation (ICEMA 3)*, Hanoi, 492–498.